Image alignment

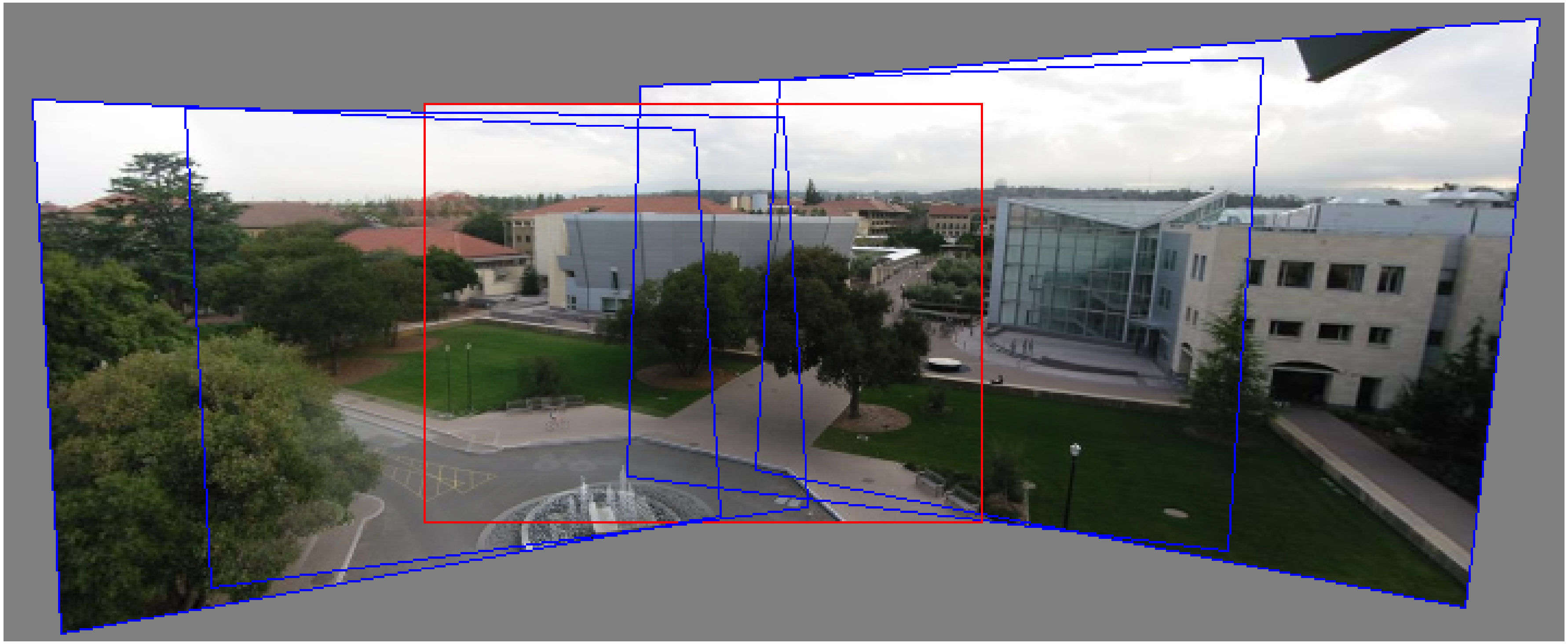
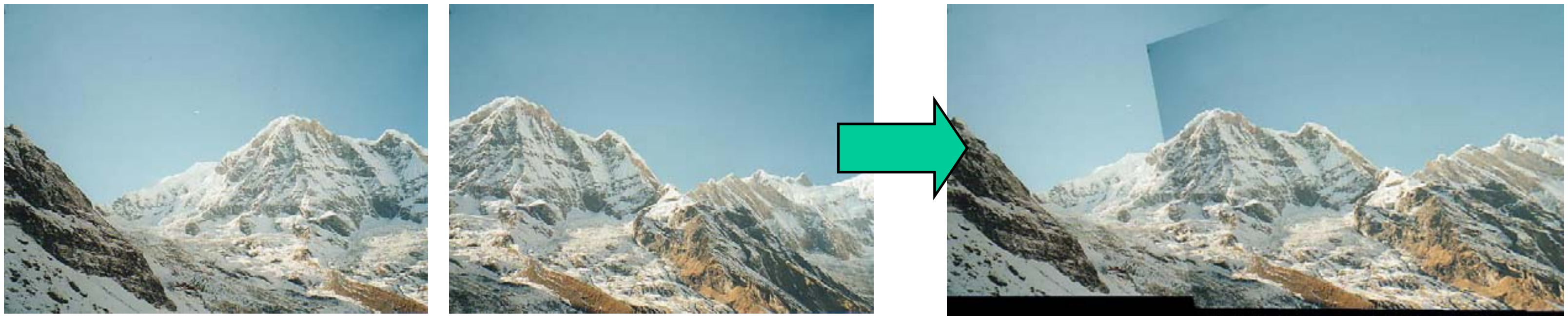
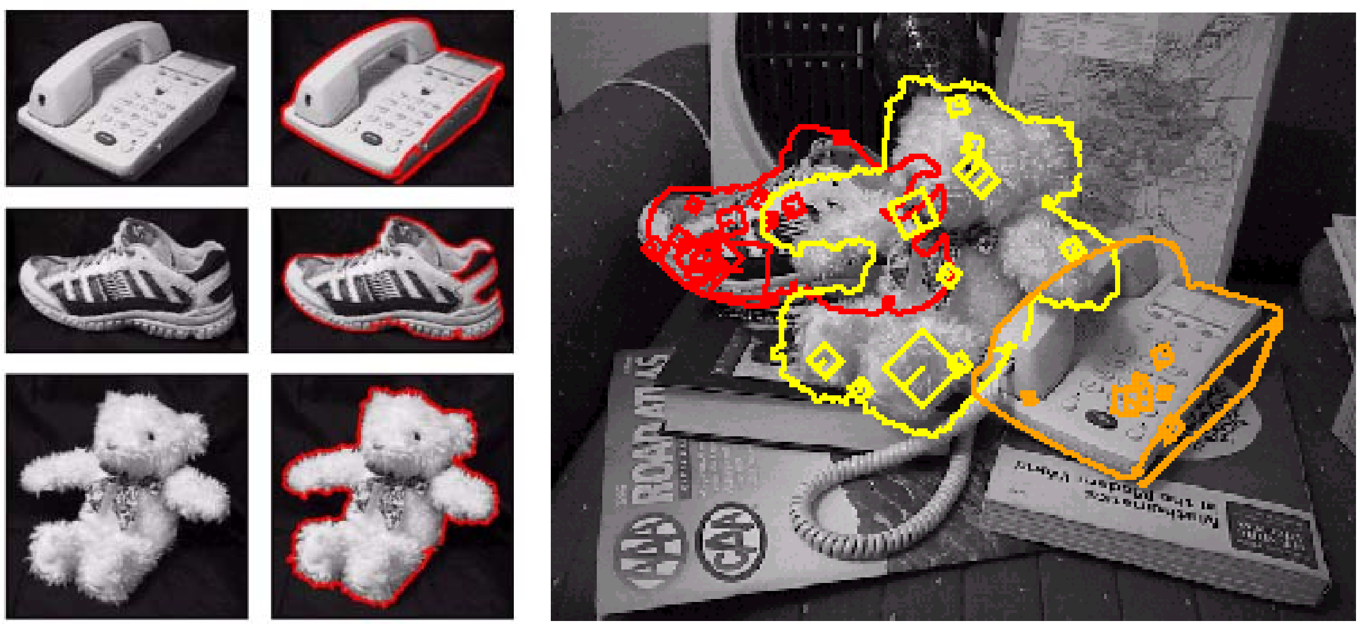


