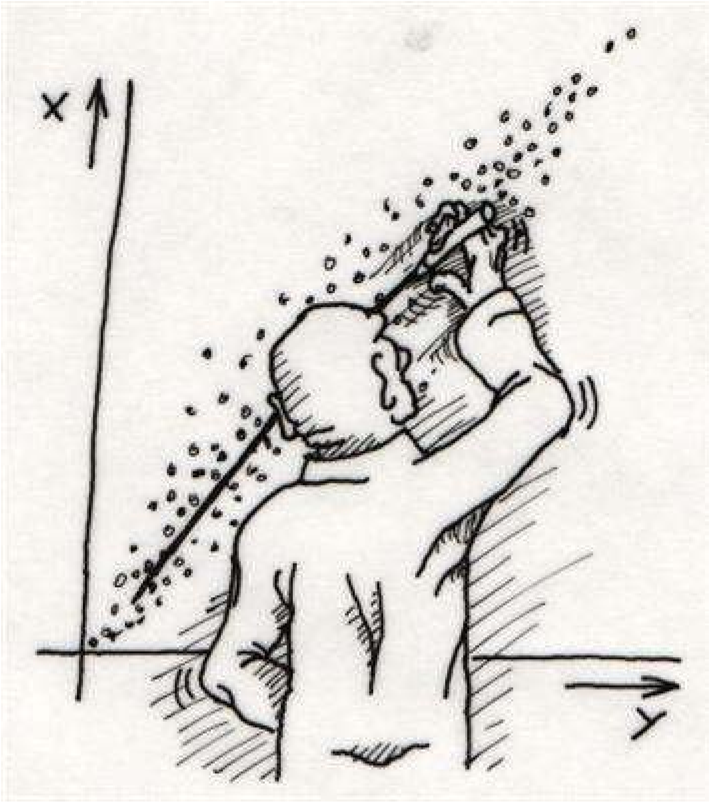
Fitting: The Hough transform

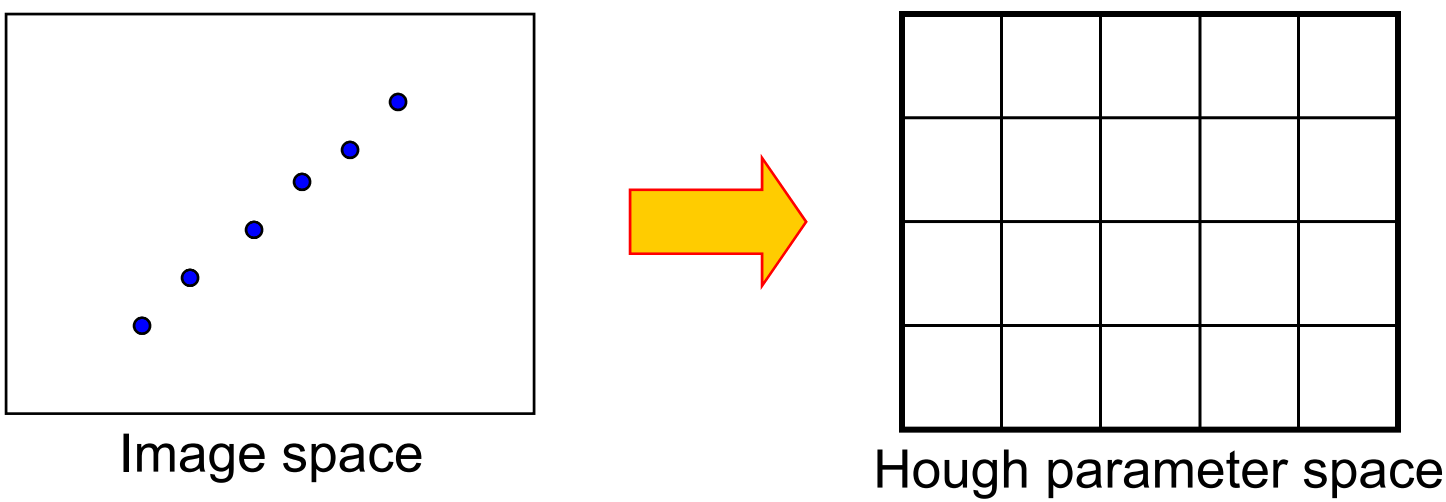


Voting schemes

- Let each feature vote for all the models that are compatible with it
- Hopefully the noise features will not vote consistently for any single model
- Missing data doesn’t matter as long as there are enough features remaining to agree on a good model

