Stereo

Many slides adapted from Steve Seitz

Binocular stereo

- Given a calibrated binocular stereo pair, fuse it to produce a depth image

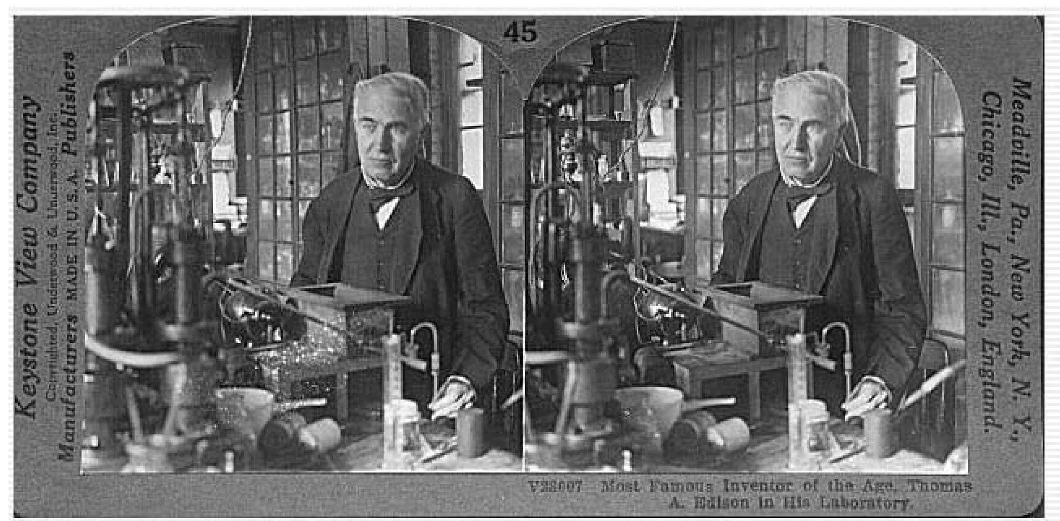
image 1

image 2

Dense depth map

Binocular stereo

- Given a calibrated binocular stereo pair, fuse it to produce a depth image
  - Humans can do it



Stereograms: Invented by Sir Charles Wheatstone, 1838

Binocular stereo

- Given a calibrated binocular stereo pair, fuse it to produce a depth image
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Autostereograms: www.magiceye.com

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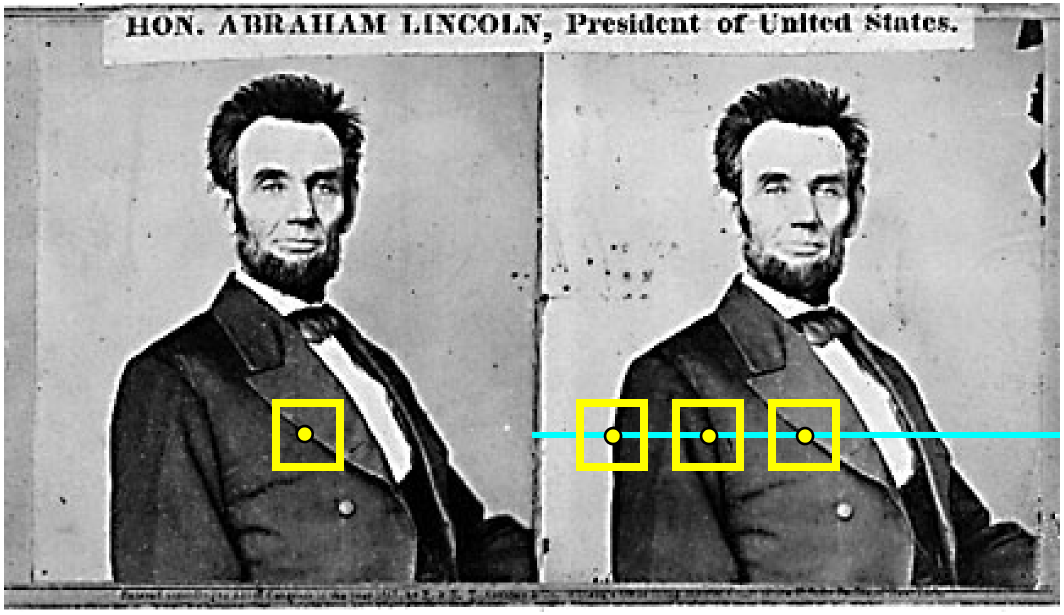
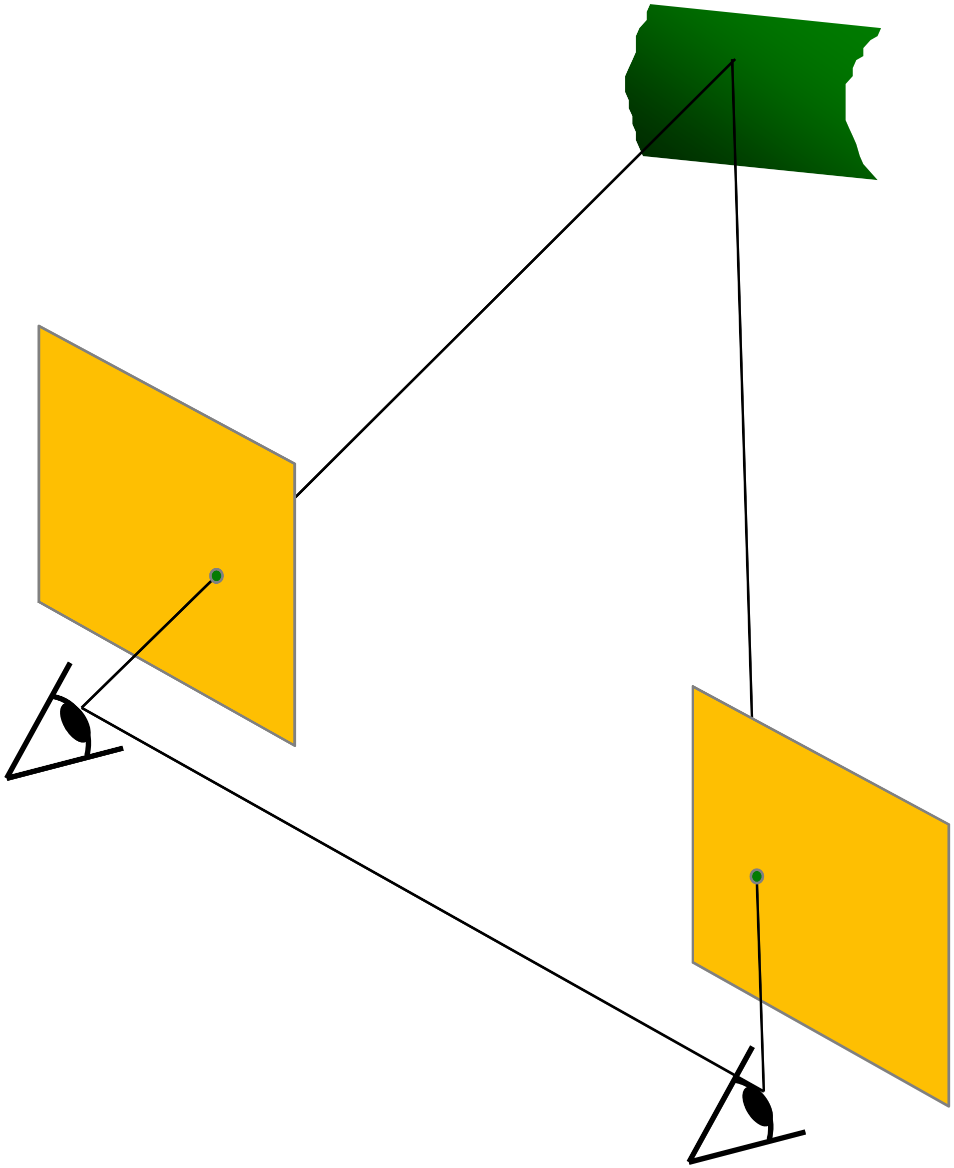
Basic stereo matching algorithm



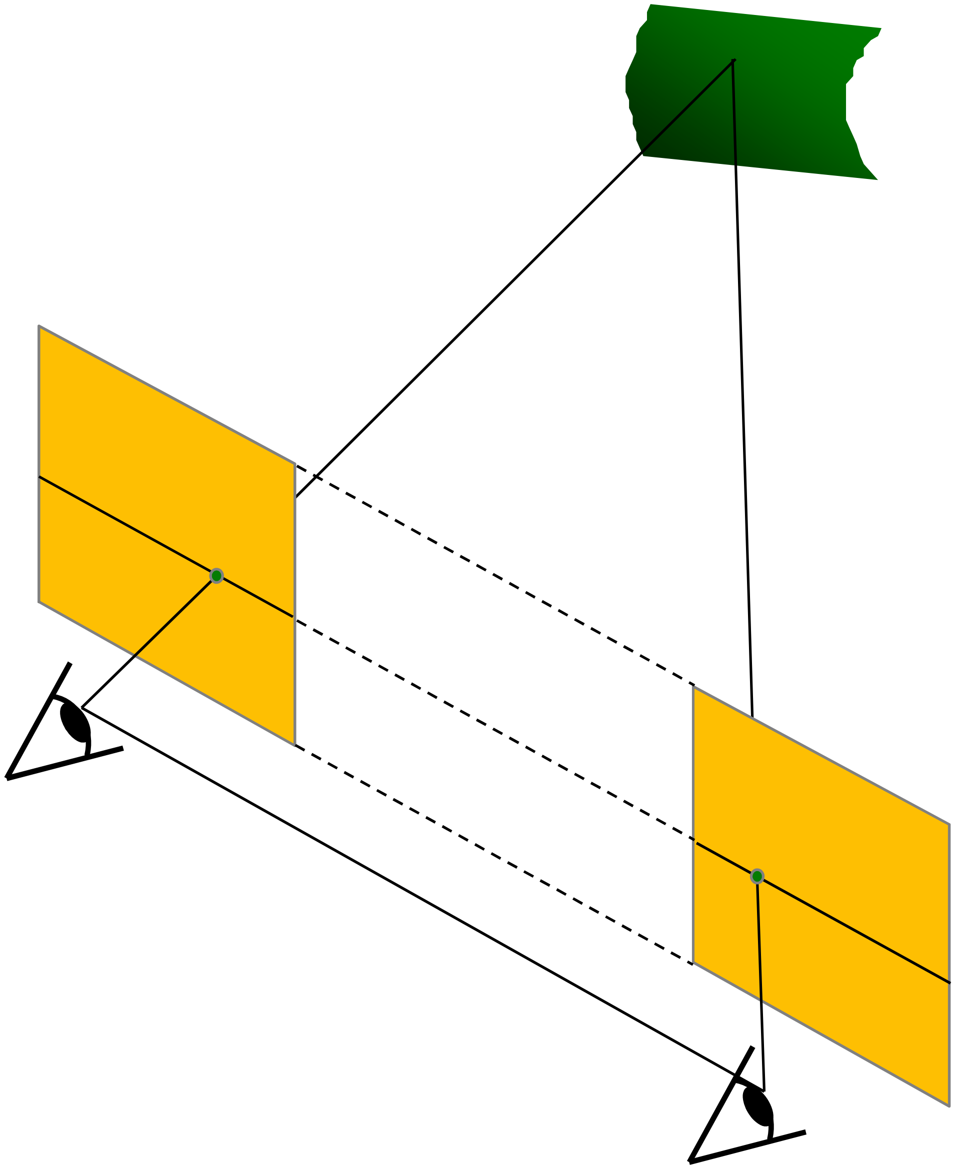
- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are scanlines
  - When does this happen?