Image alignment: Motivation

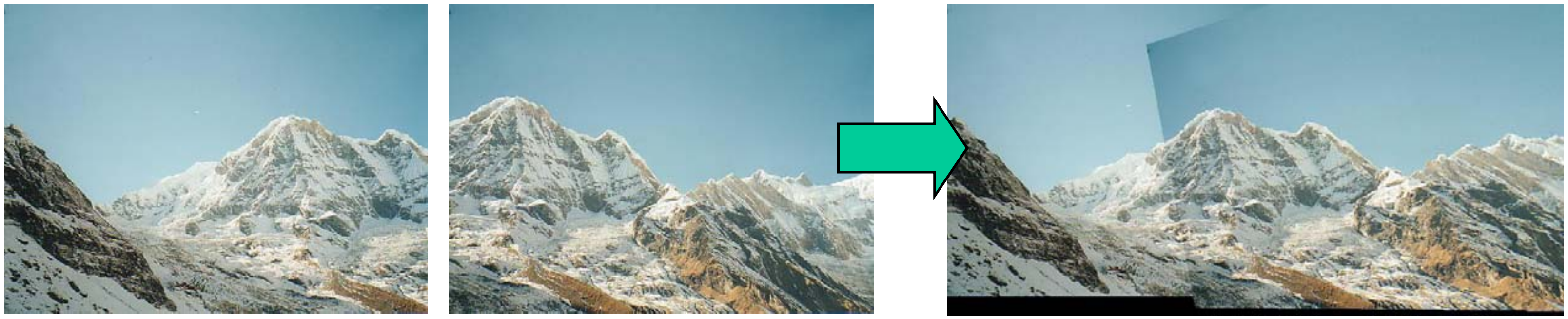


Panorama stitching

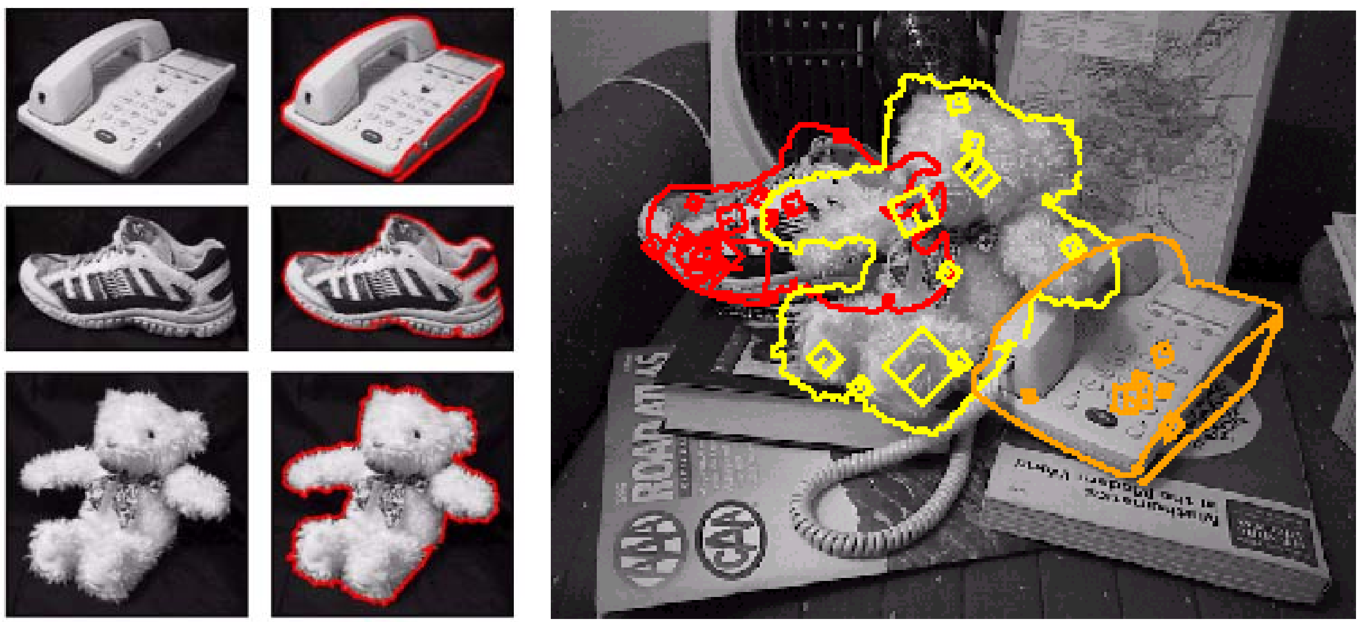


Recognition of object instances

Image alignment: Challenges

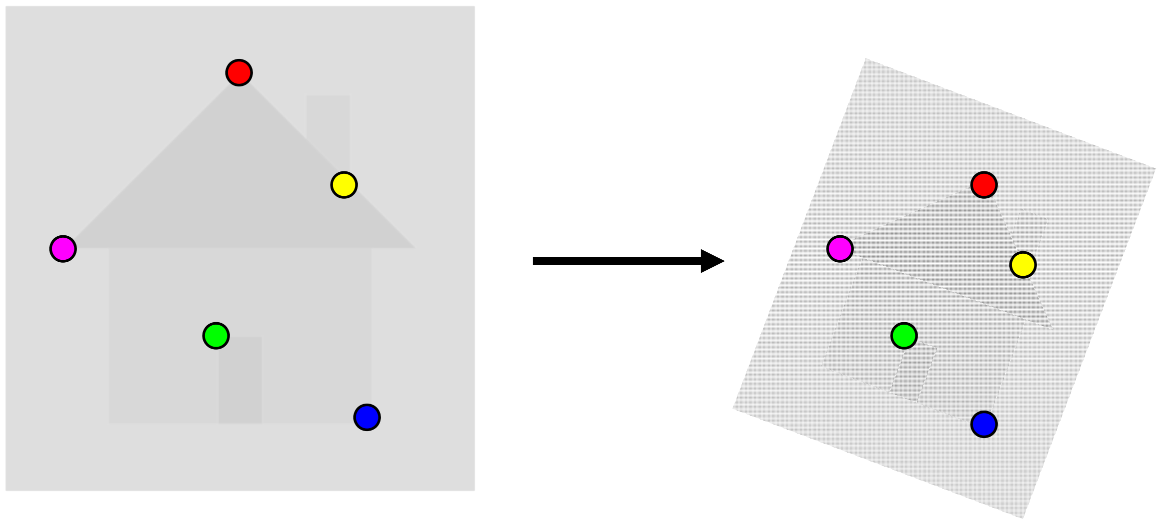


Small degree of overlap



Occlusion, clutter

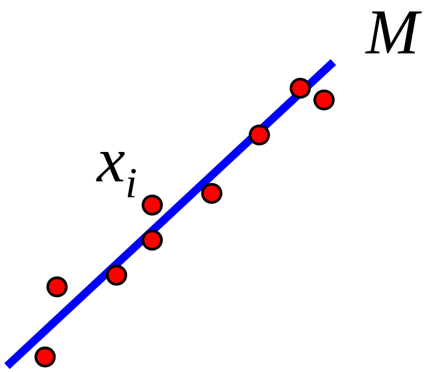
Image alignment



- Two broad approaches:
  - Direct (pixel-based) alignment
    - – Search for alignment where most pixels agree
  - Feature-based alignment
    - – Search for alignment where *extracted features* agree
    - – Can be verified using pixel-based alignment

Alignment as fitting

- Previous lectures: fitting a model to features in one image

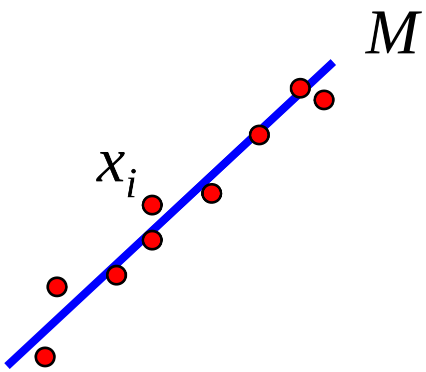


Find model *M* that minimizes

$$\sum_i \text{residual}(x_i, M)$$

Alignment as fitting

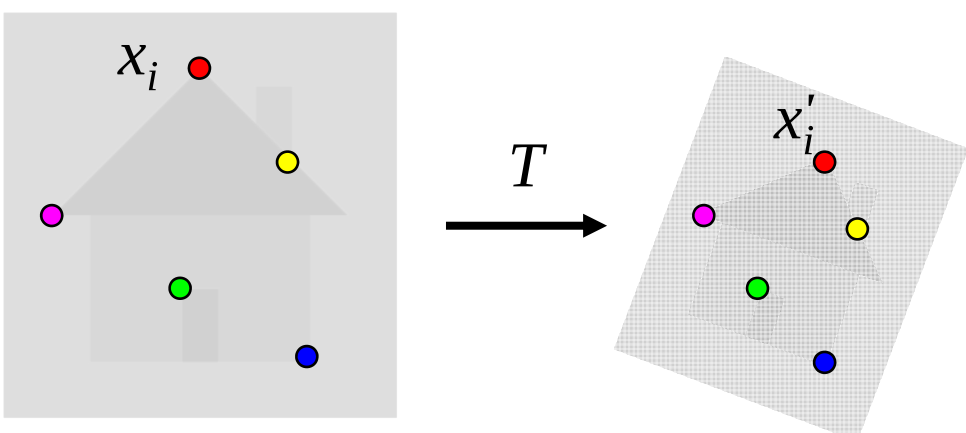
- Previous lectures: fitting a model to features in one image



Find model M that minimizes

$$\sum_i \text{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images

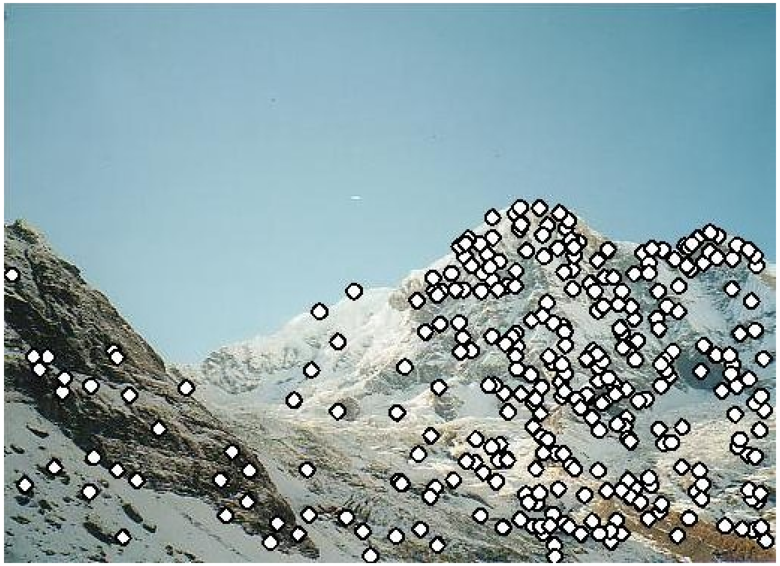
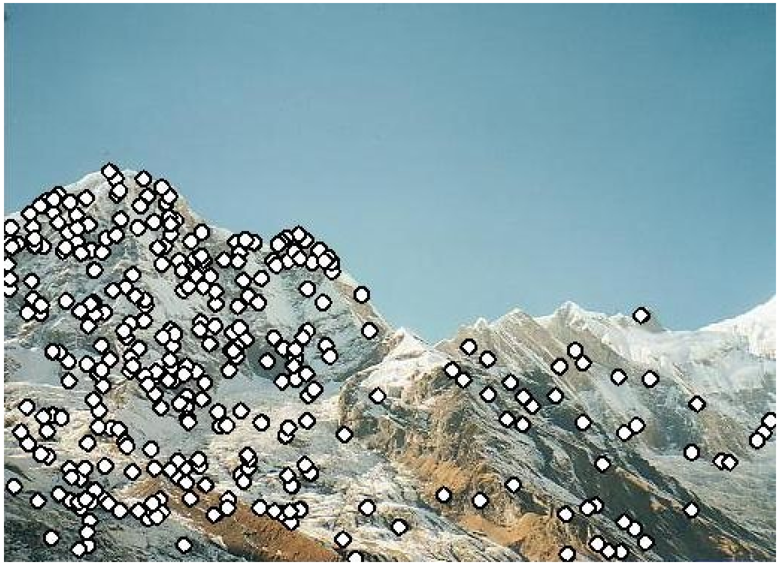


Find transformation T that minimizes

$$\sum_i \text{residual}(T(x_i), x_i')$$

Feature-based alignment outline

Feature-based alignment outline

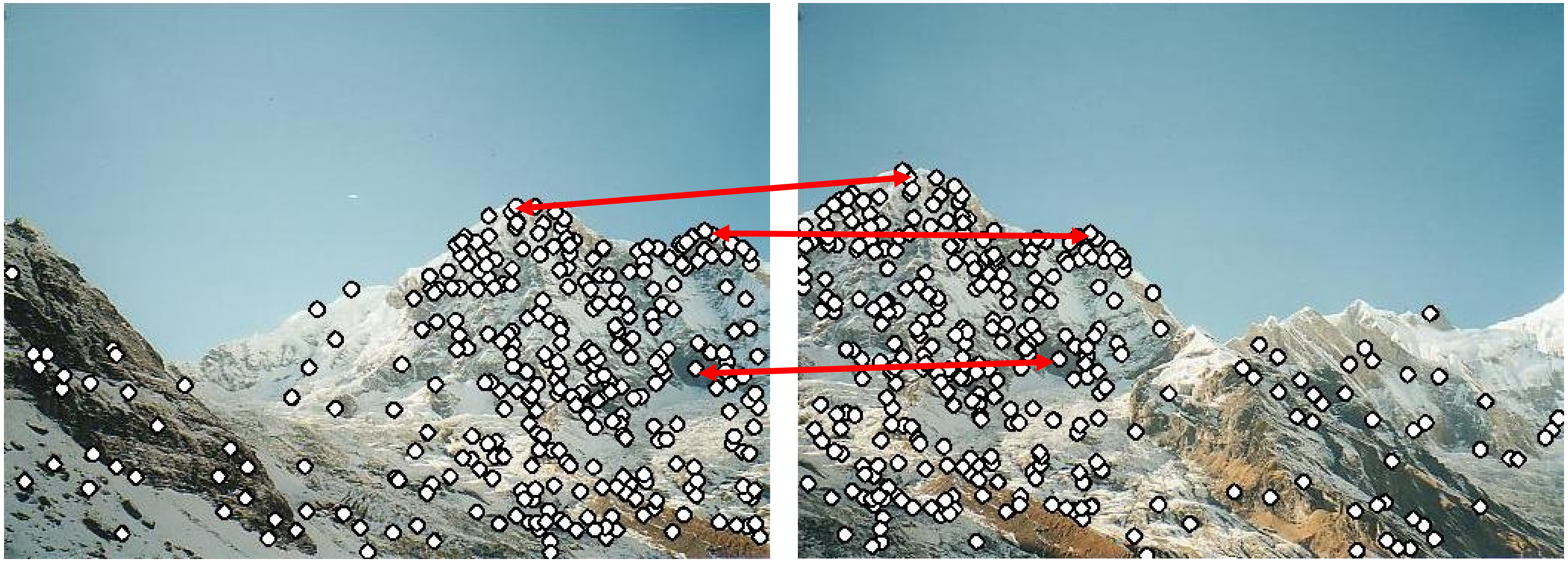
 

- Extract features

Feature-based alignment outline

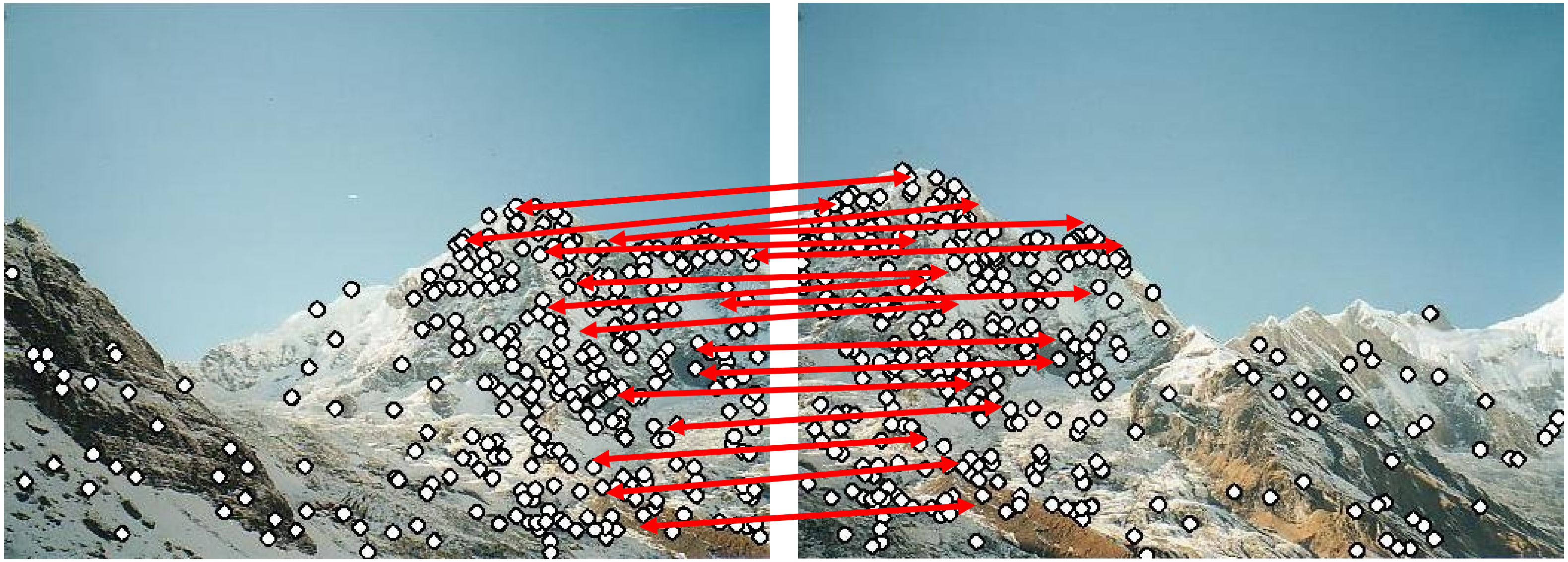
- Extract features
- Compute *putative matches*

Feature-based alignment outline



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)

Feature-based alignment outline



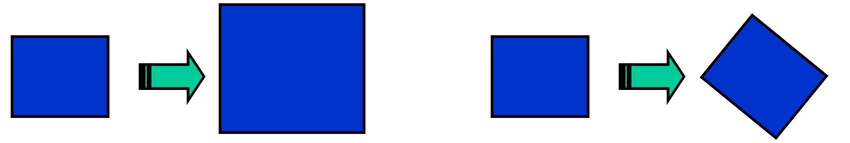
- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Feature-based alignment outline

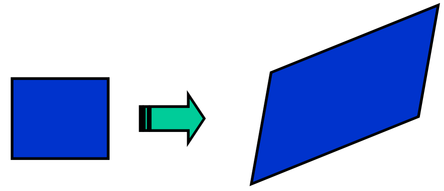
- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

2D transformation models

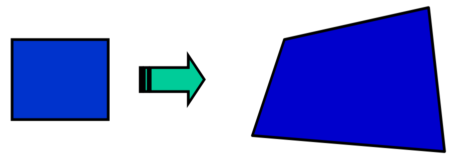
- Similarity (translation, scale, rotation)



- Affine



- Projective (homography)

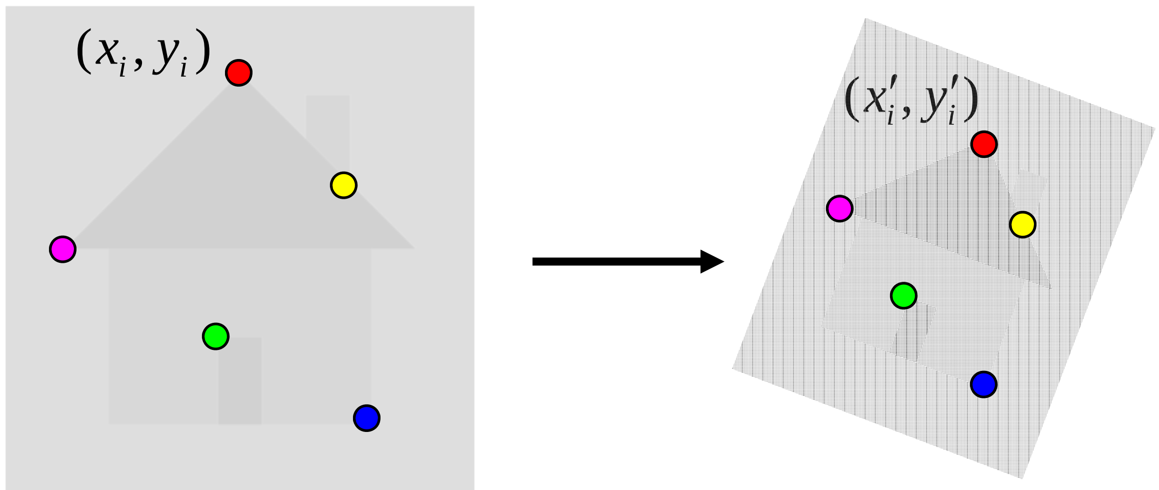


Let’s start with affine transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models

Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

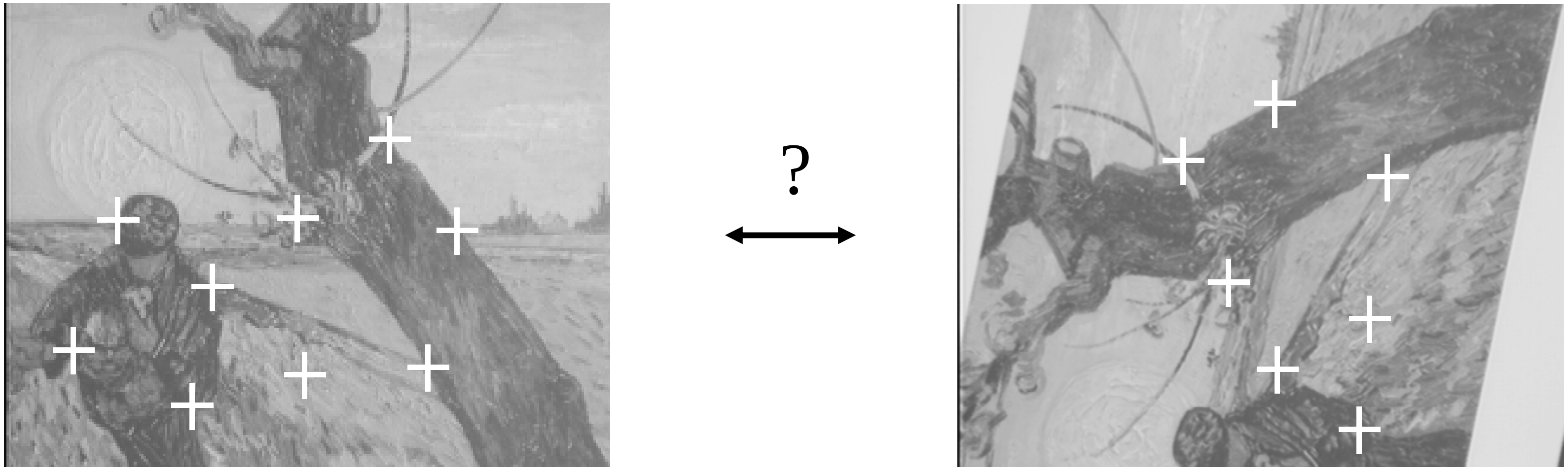
$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

Fitting an affine transformation

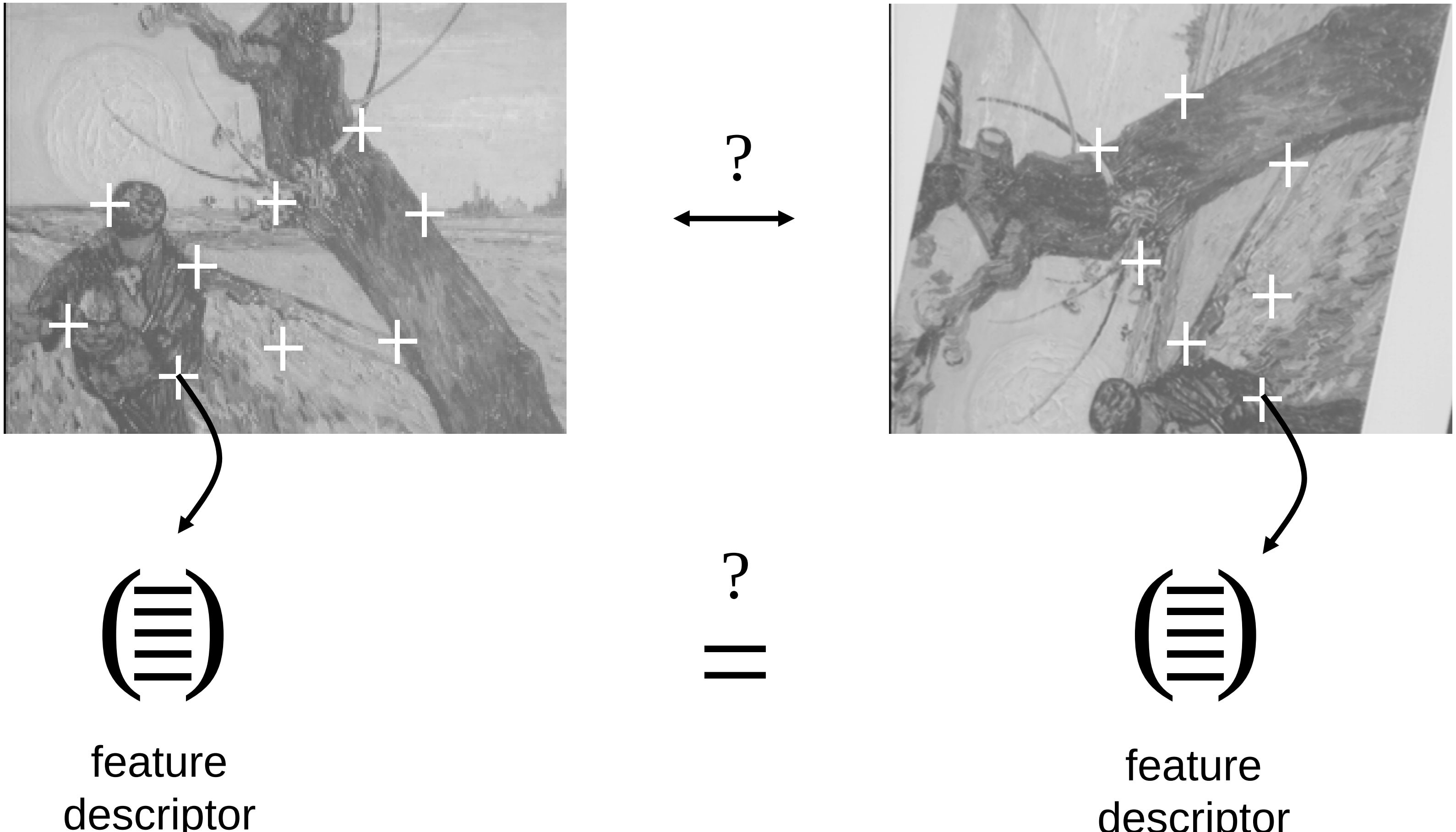
$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters

What if we don’t know the correspondences?



What if we don’t know the correspondences?