Simplest Case: Parallel images



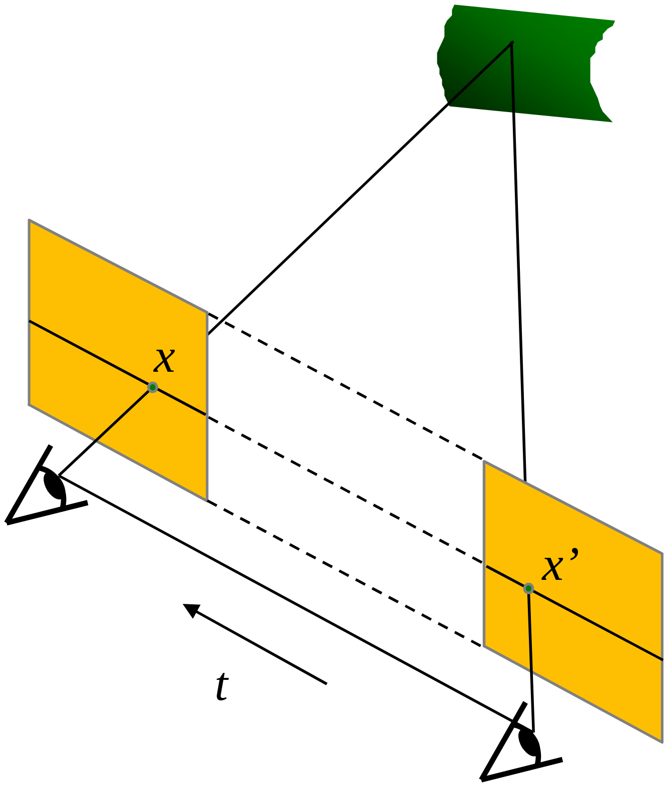
- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same

Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same
- Then, epipolar lines fall along the horizontal scan lines of the images

Essential matrix for parallel images



Epipolar constraint:

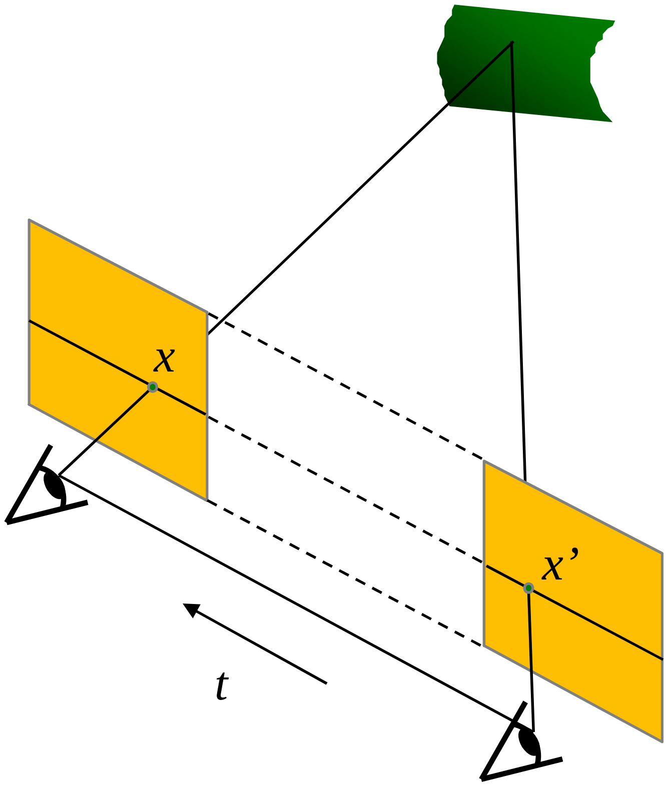
$$x^T E x' = 0, \quad E = [t_\times]R$$

$$R = I \qquad t = (T, 0, 0)$$

$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$[a_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Essential matrix for parallel images



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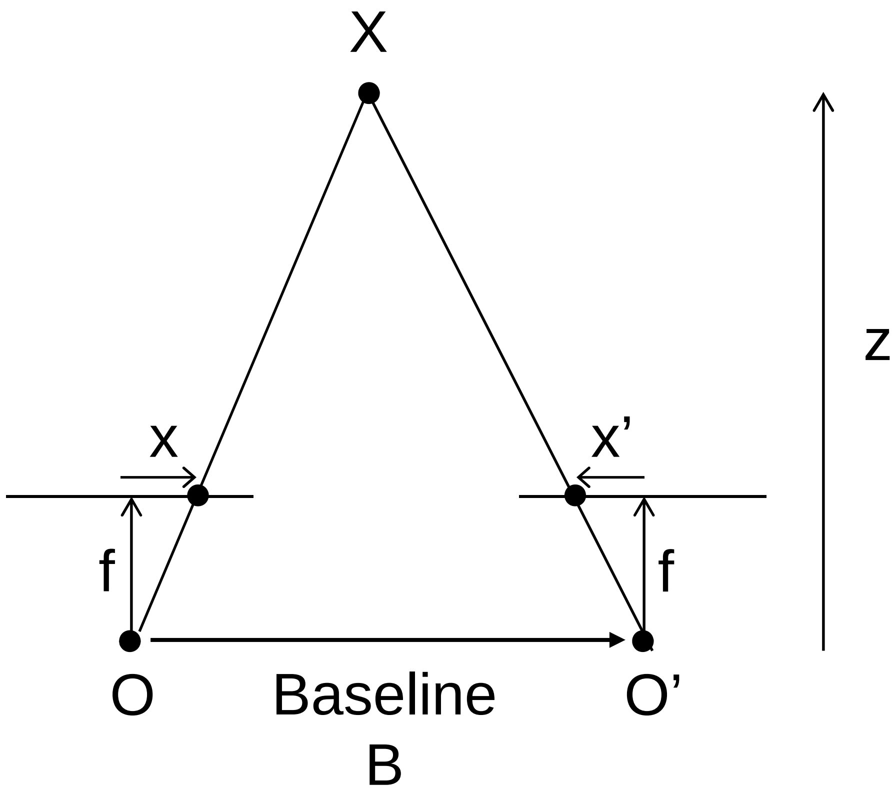
$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

The y-coordinates of corresponding points are the same!

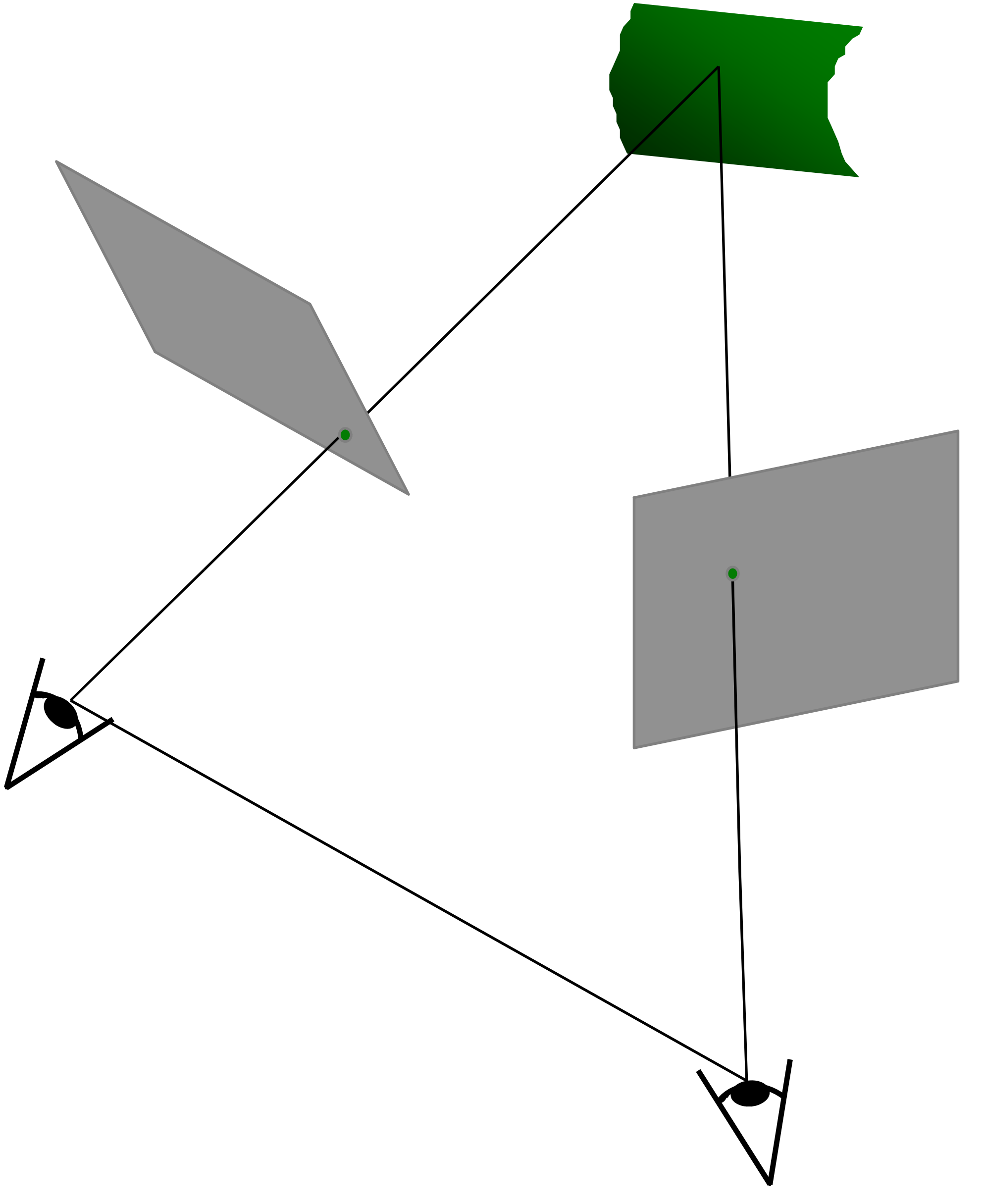
Depth from disparity



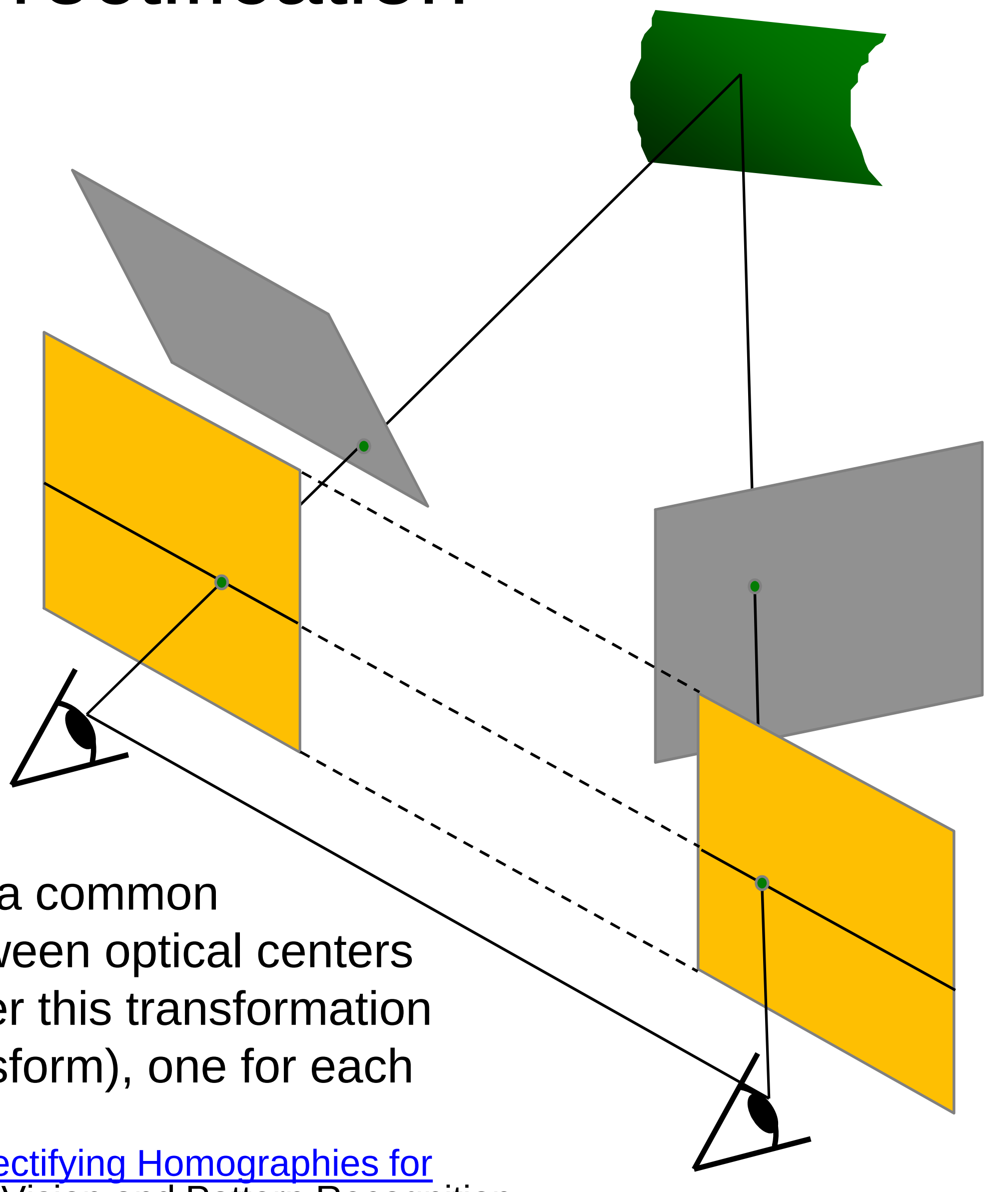
$$disparity = x - x' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth!