- Need to compare *feature descriptors* of local patches surrounding interest points

Feature descriptors

- Assuming the patches are already normalized (i.e., the local effect of the geometric transformation is factored out), how do we compute their similarity?
- Want invariance to intensity changes, noise, perceptually insignificant changes of the pixel pattern

Feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors?
  - Sum of squared differences (SSD)

$$\mathrm{SSD}(u,v) = \sum_i \left(u_i - v_i\right)^2$$

    – Not invariant to intensity change

  - Normalized correlation

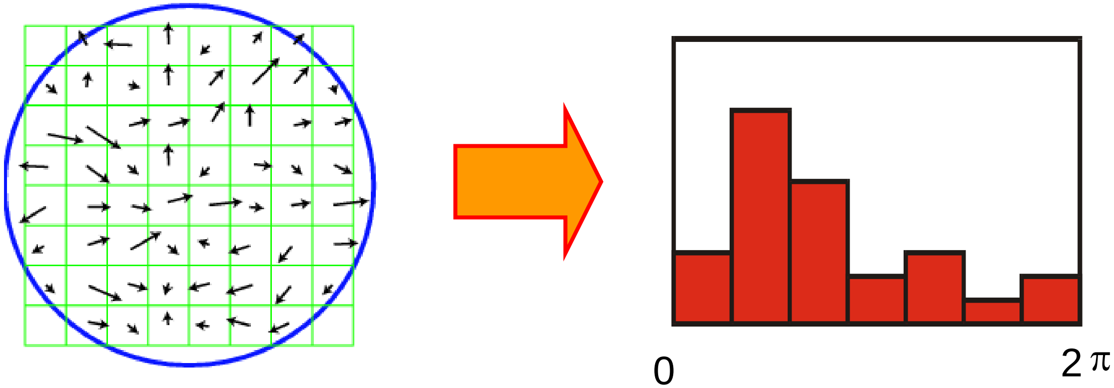
$$\rho(u,v) = \frac{\sum_i (u_i - \overline{u})(v_i - \overline{v})}{\sqrt{\left(\sum_j (u_j - \overline{u})^2\right)\left(\sum_j (v_j - \overline{v})^2\right)}}$$

    – Invariant to affine intensity change

Feature descriptors

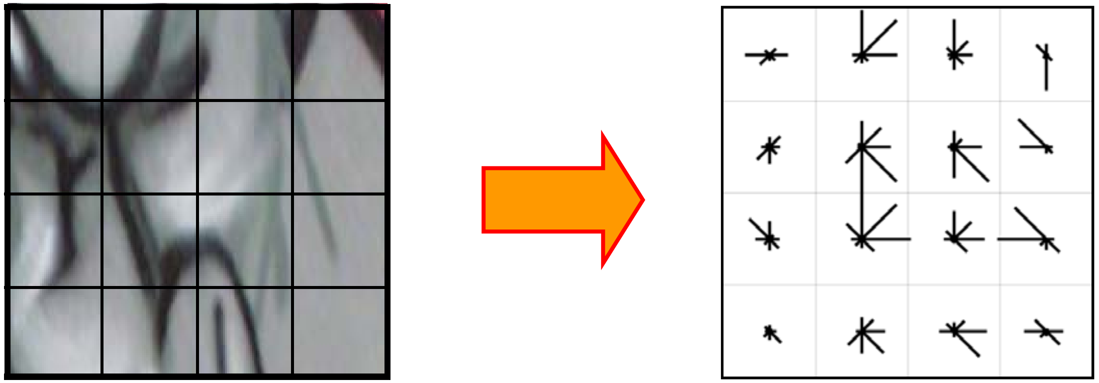
- Disadvantage of patches as descriptors:
  - Small shifts can affect matching score a lot

- Solution: histograms



Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions



David G. Lowe. ["Distinctive image features from scale-invariant keypoints."]() *IJCV* 60 (2), pp. 91-110, 2004.

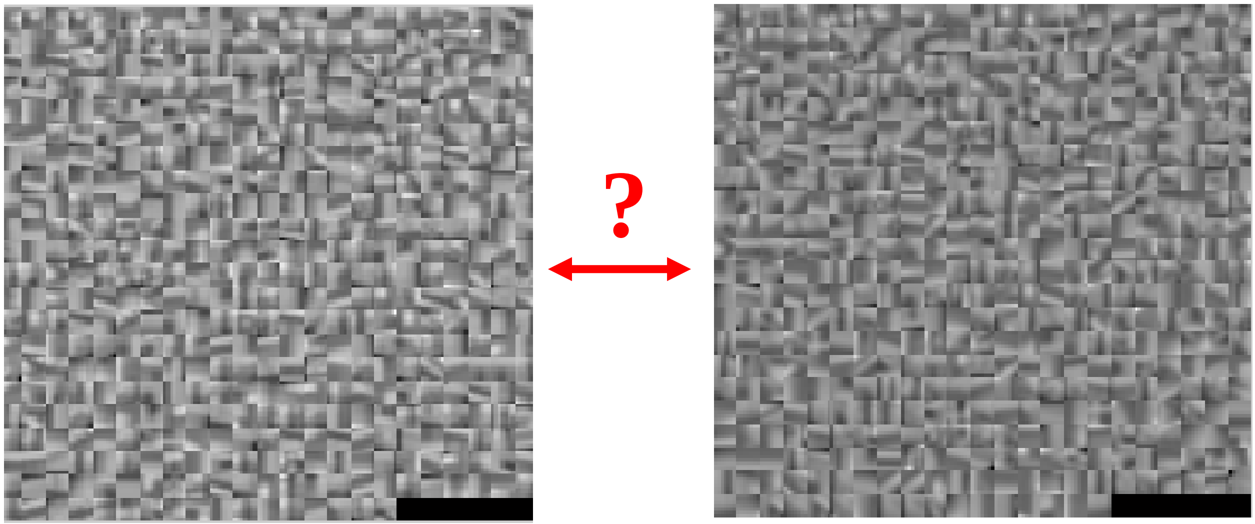
Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions
- Advantage over raw vectors of pixel values
  - Gradients less sensitive to illumination change
  - “Subdivide and disorder” strategy achieves robustness to small shifts, but still preserves some spatial information

David G. Lowe. "Distinctive image features from scale-invariant keypoints.” *IJCV* 60 (2), pp. 91-110, 2004.

Feature matching

- Generating *putative matches*: for each patch in one image, find a short list of patches in the other image that could match it based solely on appearance

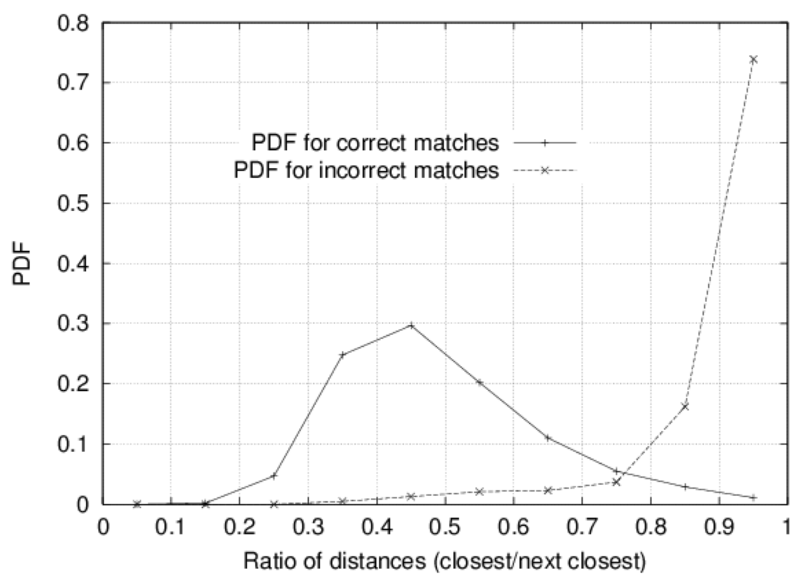


Feature matching

- Generating *putative matches*: for each patch in one image, find a short list of patches in the other image that could match it based solely on appearance
  - Exhaustive search
    - For each feature in one image, compute the distance to *all* features in the other image and find the “closest” ones (threshold or fixed number of top matches)
  - Fast approximate nearest neighbor search
    - Hierarchical spatial data structures (kd-trees, vocabulary trees)
    - Hashing

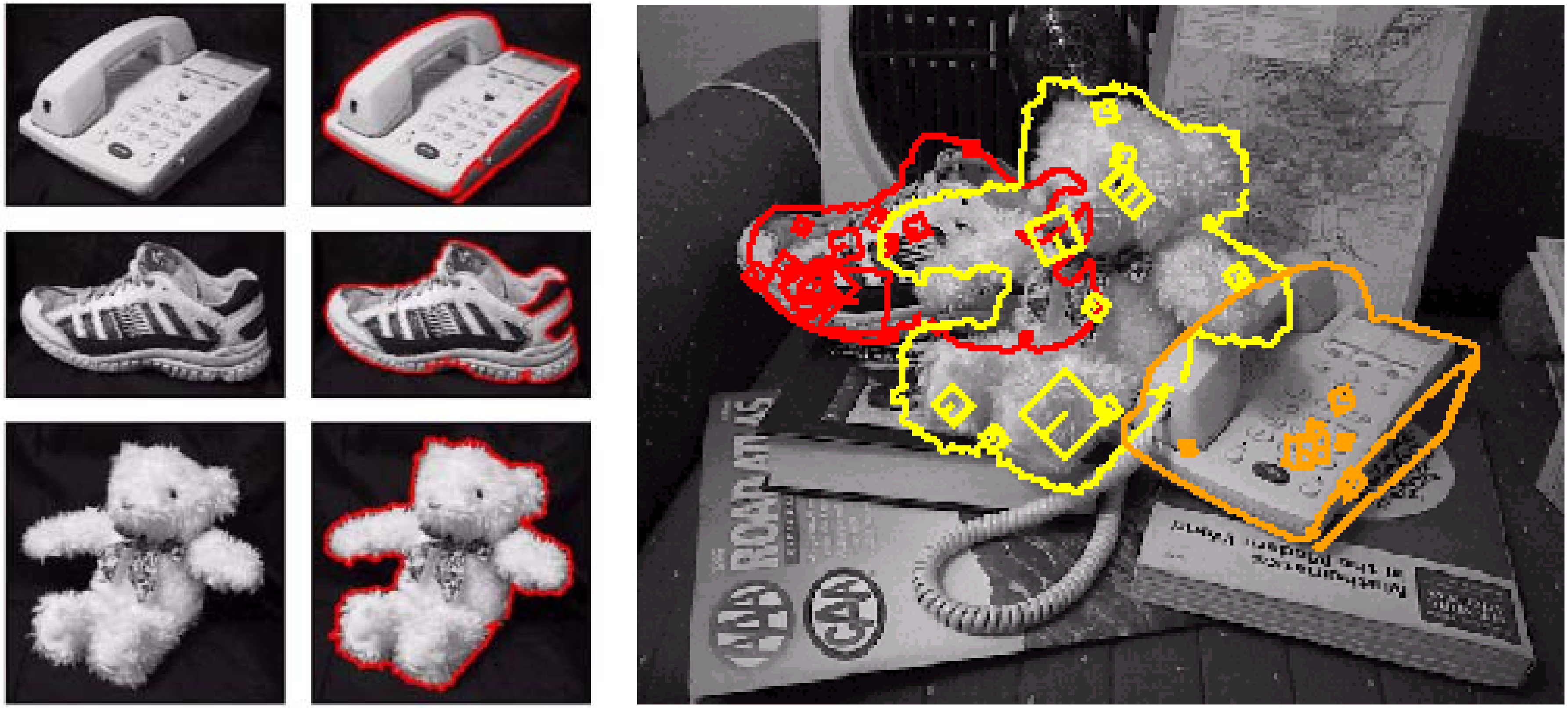
Feature space outlier rejection

- How can we tell which putative matches are more reliable?
- Heuristic: compare distance of **nearest** neighbor to that of **second** nearest neighbor
  - Ratio will be high for features that are not distinctive
  - Threshold of 0.8 provides good separation



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Reading



David G. Lowe. **["Distinctive image features from scale-invariant keypoints."]** *IJCV* 60 (2), pp. 91-110, 2004.