Stereo image rectification



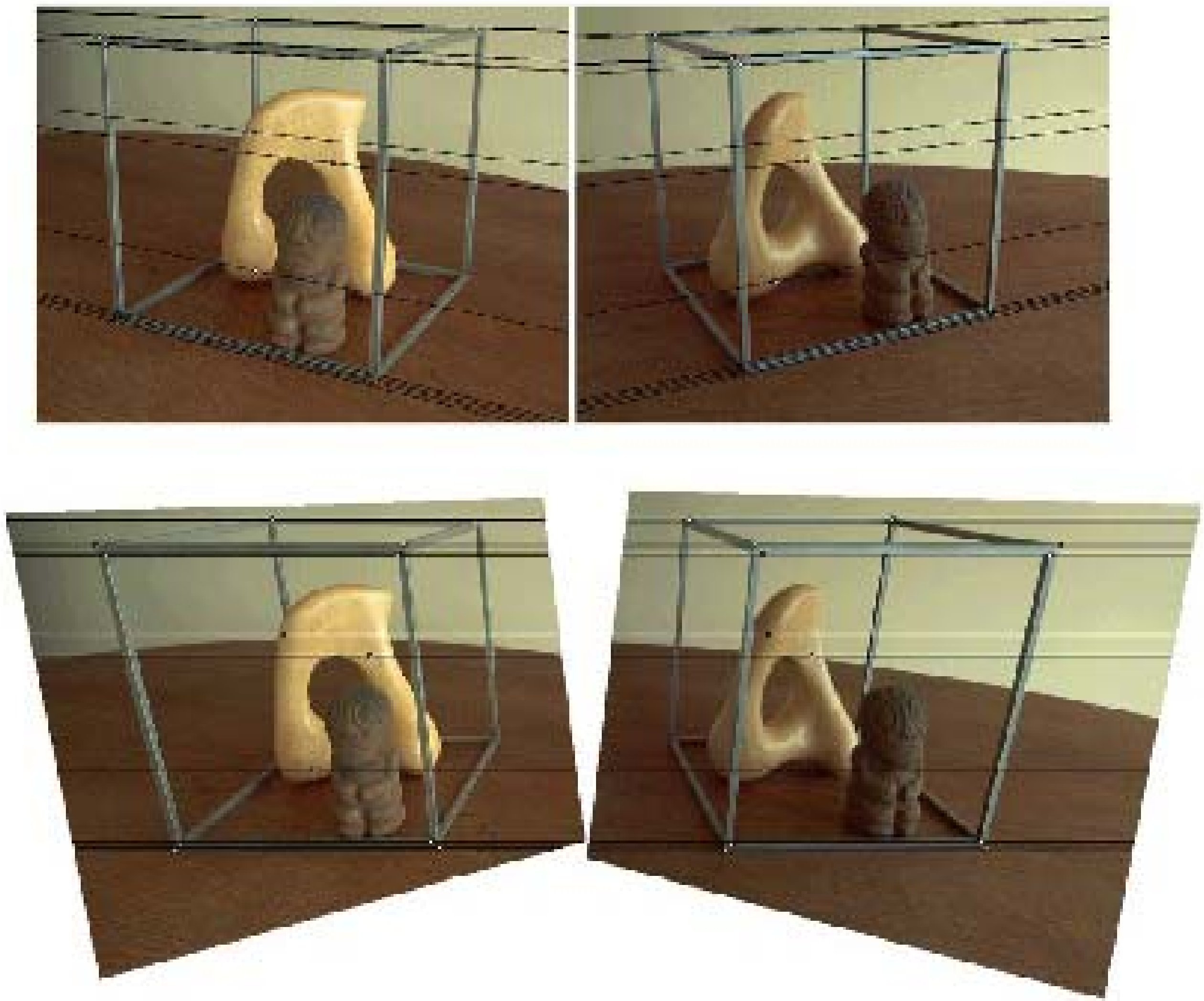
Stereo image rectification



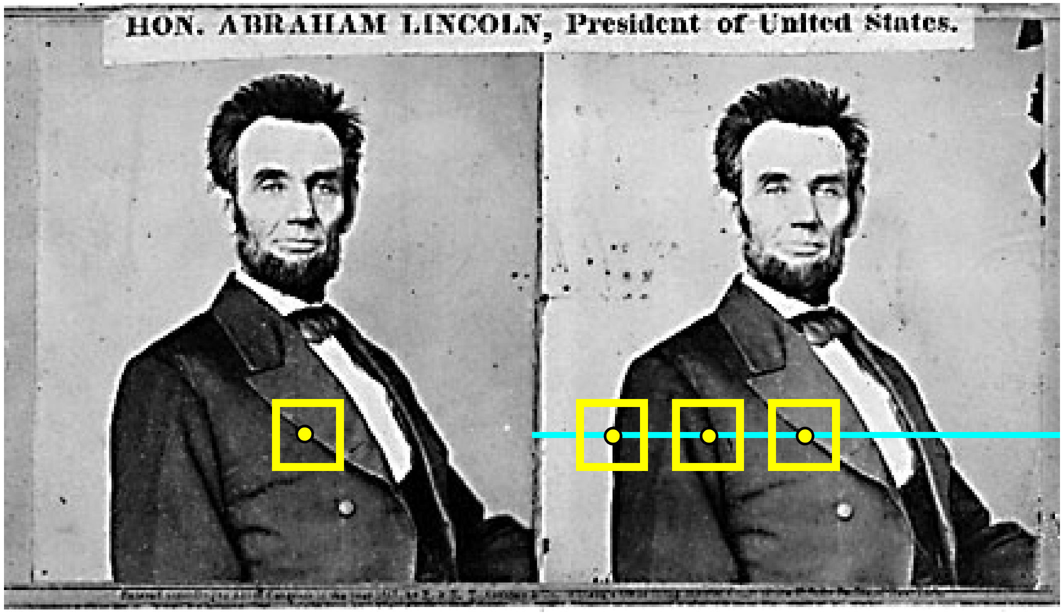
- reproject image planes onto a common plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- two homographies (3x3 transform), one for each input image reprojection

➢ C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision]. IEEE Conf. Computer Vision and Pattern Recognition, 1999.

Rectification example

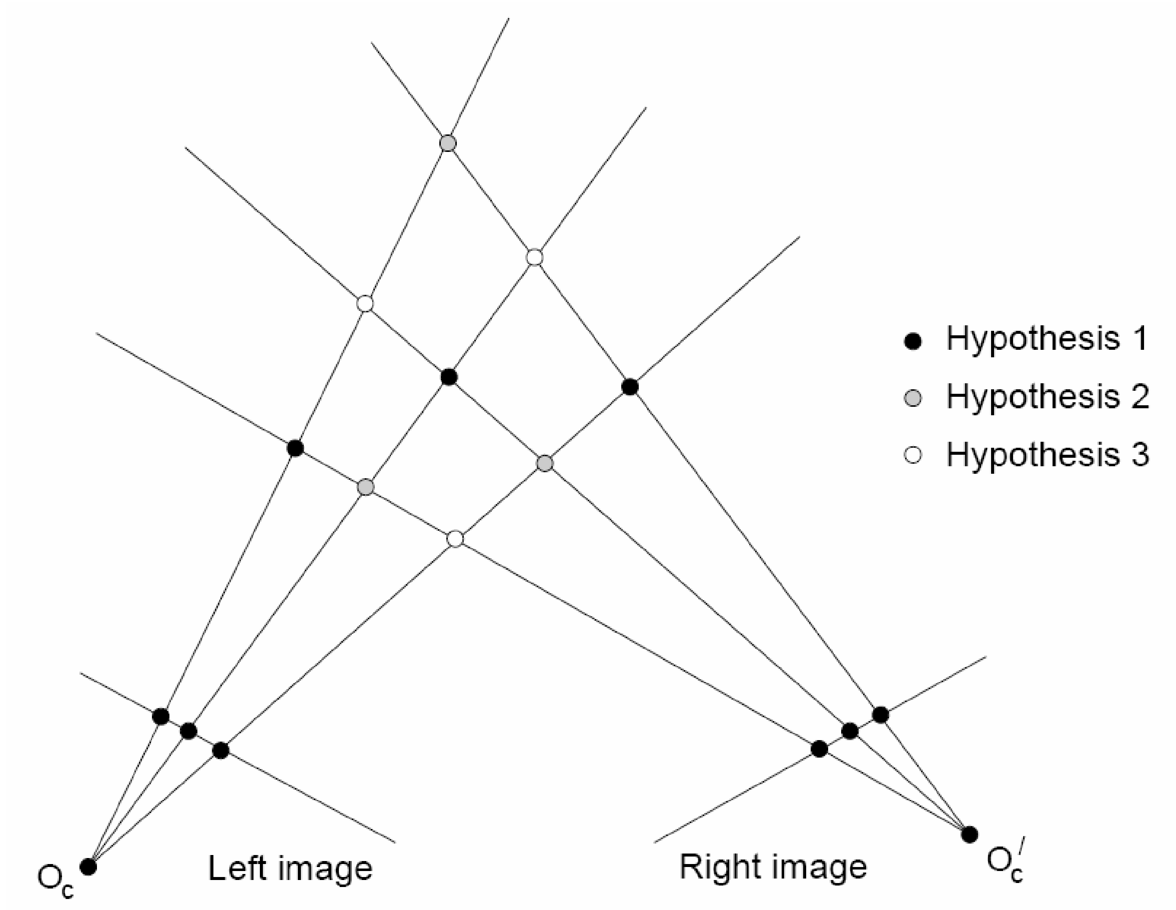


Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match x'
  - Compute disparity x-x' and set depth(x) = 1/(x-x')

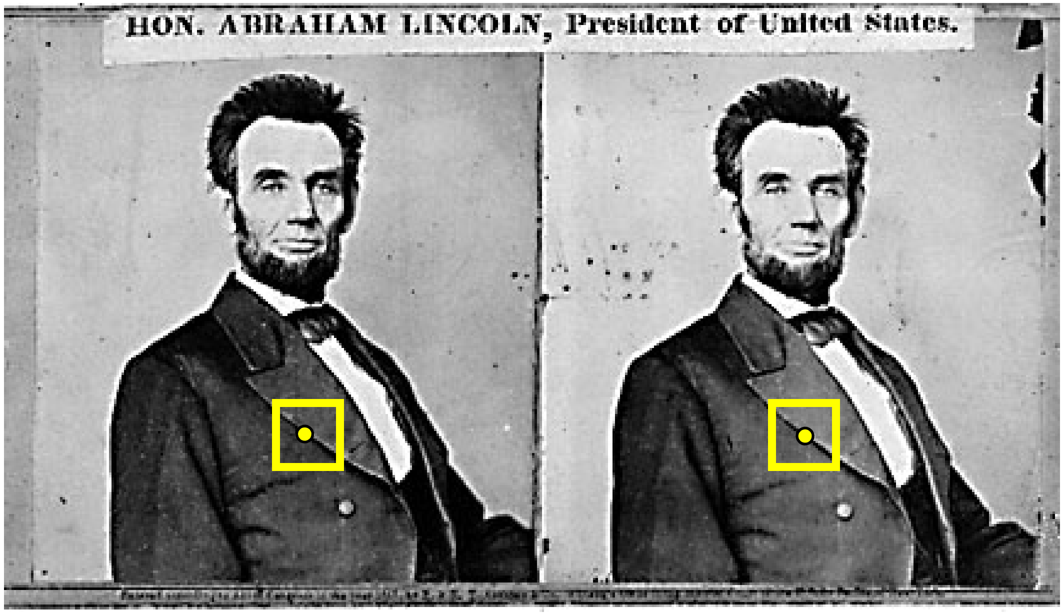
Correspondence problem



Multiple matching hypotheses satisfy the epipolar constraint, but which one is correct?

Correspondence problem

- Let's make some assumptions to simplify the matching problem
  - The baseline is relatively small (compared to the depth of scene points)
  - Then most scene points are visible in both views
  - Also, matching regions are similar in appearance