Dealing with outliers

- The set of putative matches contains a very high percentage of outliers
- Heuristics for feature-space outlier rejection
- Geometric fitting strategies:
  - RANSAC
  - Incremental alignment
  - Hough transform
  - Hashing

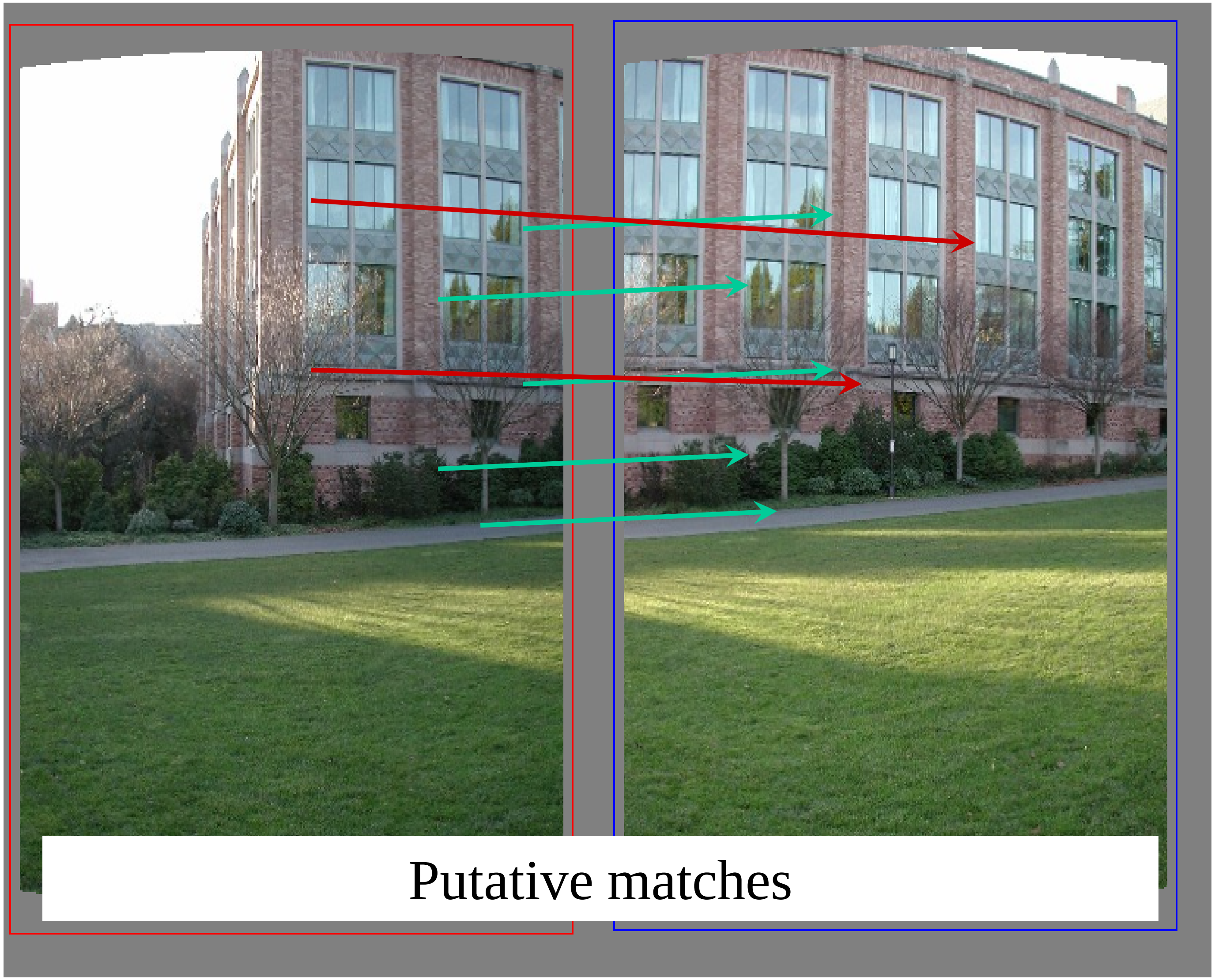
Strategy 1: RANSAC

RANSAC loop:

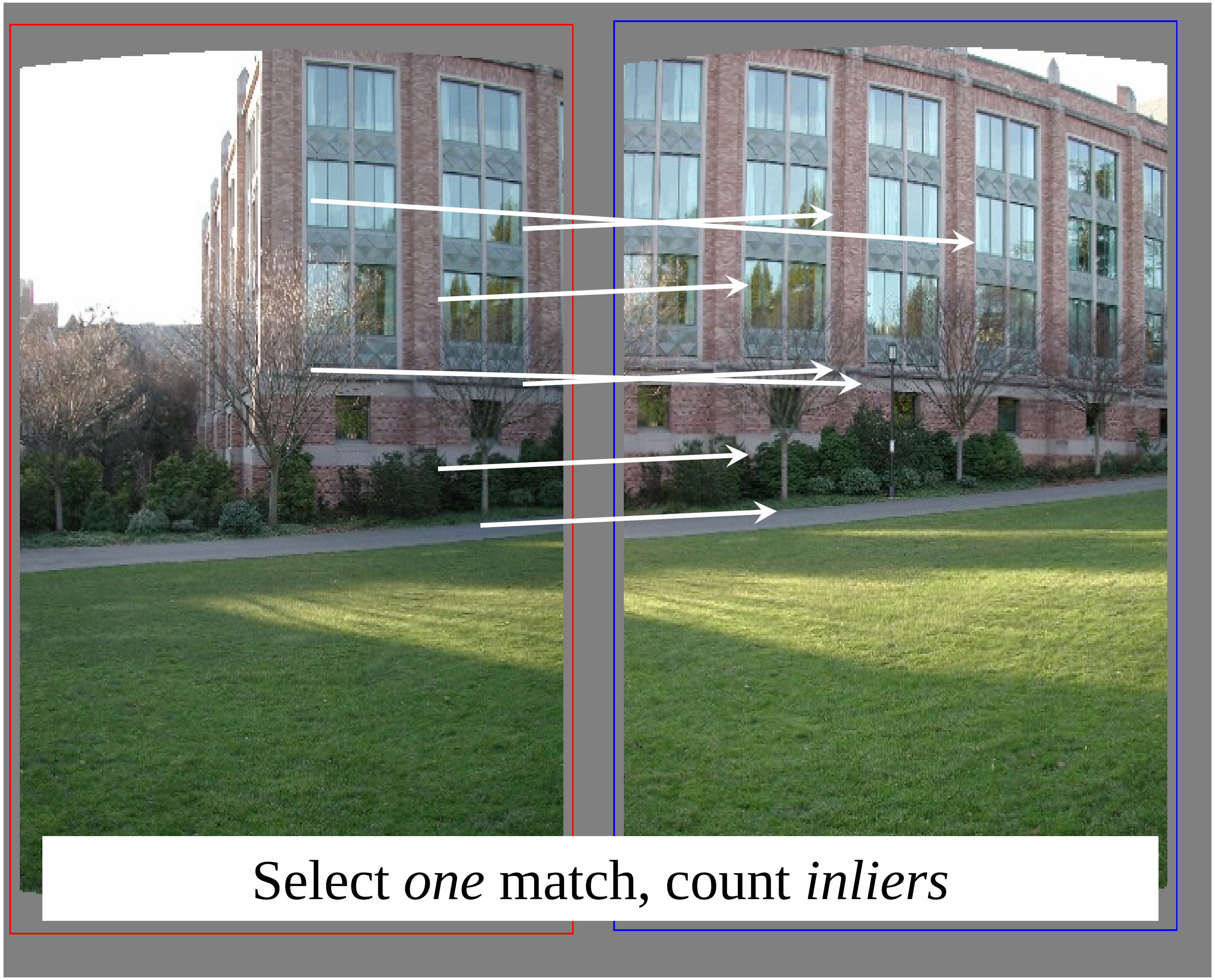
1. Randomly select a *seed group* of matches
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