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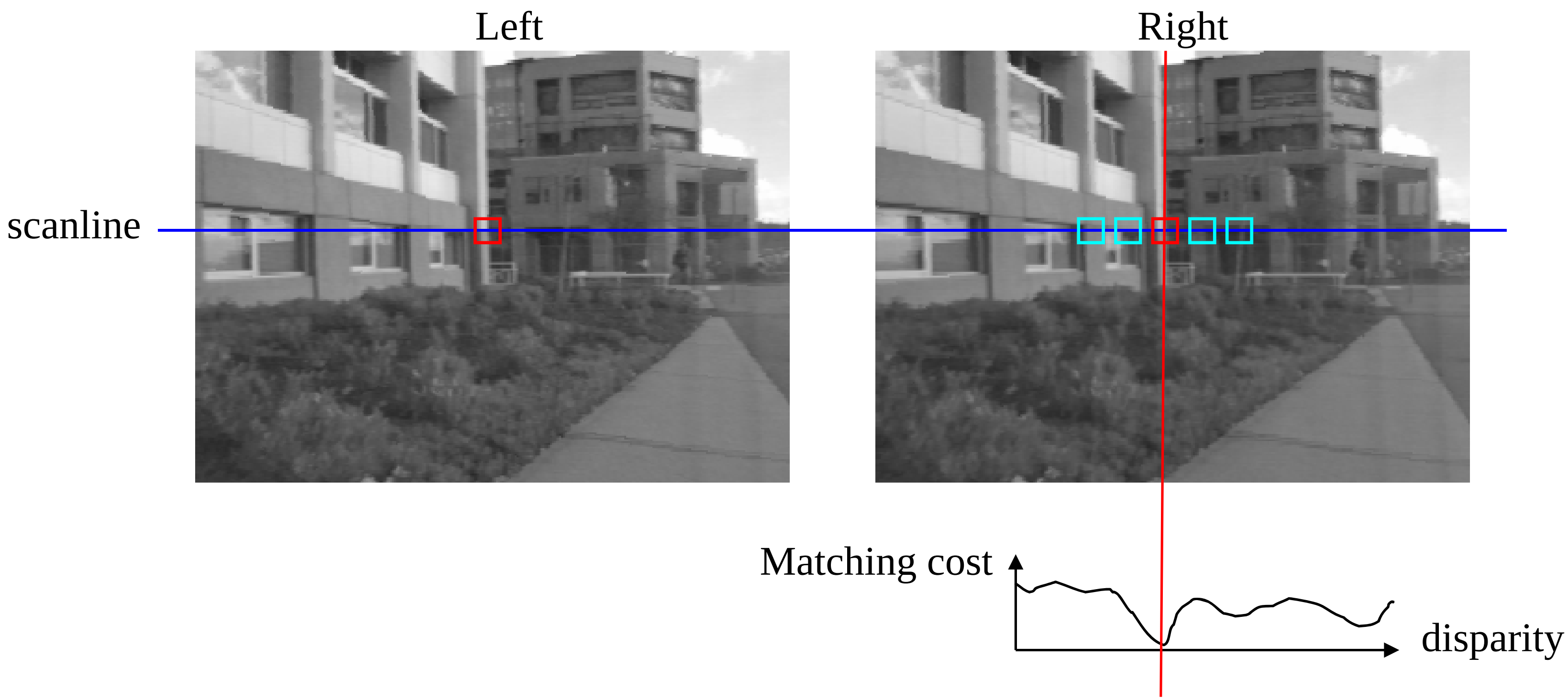
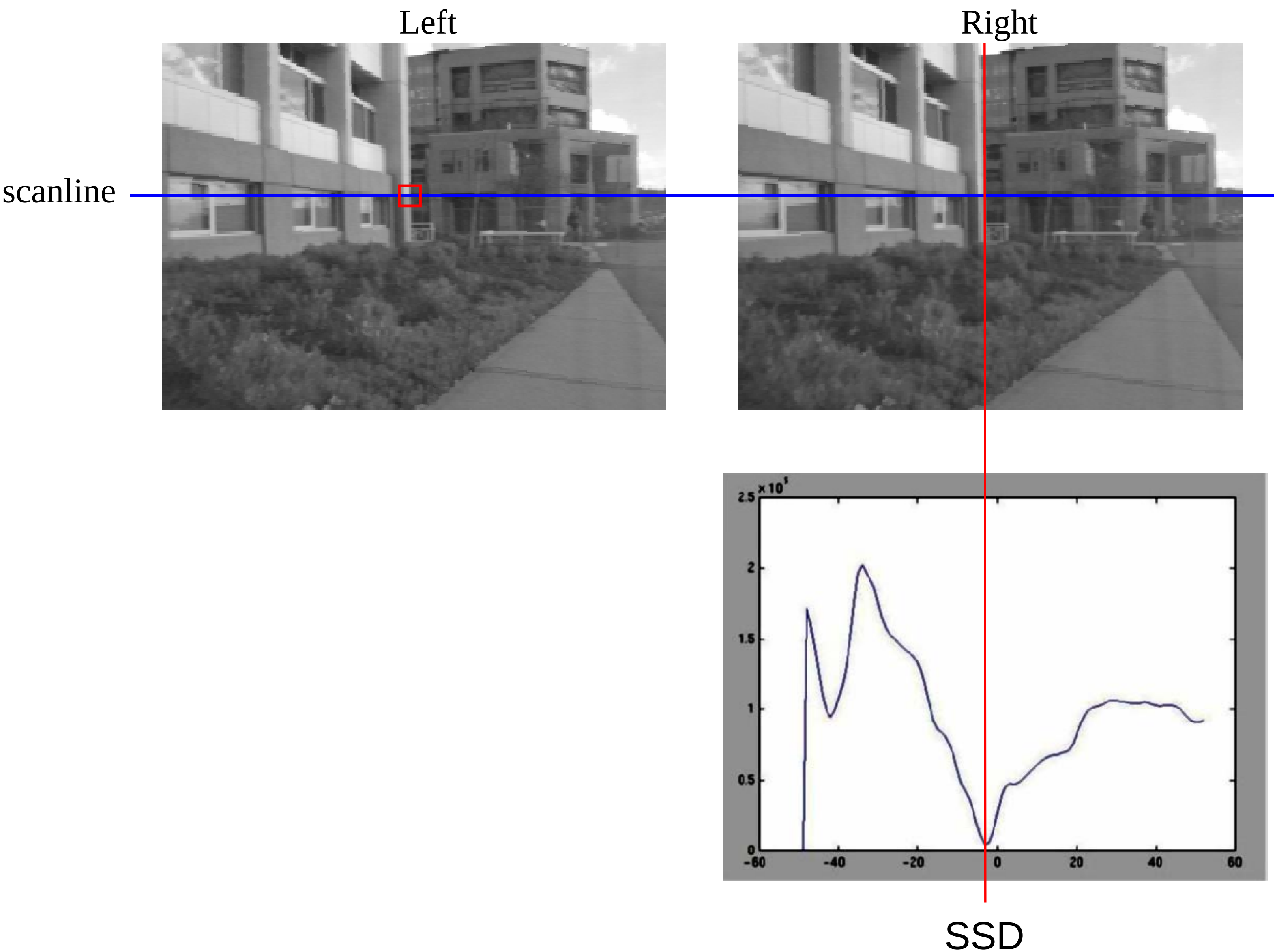
Correspondence search with similarity constraint

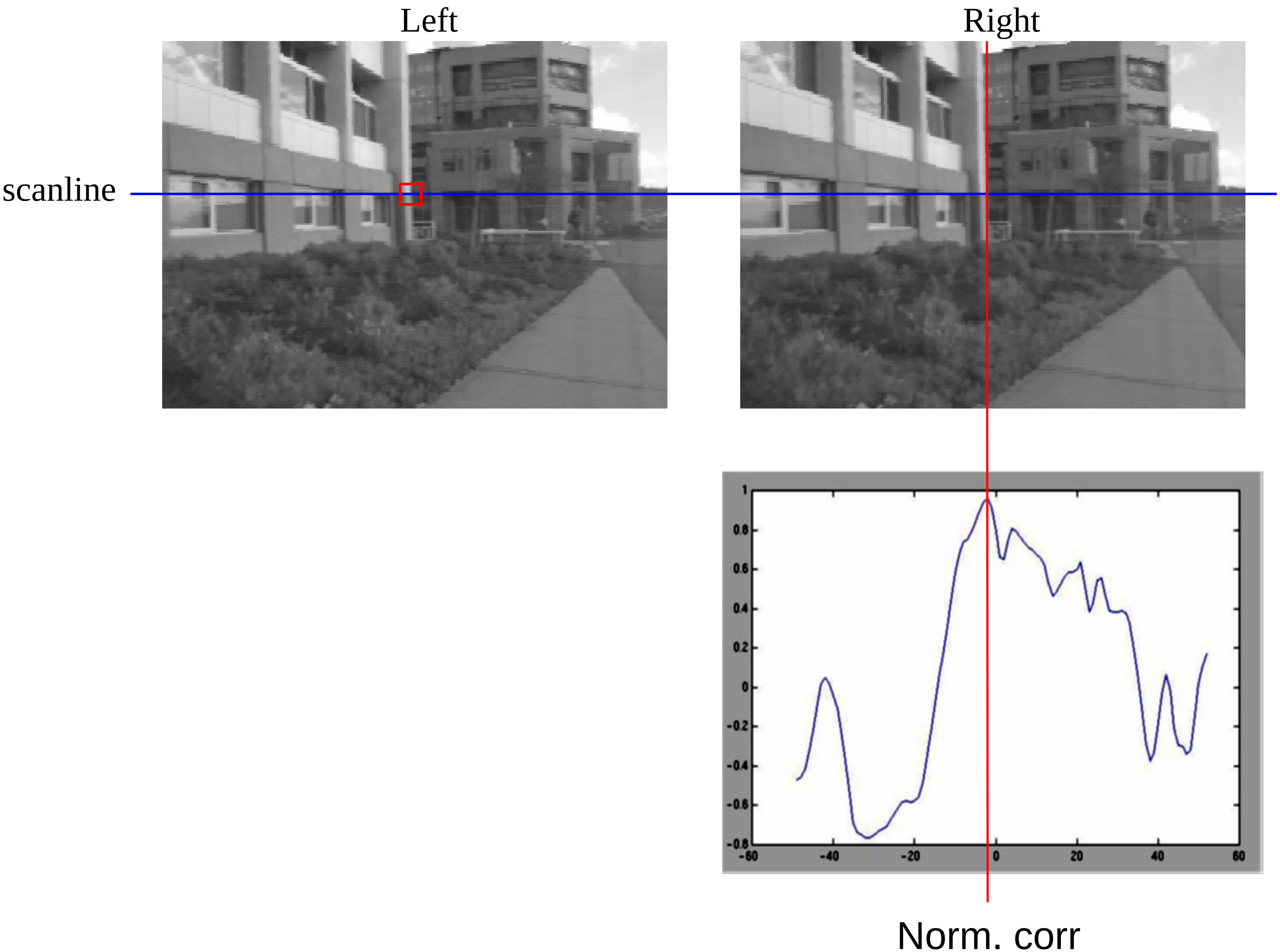


- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

Correspondence search with similarity constraint

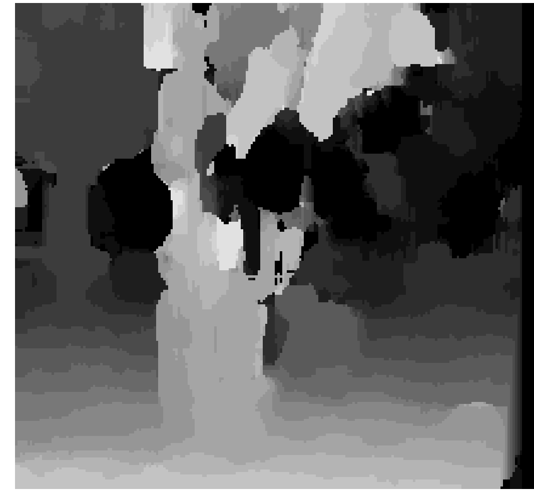


Correspondence search with similarity constraint



Effect of window size

W = 3

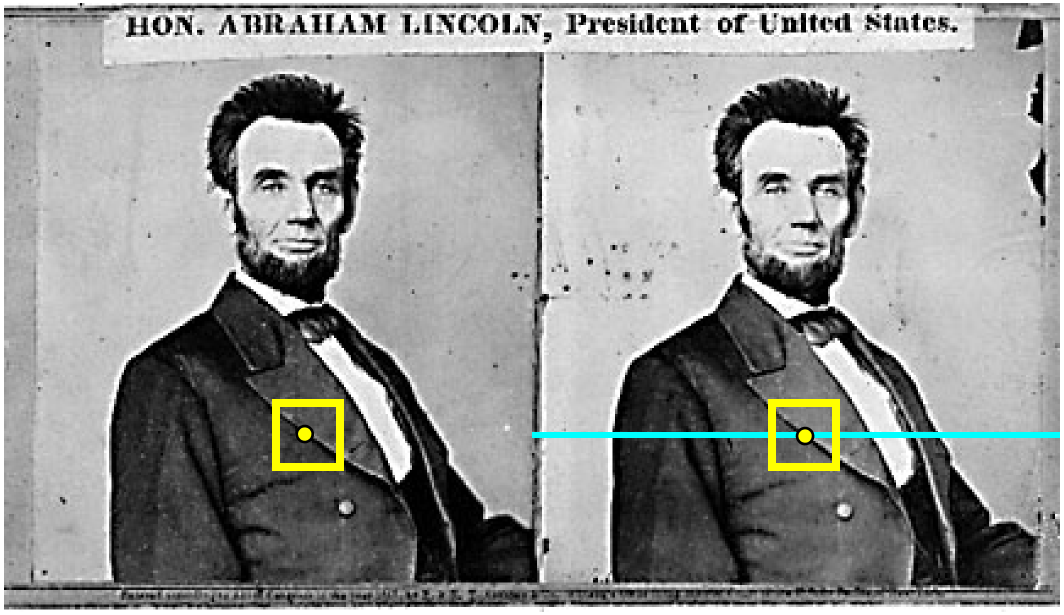


W = 20

- Smaller window
  - \+ More detail
  - – More noise

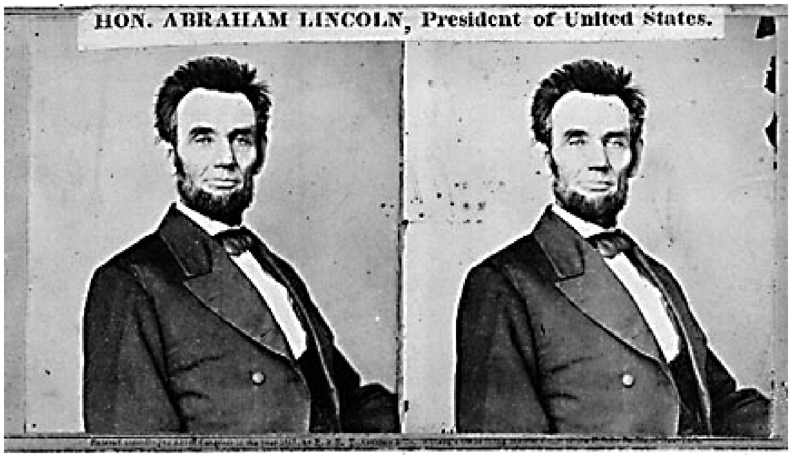
- Larger window
  - \+ Smoother disparity maps
  - – Less detail

The similarity constraint



- Corresponding regions in two images should be similar in appearance
- …and non-corresponding regions should be different
- When will the similarity constraint fail?

Limitations of similarity constraint



Textureless surfaces

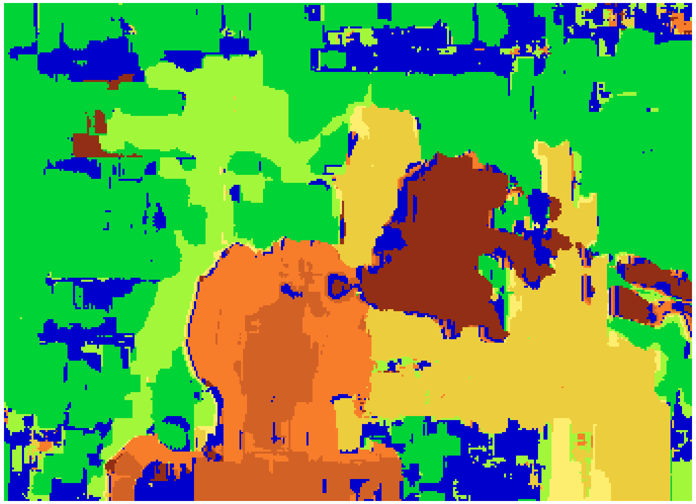
Occlusions, repetition

Non-Lambertian surfaces, specularities

Results with window search

Data

Window-based matching



Ground truth

Better methods exist...

Graph cuts

Ground truth

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

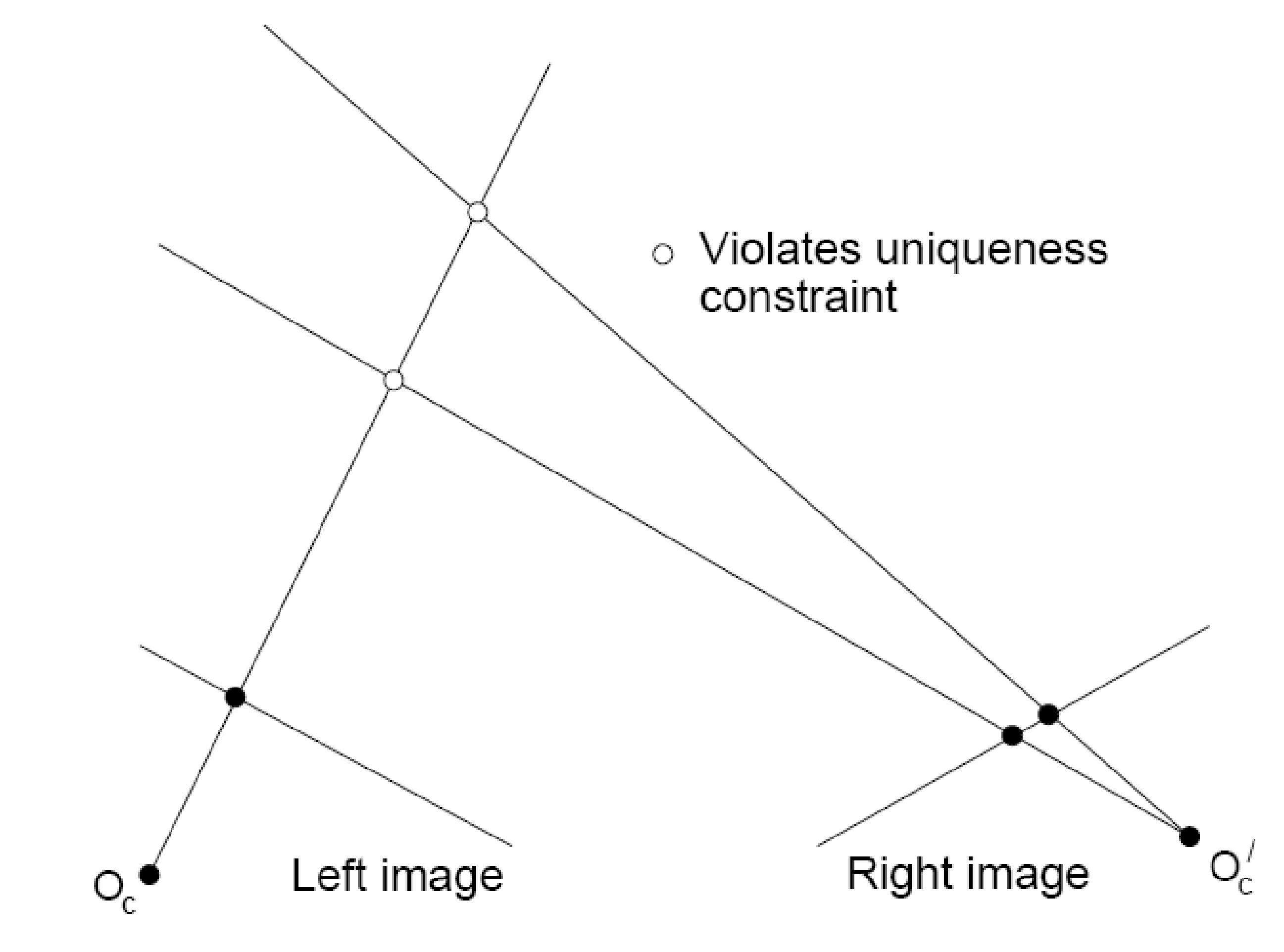
For the latest and greatest: http://www.middlebury.edu/stereo/

How can we improve window-based matching?

- The similarity constraint is **local** (each reference window is matched independently)
- Need to enforce **non-local** correspondence constraints

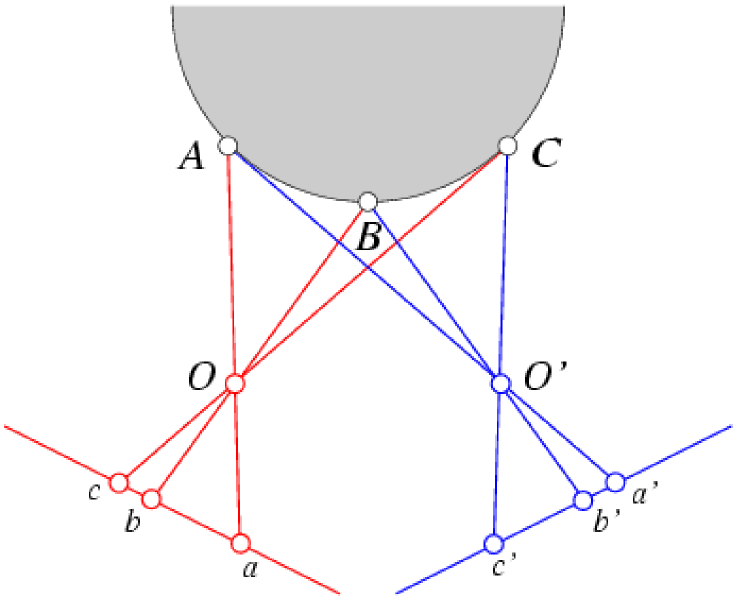
Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image



Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views



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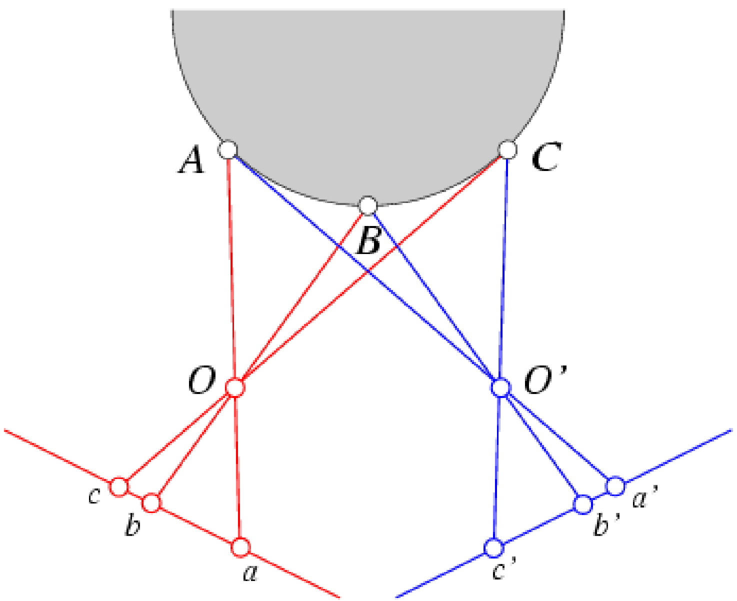
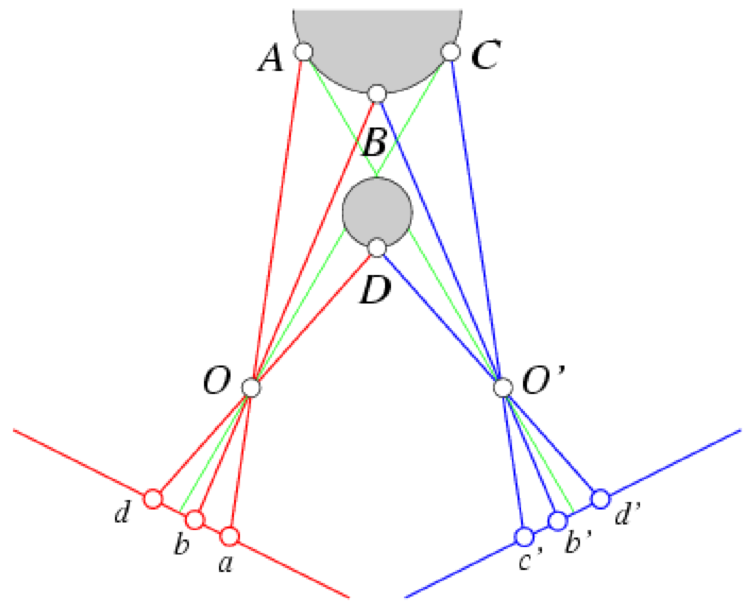




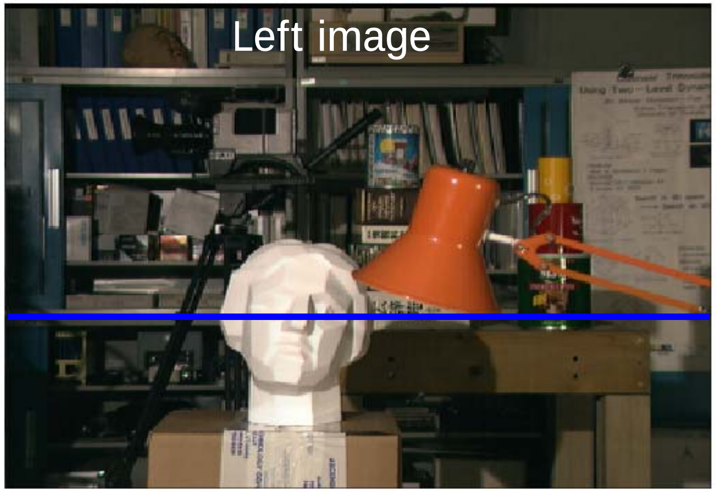
Ordering constraint doesn't hold

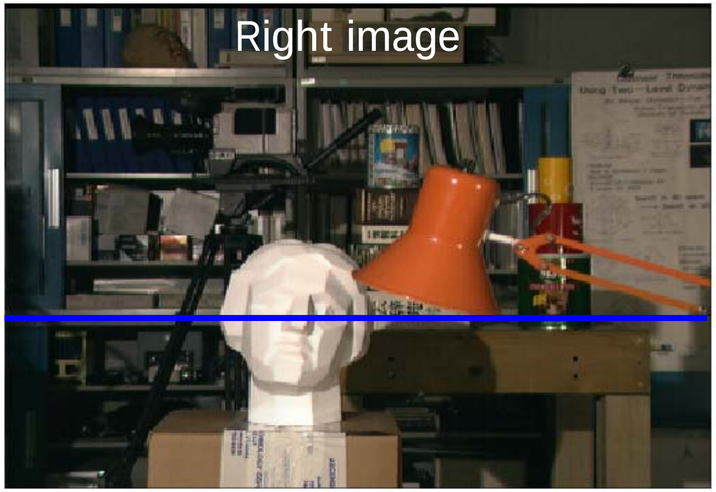
Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part)