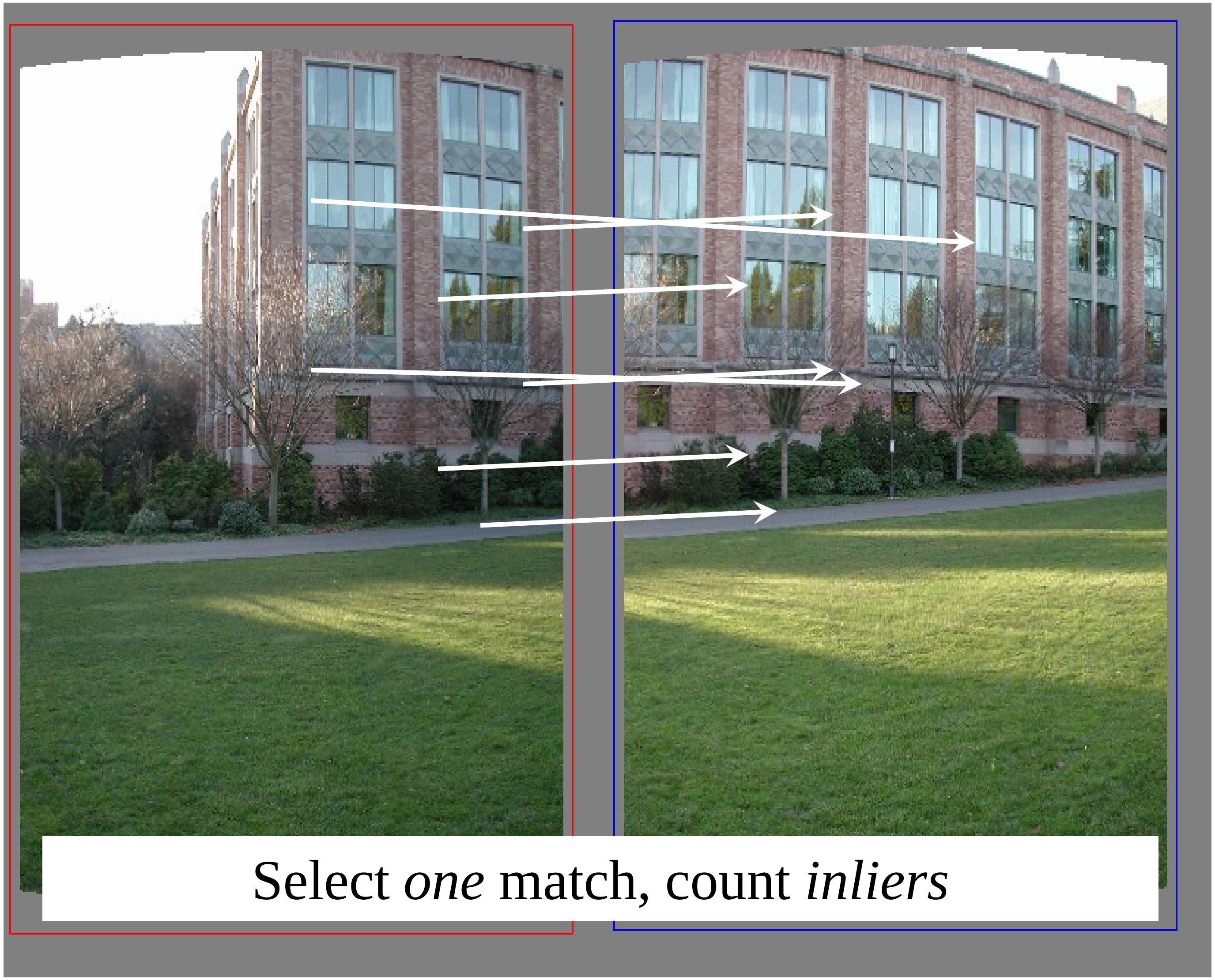
RANSAC example: Translation



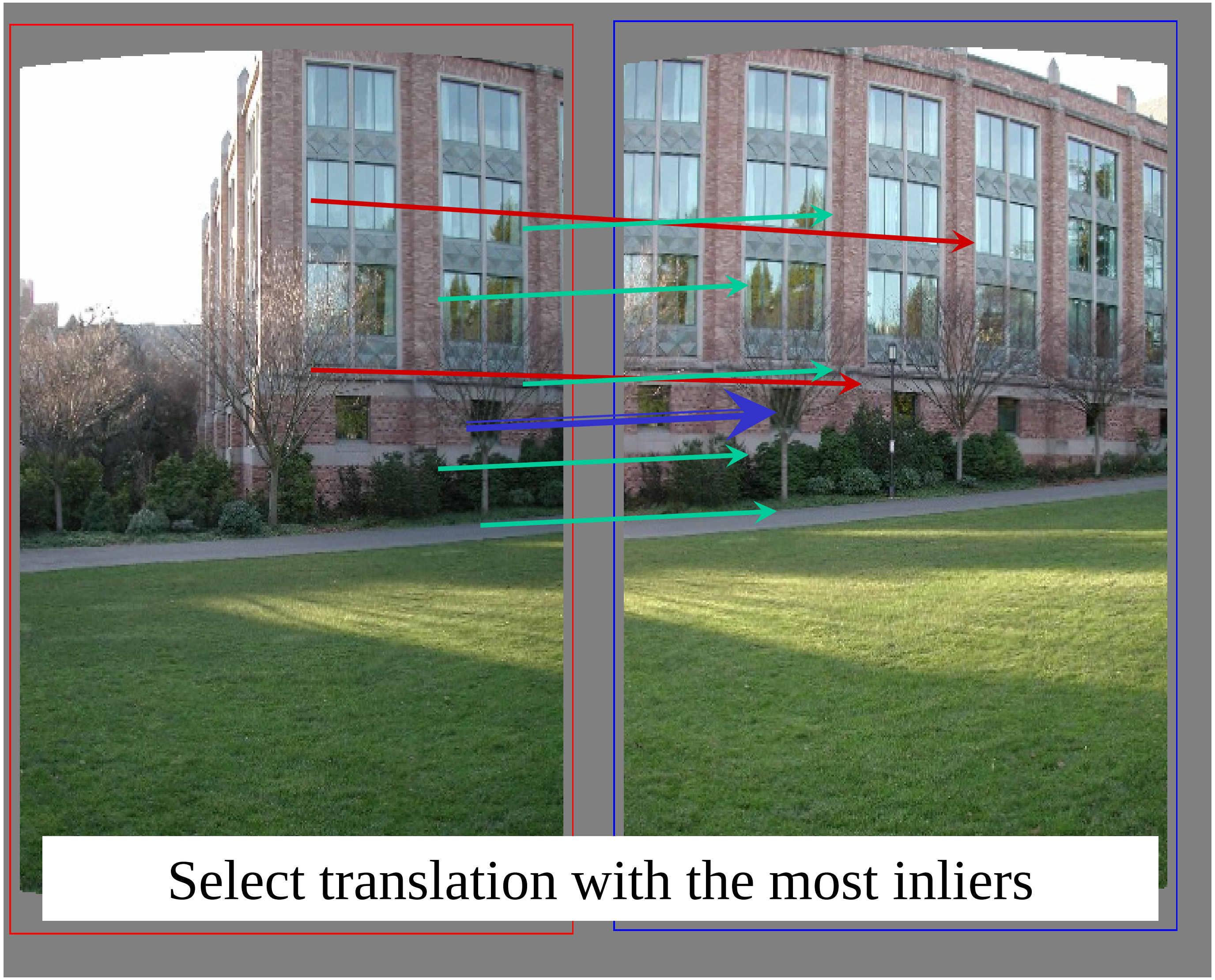
RANSAC example: Translation



RANSAC example: Translation



RANSAC example: Translation



Problem with RANSAC

- In many practical situations, the percentage of outliers (incorrect putative matches) is often very high (90% or above)
- Alternative strategy: restrict search space by using strong locality constraints on seed groups and inliers
  - Incremental alignment

Strategy 2: Incremental alignment

- Take advantage of strong locality constraints: only pick close-by matches to start with, and gradually add more matches in the same neighborhood

S. Lazebnik, C. Schmid and J. Ponce, **"Semi-local affine parts for object recognition,"** BMVC 2004.

Strategy 2: Incremental alignment

- Take advantage of strong locality constraints: only pick close-by matches to start with, and gradually add more matches in the same neighborhood

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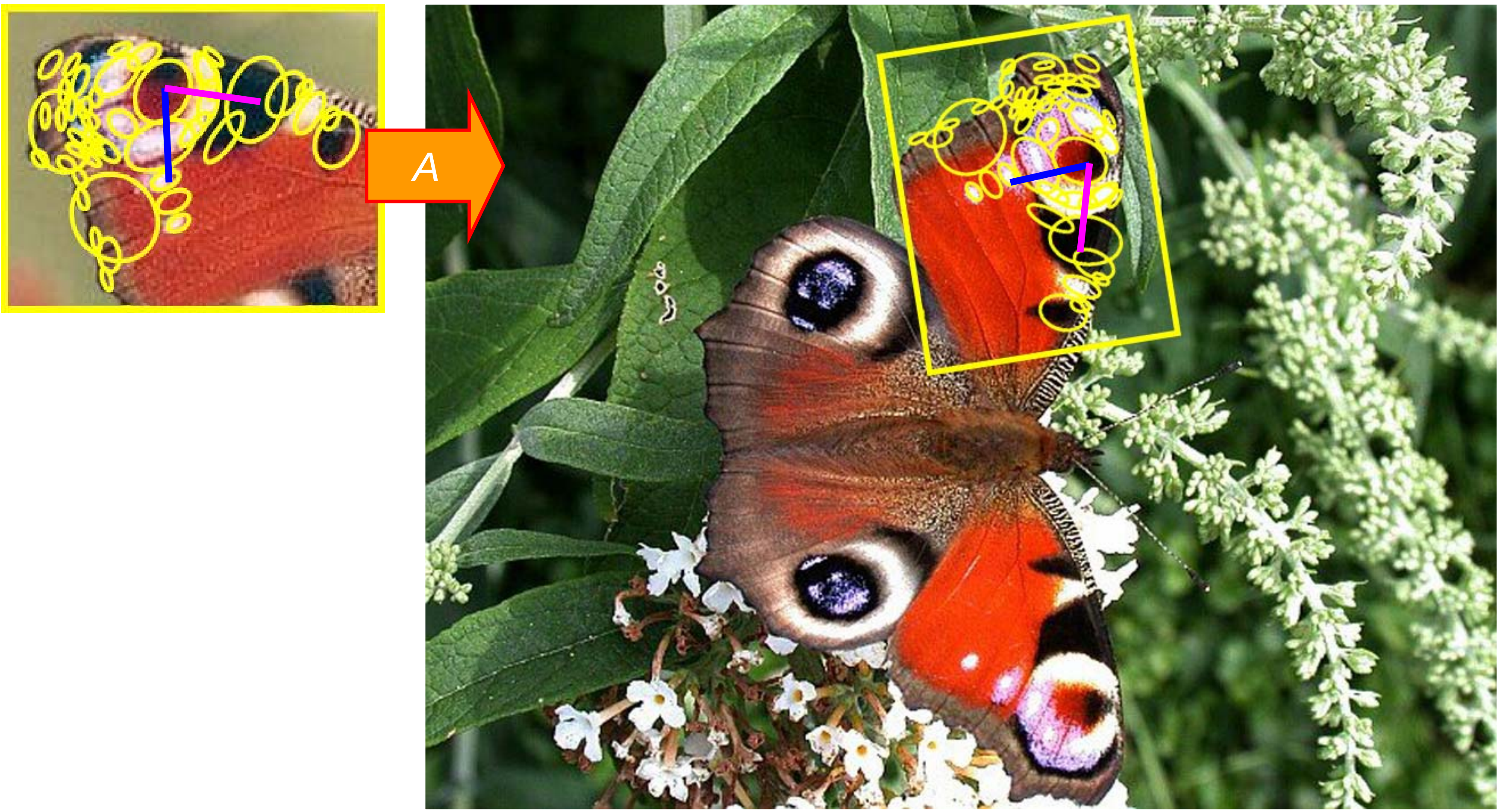
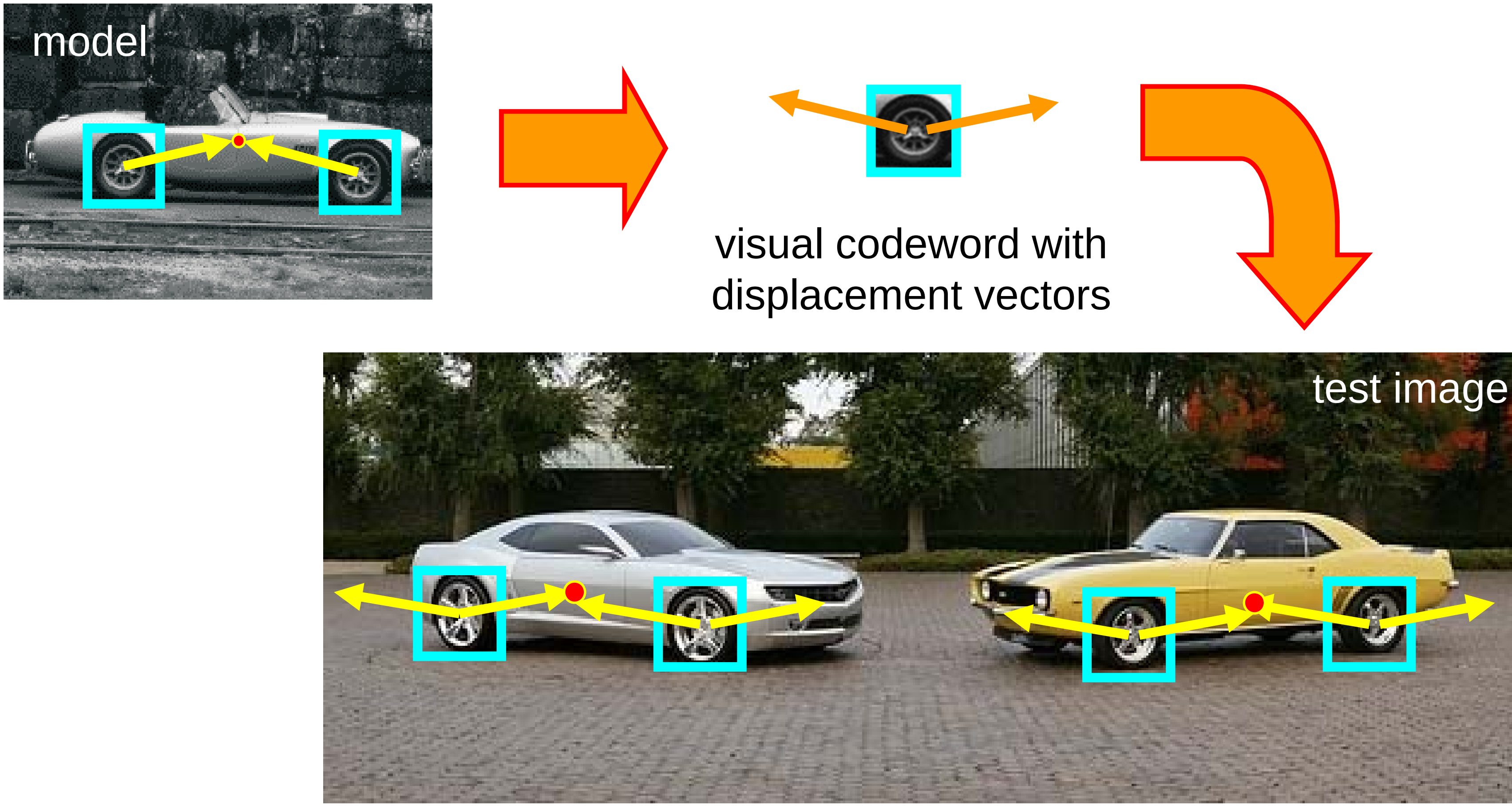


Strategy 3: Hough transform

- Recall: Generalized Hough transform

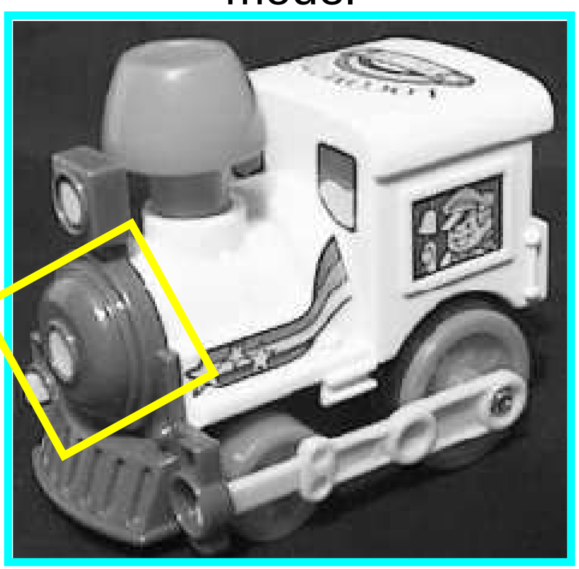


B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Strategy 3: Hough transform