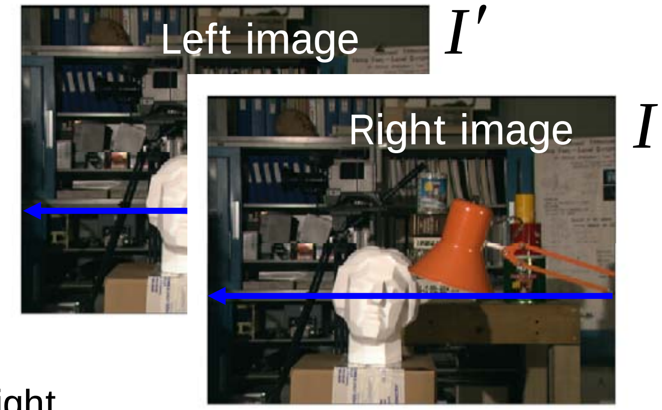
Scanline stereo

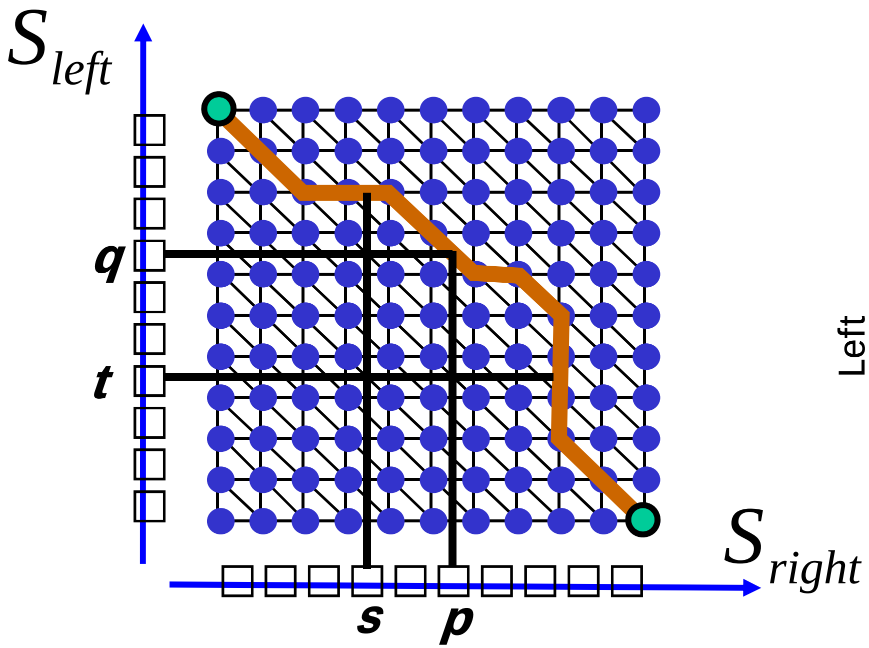
- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently

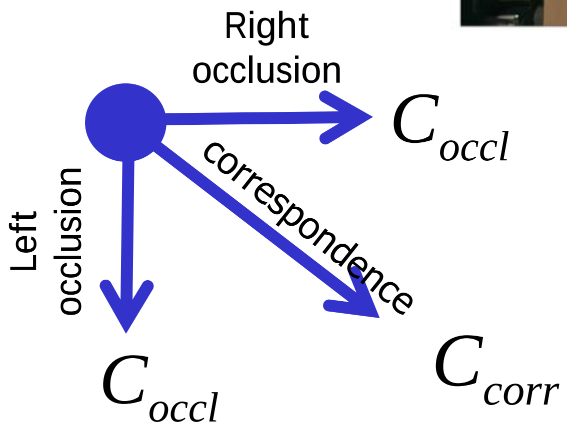




"Shortest paths" for scan-line stereo







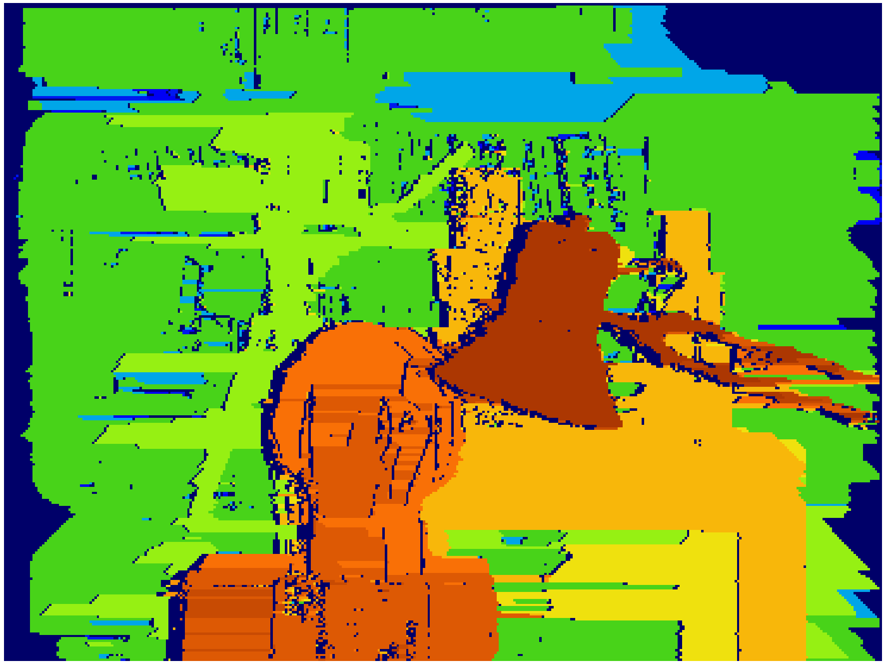
Can be implemented with dynamic programming

Ohta & Kanade '85, Cox et al. '96

Slide credit: Y. Boykov

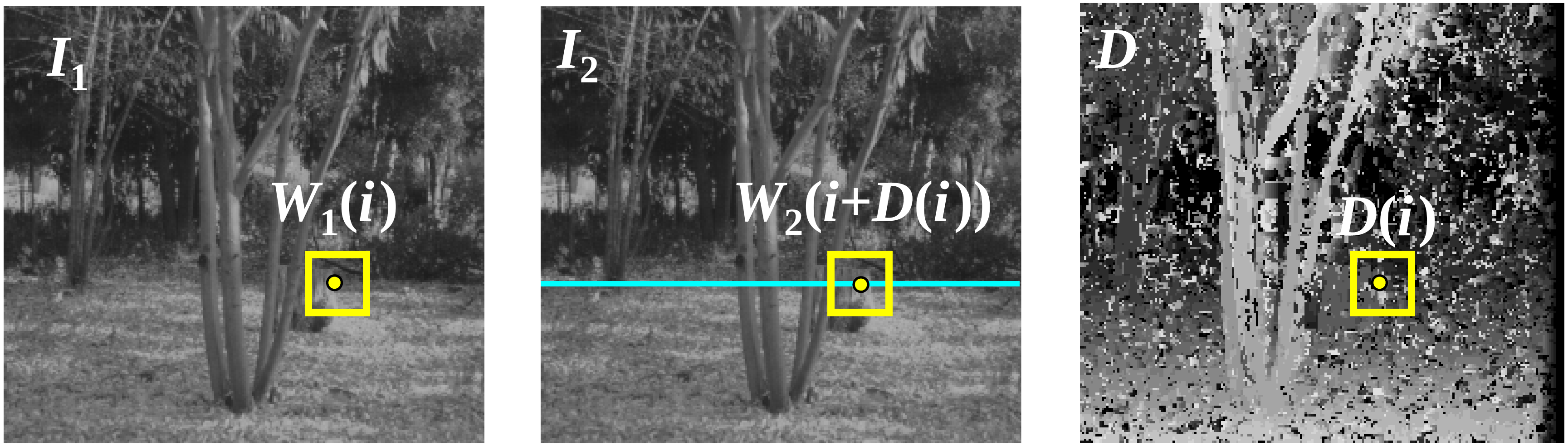
Coherent stereo on 2D grid

- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid

Stereo matching as energy minimization



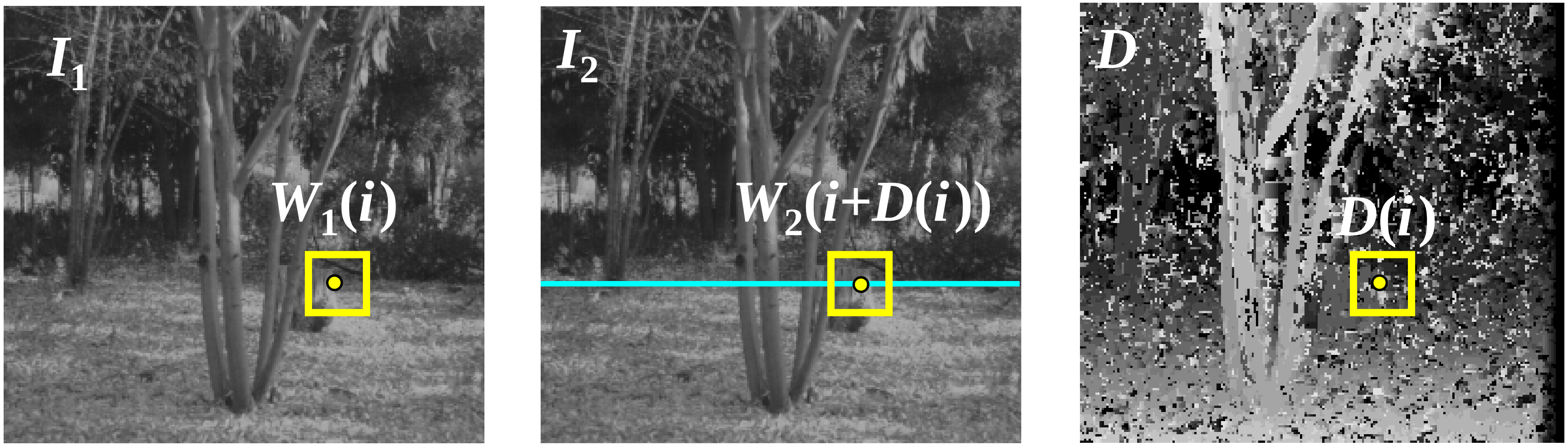
$$E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)$$

$$E_{\text{data}} = \sum_i \left(W_1(i) - W_2(i + D(i))\right)^2 \qquad E_{\text{smooth}} = \sum_{\text{neighbors } i,j} \rho\left(D(i) - D(j)\right)$$

- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

Stereo matching as energy minimization



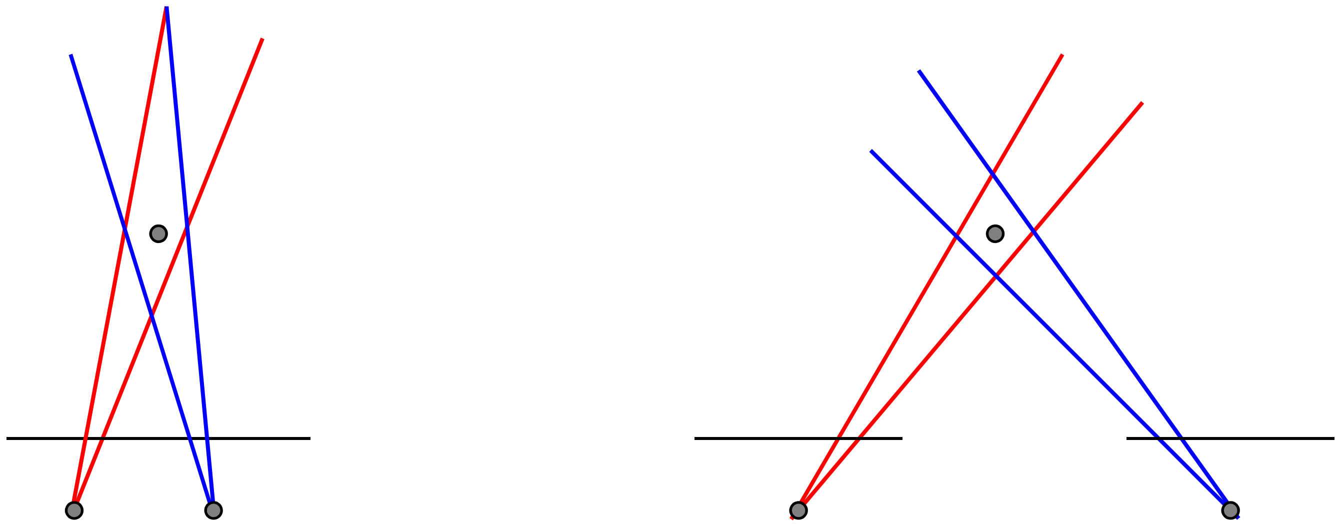
- Probabilistic interpretation: we want to find a Maximum A Posteriori (MAP) estimate of disparity image *D*:

$$P(D \mid I_1, I_2) \propto P(I_1, I_2 \mid D) P(D)$$

$$-\log P(D \mid I_1, I_2) \propto -\log P(I_1, I_2 \mid D) - \log P(D)$$

$$\boxed{E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)}$$

The role of the baseline



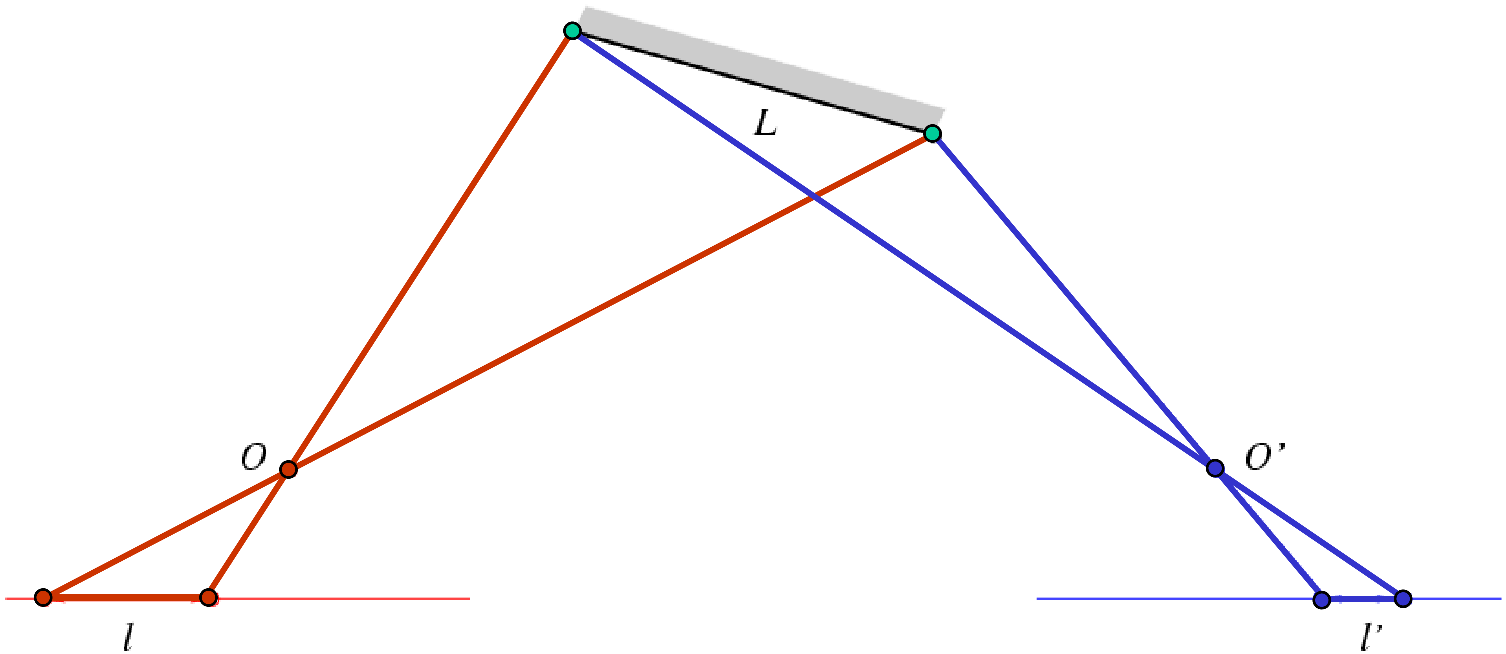
Small Baseline **Large Baseline**

Small baseline: large depth error

Large baseline: difficult search problem

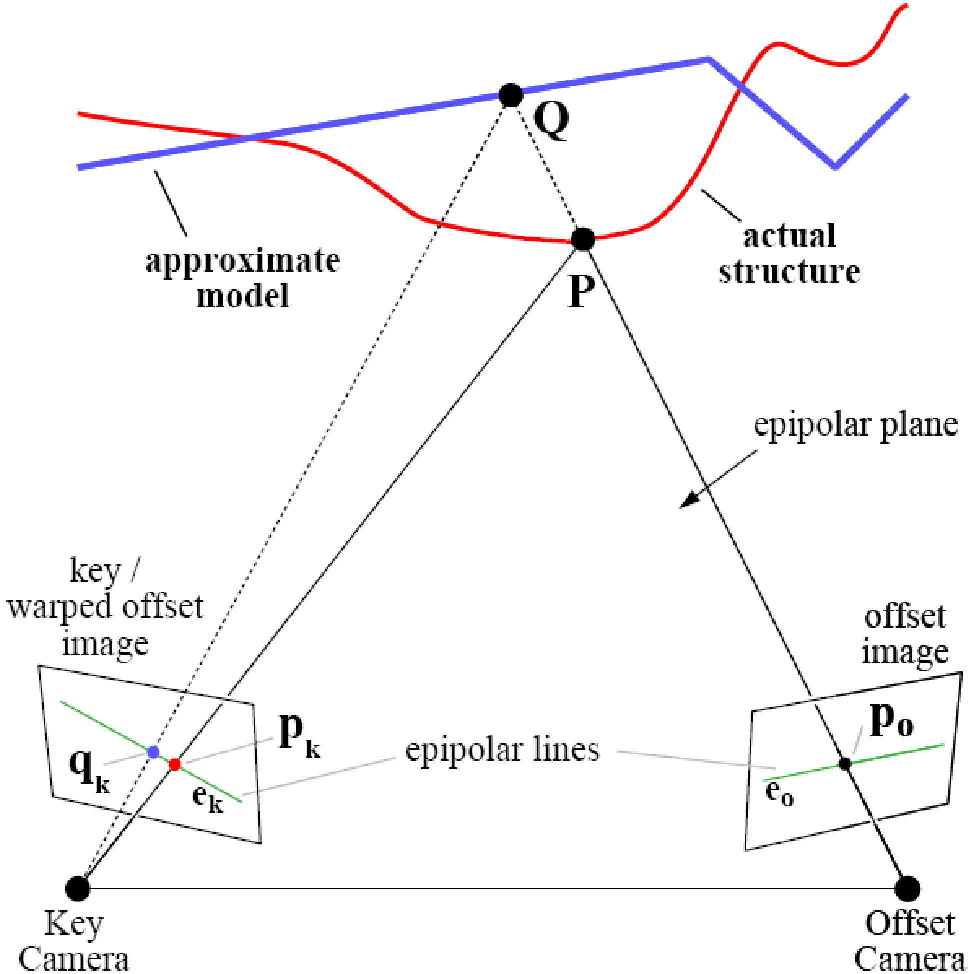
Source: S. Seitz

Problem for wide baselines: Foreshortening



- Matching with fixed-size windows will fail!
- Possible solution: adaptively vary window size
- Another solution: *model-based stereo*

Model-based stereo



Paul E. Debevec, Camillo J. Taylor, and Jitendra Malik. [Modeling and Rendering Architecture from Photographs.](#) SIGGRAPH 1996.

Model-based stereo

key image

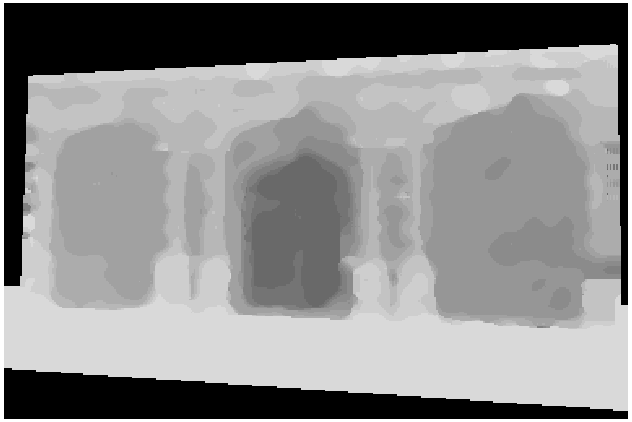
offset image

Paul E. Debevec, Camillo J. Taylor, and Jitendra Malik. [Modeling and Rendering Architecture from Photographs.](#) SIGGRAPH 1996.

Model-based stereo

key image

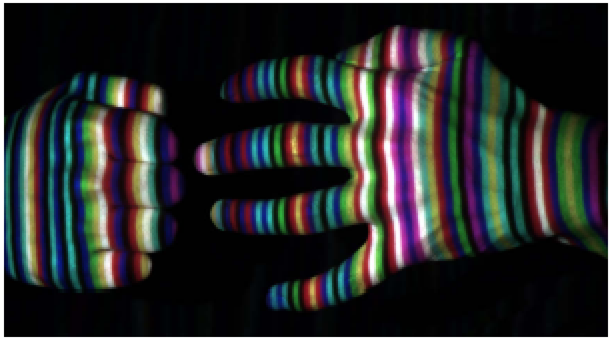
warped offset image

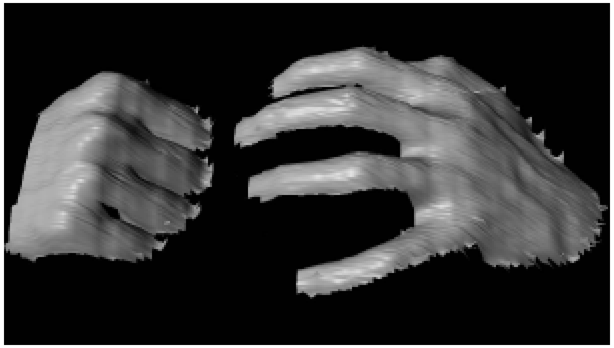


displacement map

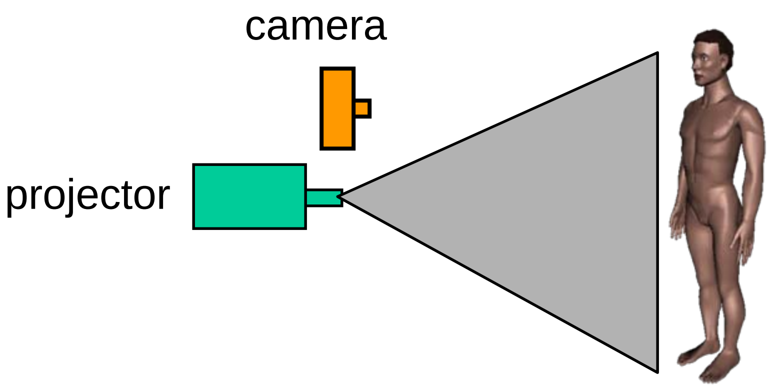
Paul E. Debevec, Camillo J. Taylor, and Jitendra Malik. [Modeling and Rendering Architecture from Photographs.]() SIGGRAPH 1996.

Active stereo with structured light



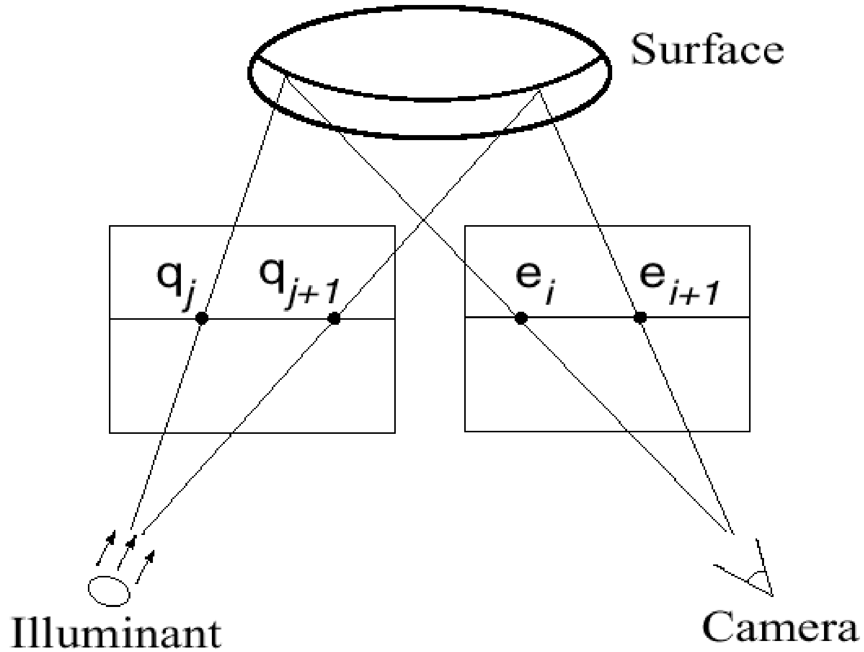


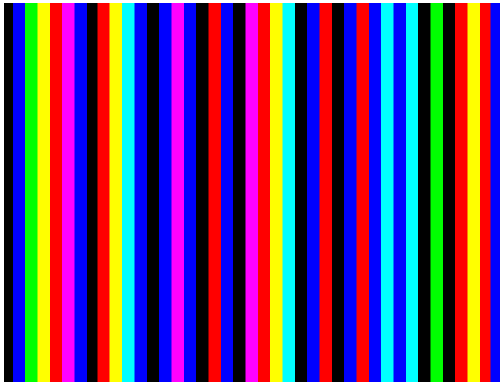
- Project “structured” light patterns onto the object
  - Simplifies the correspondence problem
  - Allows us to use only one camera