- Suppose our features are adapted to scale and rotation
  - Then a single feature match provides an alignment hypothesis (translation, scale, orientation)

model

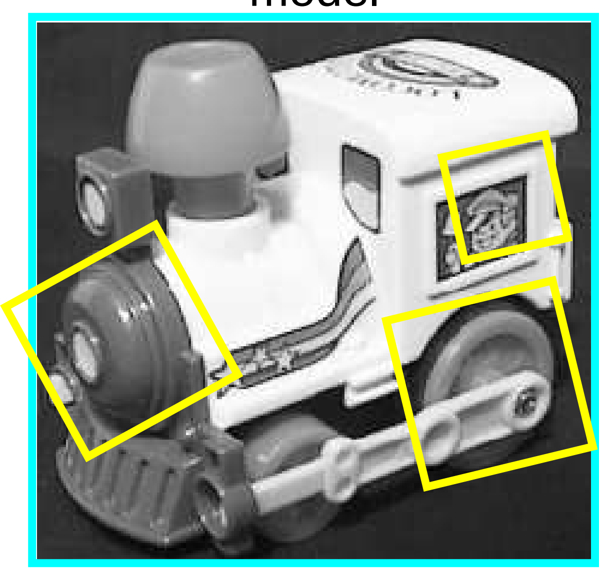


David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Strategy 3: Hough transform

- Suppose our features are adapted to scale and rotation
  - Then a single feature match provides an alignment hypothesis (translation, scale, orientation)
  - Of course, a hypothesis obtained from a single match is unreliable
  - Solution: let each match vote for its hypothesis in a Hough space with very coarse bins

model



David G. Lowe. **["Distinctive image features from scale-invariant keypoints."]()** *IJCV* 60 (2), pp. 91-110, 2004.

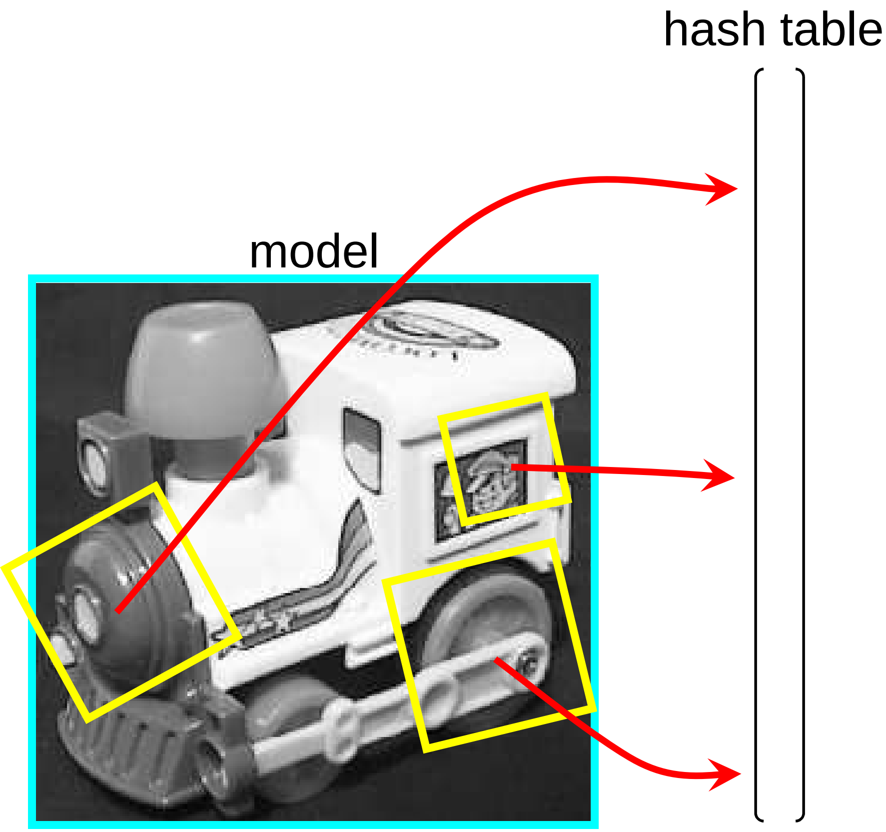
Hough transform details (D. Lowe's system)

- **Training phase:** For each model feature, record 2D location, scale, and orientation of model (relative to normalized feature frame)
- **Test phase:** Let each match between a test and a model feature vote in a 4D Hough space
  - Use broad bin sizes of 30 degrees for orientation, a factor of 2 for scale, and 0.25 times image size for location
  - Vote for two closest bins in each dimension
- Find all bins with at least three votes and perform geometric verification
  - Estimate least squares *affine* transformation
  - Use stricter thresholds on transformation residual
  - Search for additional features that agree with the alignment

David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

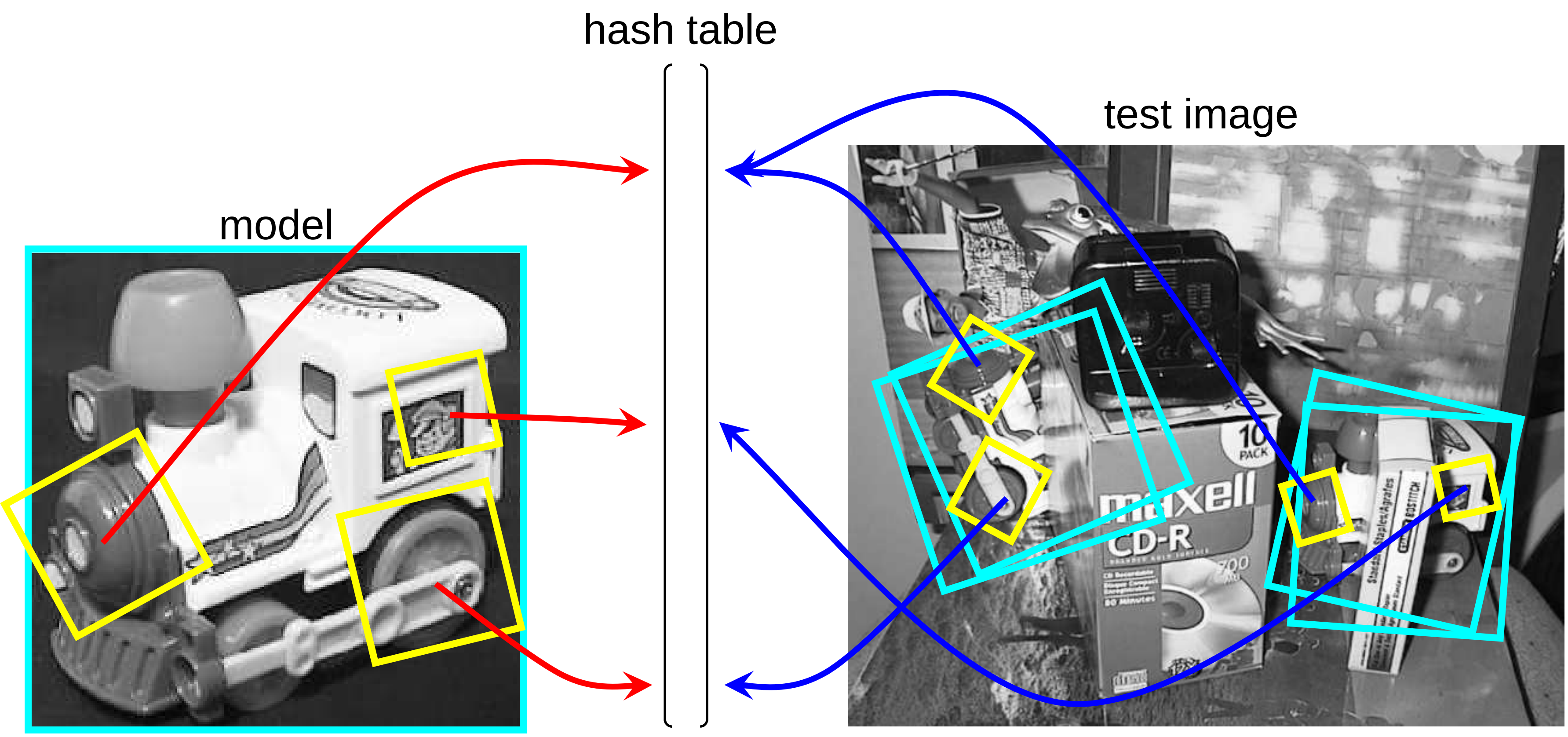
Strategy 4: Hashing

- Make each image feature into a low-dimensional “key” that indexes into a table of hypotheses



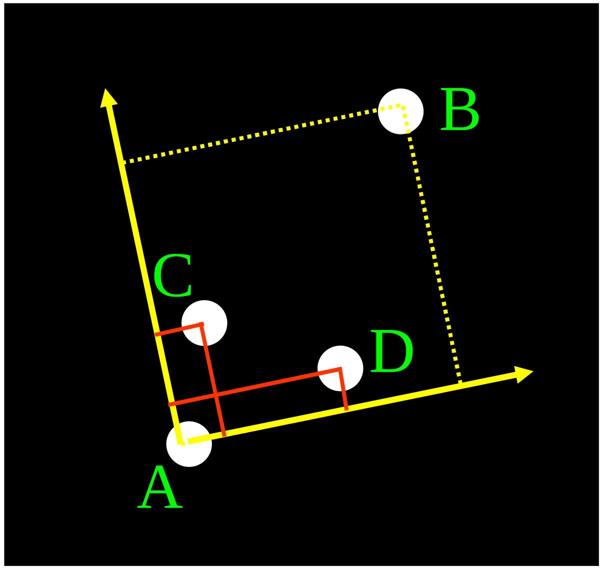
Strategy 4: Hashing

- Make each image feature into a low-dimensional "key" that indexes into a table of hypotheses
- Given a new test image, compute the hash keys for all features found in that image, access the table, and look for consistent hypotheses



Strategy 4: Hashing

- Make each image feature into a low-dimensional “key” that indexes into a table of hypotheses
- Given a new test image, compute the hash keys for all features found in that image, access the table, and look for consistent hypotheses
- This can even work when we don’t have any feature descriptors: we can take n-tuples of neighboring features and compute invariant hash codes from their geometric configurations