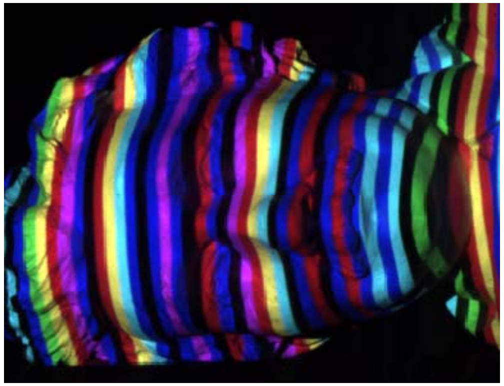


L. Zhang, B. Curless, and S. M. Seitz. [Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming.](#) *3DPVT* 2002

Active stereo with structured light

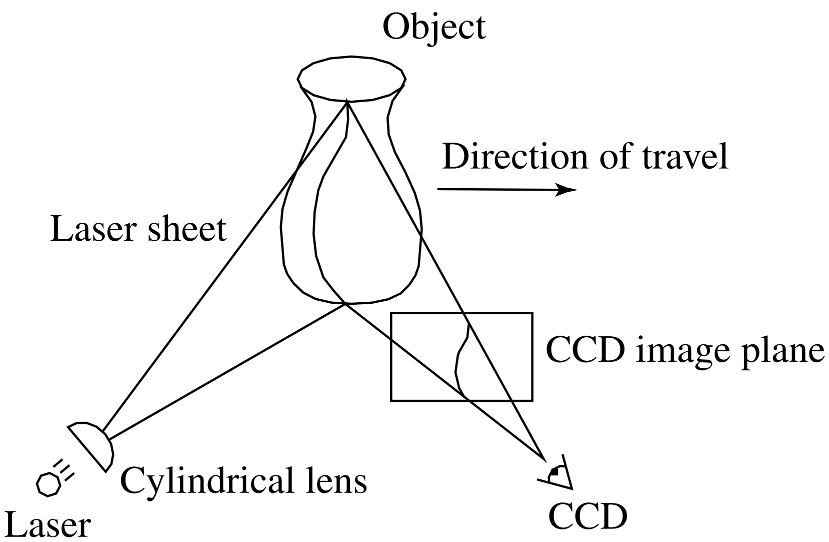


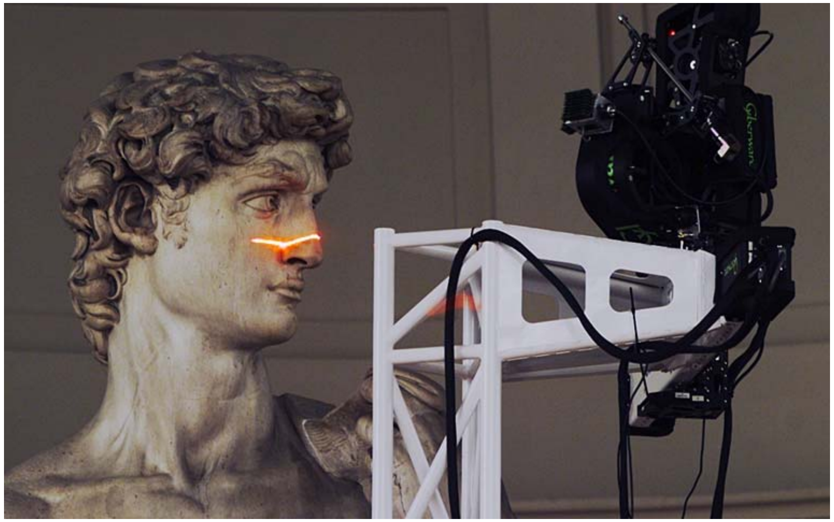




L. Zhang, B. Curless, and S. M. Seitz. [Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming.]() *3DPVT* 2002

Laser scanning





Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

Optical triangulation

- Project a single stripe of laser light
- Scan it across the surface of the object
- This is a very precise version of structured light scanning

Source: S. Seitz

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

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The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

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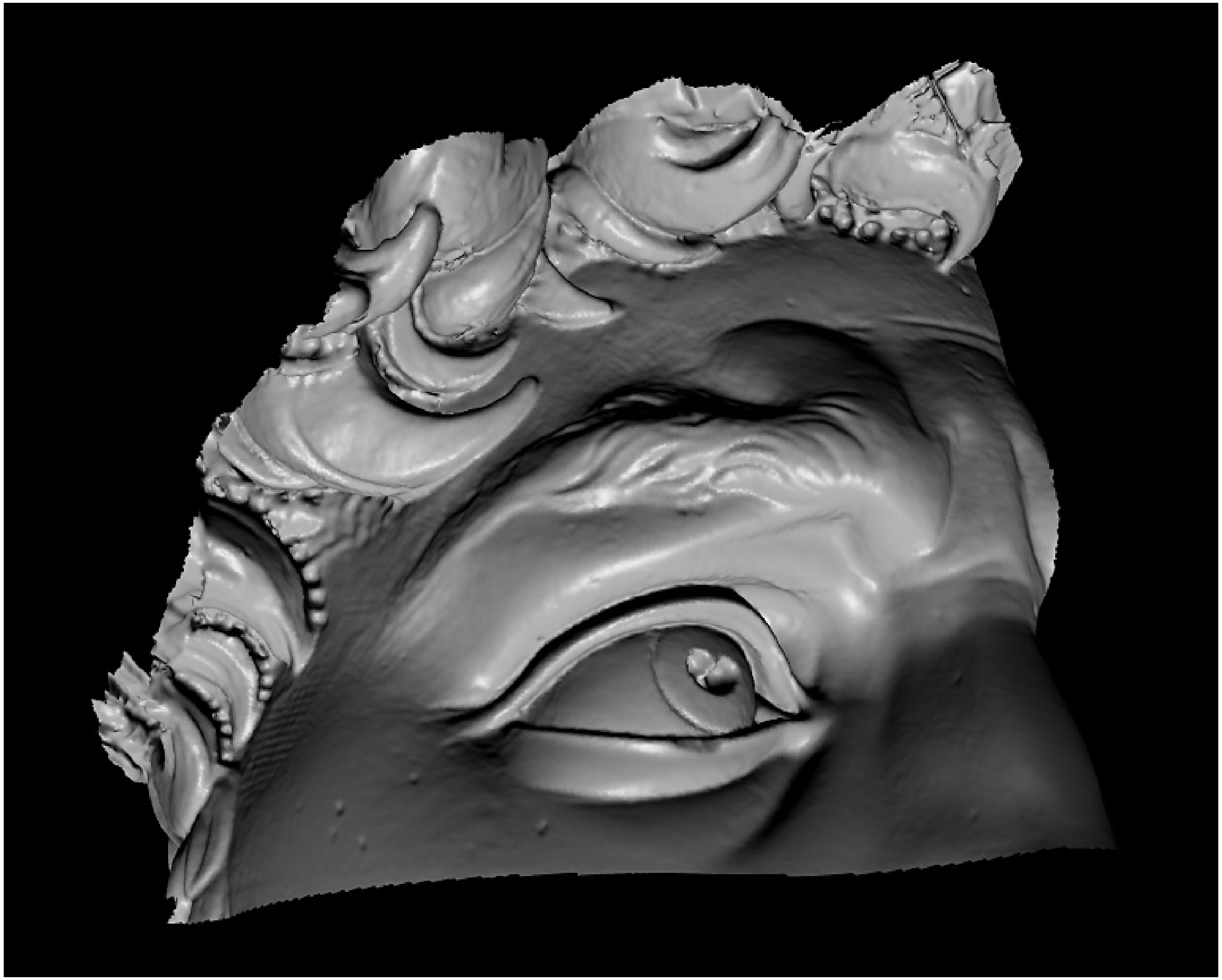
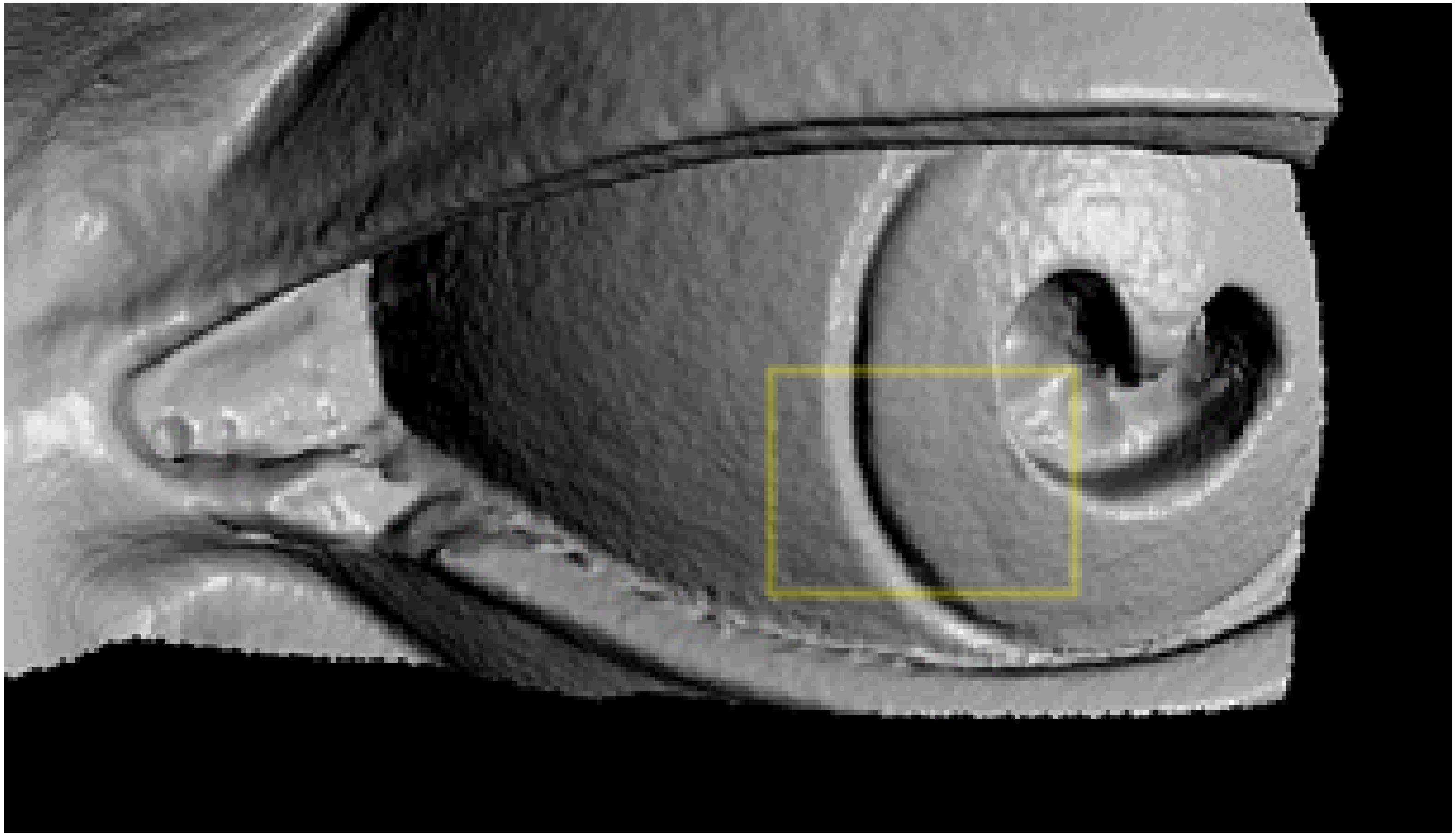


The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Laser scanned models

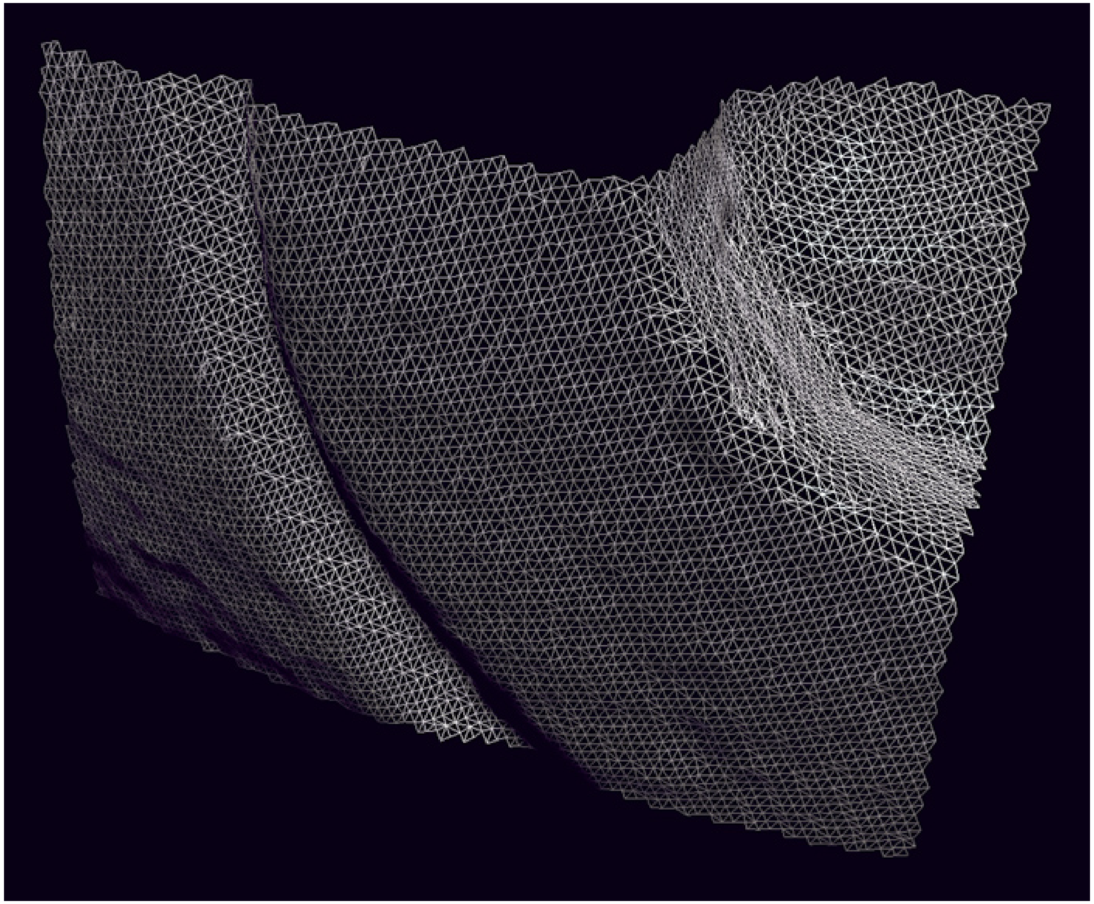


The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Laser scanned models

1.0 mm resolution (56 million triangles)



The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Aligning range images

- A single range scan is not sufficient to describe a complex surface
- Need techniques to register multiple range images

B. Curless and M. Levoy, [A Volumetric Method for Building Complex Models from Range Images](), SIGGRAPH 1996

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… which brings us to *multi-view stereo*