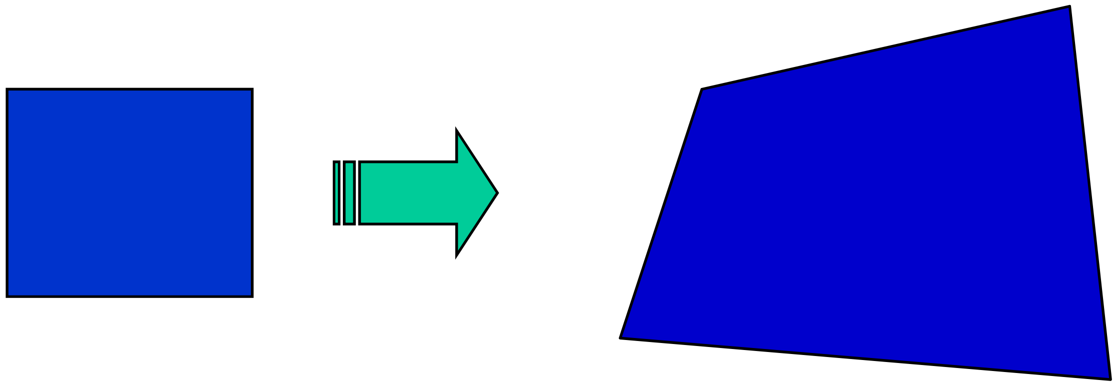


Application: Searching the sky

http://www.astrometry.net/

Beyond affine transformations

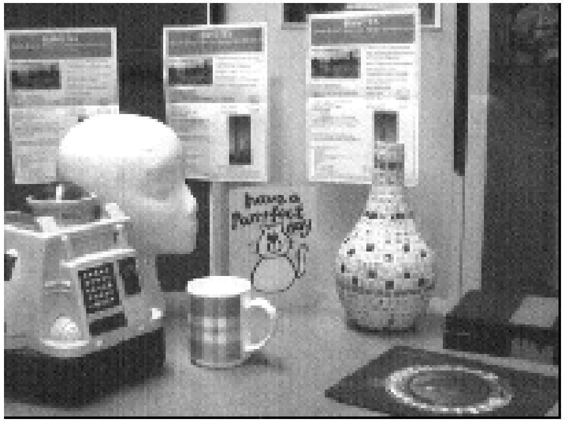
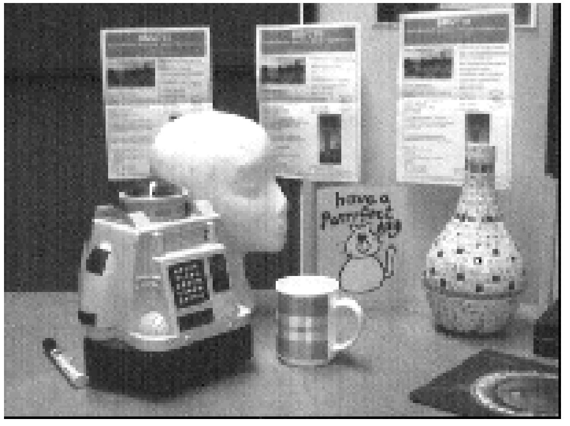
- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)



Homography

- The transformation between two views of a planar surface


- The transformation between images from two cameras that share the same center

Fitting a homography

- Recall: homogenenous coordinates

$$(x, y) \Rightarrow \left[\begin{array}{c} x \\ y \\ 1 \end{array}\right]$$

Converting *to* homogenenous image coordinates

$$\left[\begin{array}{c} x \\ y \\ w \end{array}\right] \Rightarrow (x/w, y/w)$$

Converting *from* homogenenous image coordinates

Fitting a homography

- Recall: homogenenous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogenenous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogenenous image coordinates

- Equation for homography:

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Fitting a homography

- Equation for homography:

$$\lambda \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{x}_i' = \mathbf{H}\,\mathbf{x}_i = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \mathbf{h}_3^T \end{bmatrix} \mathbf{x}_i$$

9 entries, 8 degrees of freedom
(scale is arbitrary)

$$\mathbf{x}_i' \times \mathbf{H}\,\mathbf{x}_i = 0$$

$$\mathbf{x}_i' \times \mathbf{H}\,\mathbf{x}_i = \begin{bmatrix} y_i'\,\mathbf{h}_3^T\mathbf{x}_i - \mathbf{h}_2^T\mathbf{x}_i \\ \mathbf{h}_1^T\mathbf{x}_i - x_i'\,\mathbf{h}_3^T\mathbf{x}_i \\ x_i'\,\mathbf{h}_2^T\mathbf{x}_i - y_i'\,\mathbf{h}_1^T\mathbf{x}_i \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -\mathbf{x}_i^T & y_i'\,\mathbf{x}_i^T \\ \mathbf{x}_i^T & 0^T & -x_i'\,\mathbf{x}_i^T \\ -y_i'\,\mathbf{x}_i^T & x_i'\,\mathbf{x}_i^T & 0^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

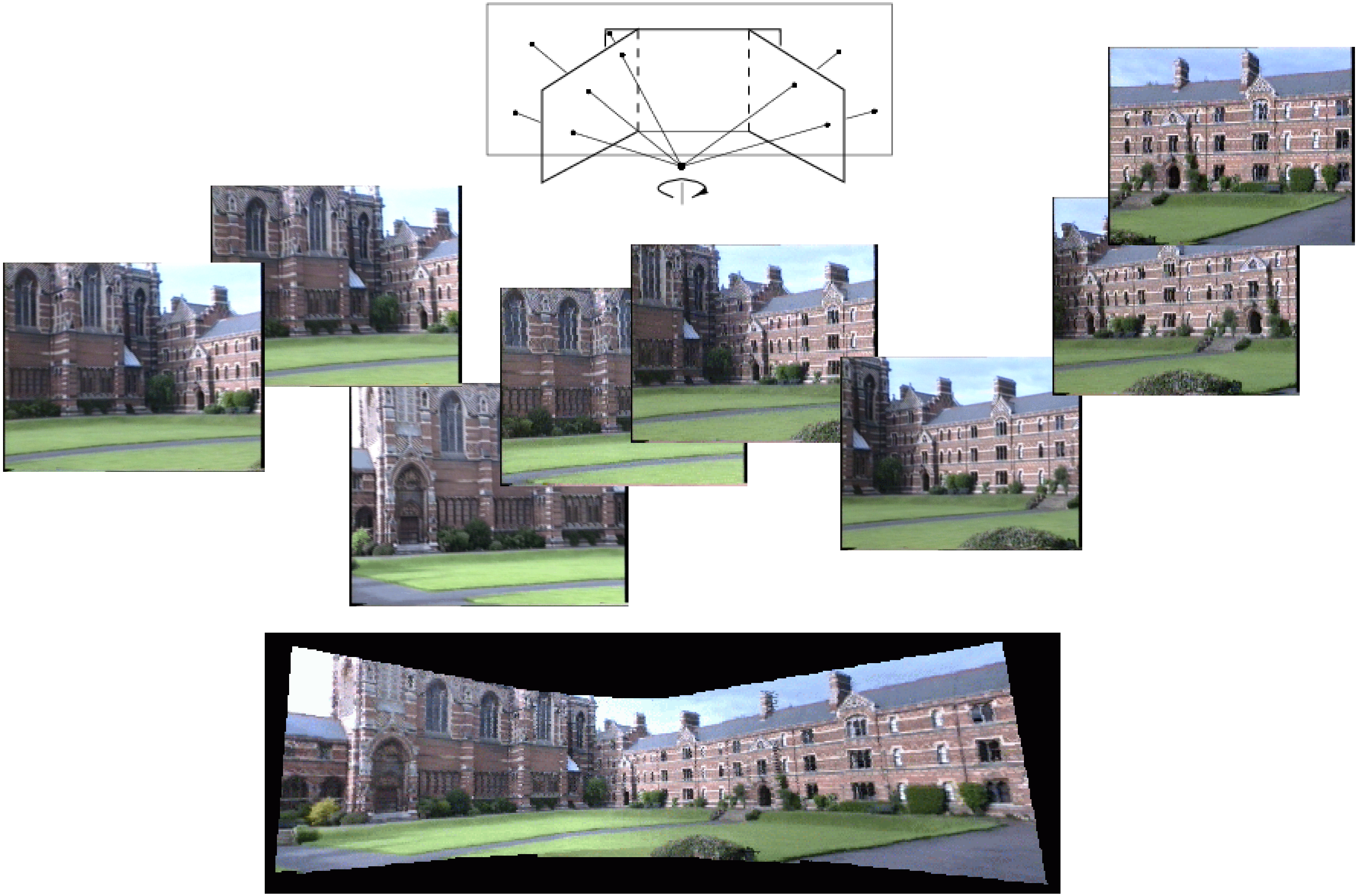
3 equations, only 2 linearly
independent

Direct linear transform

$$\begin{bmatrix} 0^T & \mathbf{x}_1^T & -y_1' \mathbf{x}_1^T \\ \mathbf{x}_1^T & 0^T & -x_1' \mathbf{x}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{x}_n^T & -y_n' \mathbf{x}_n^T \\ \mathbf{x}_n^T & 0^T & -x_n' \mathbf{x}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0 \qquad \mathbf{A}\,\mathbf{h} = 0$$

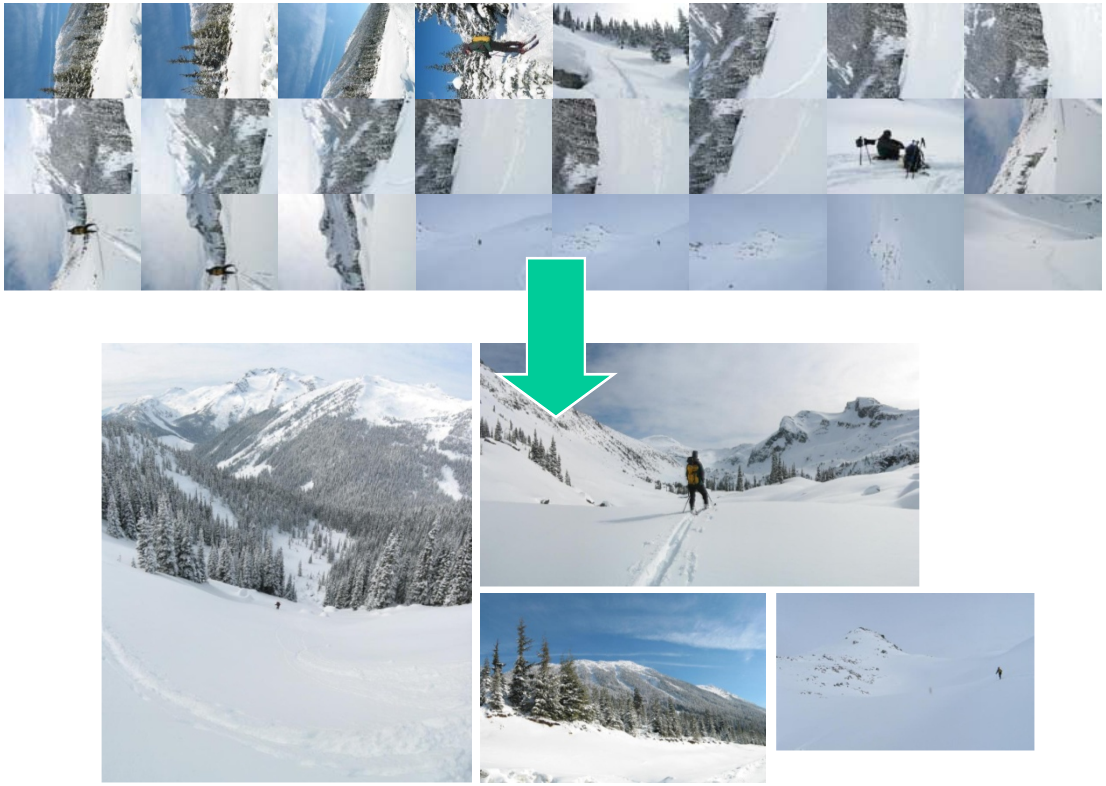
- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Four matches needed for a minimal solution (null space of 8x9 matrix)
- More than four: homogeneous least squares

Application: Panorama stitching



Recognizing panoramas

- Given contents of a camera memory card, automatically figure out which pictures go together and stitch them together into panoramas



M. Brown and D. Lowe, ***"Recognizing Panoramas,"*** ICCV 2003.
http://www.cs.ubc.ca/~mbrown/panorama/panorama.html

Issues in alignment-based applications

- Choosing the geometric alignment model
  - Tradeoff between “correctness” and robustness (also, efficiency)
- Choosing the descriptor
  - “Rich” imagery (natural images): high-dimensional patch-based descriptors (e.g., SIFT)
  - “Impoverished” imagery (e.g., star fields): need to create invariant geometric descriptors from k-tuples of point-based features
- Strategy for finding putative matches
  - Small number of images, one-time computation (e.g., panorama stitching): brute force search
  - Large database of model images, frequent queries: indexing or hashing
  - Heuristics for feature-space pruning of putative matches

Issues in alignment-based applications

- Choosing the geometric alignment model
- Choosing the descriptor
- Strategy for finding putative matches
- Hypothesis generation strategy
  - Relatively large inlier ratio: RANSAC
  - Small inlier ratio: locality constraints, Hough transform
- Hypothesis verification strategy
  - Size of consensus set, residual tolerance depend on inlier ratio and expected accuracy of the model
  - Possible refinement of geometric model
  - Dense verification

Next time: Single-view geometry