Hough transform

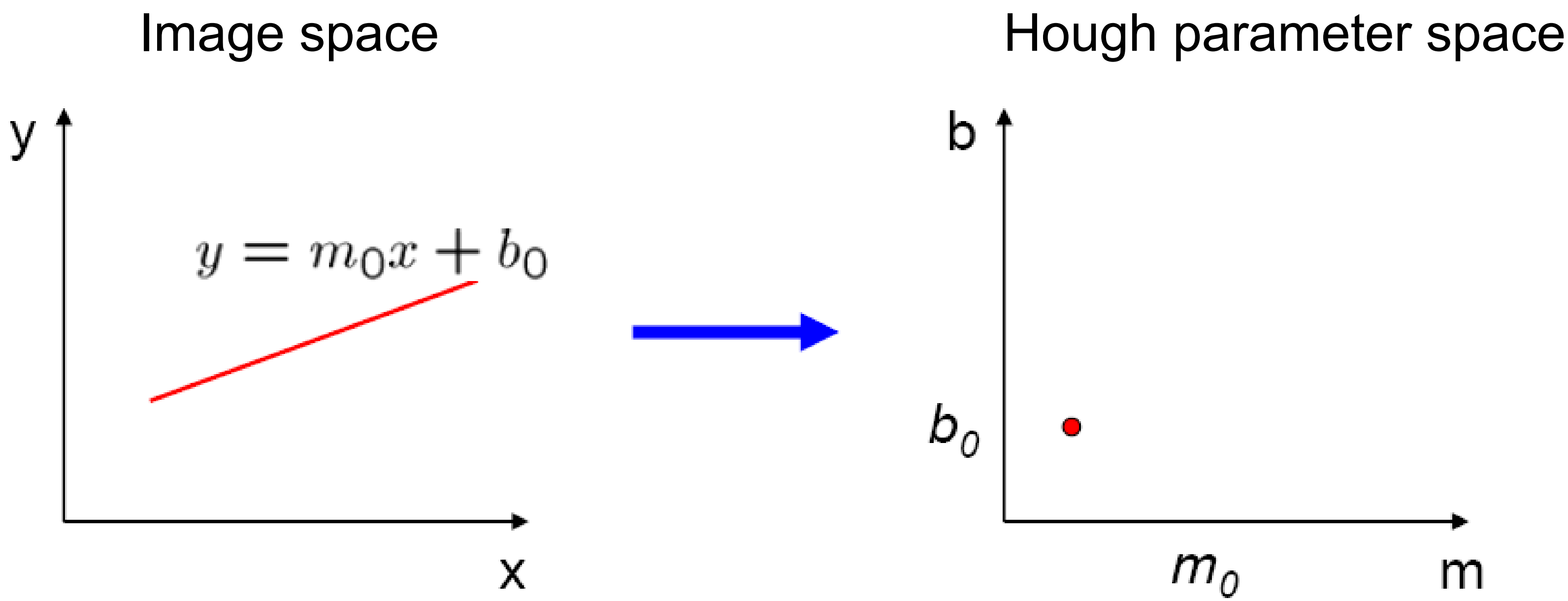
- An early type of voting scheme
- General outline:
  - Discretize parameter space into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes



P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Parameter space representation

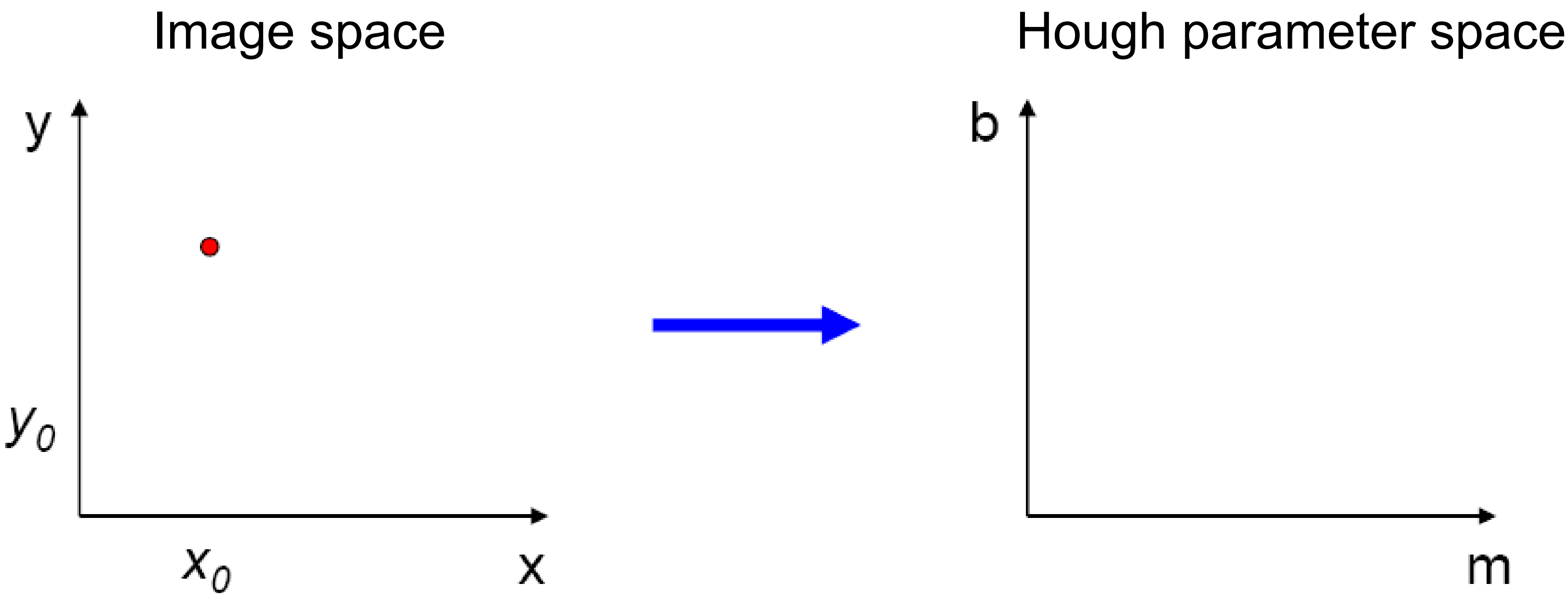
- A line in the image corresponds to a point in Hough space



Source: S. Seitz

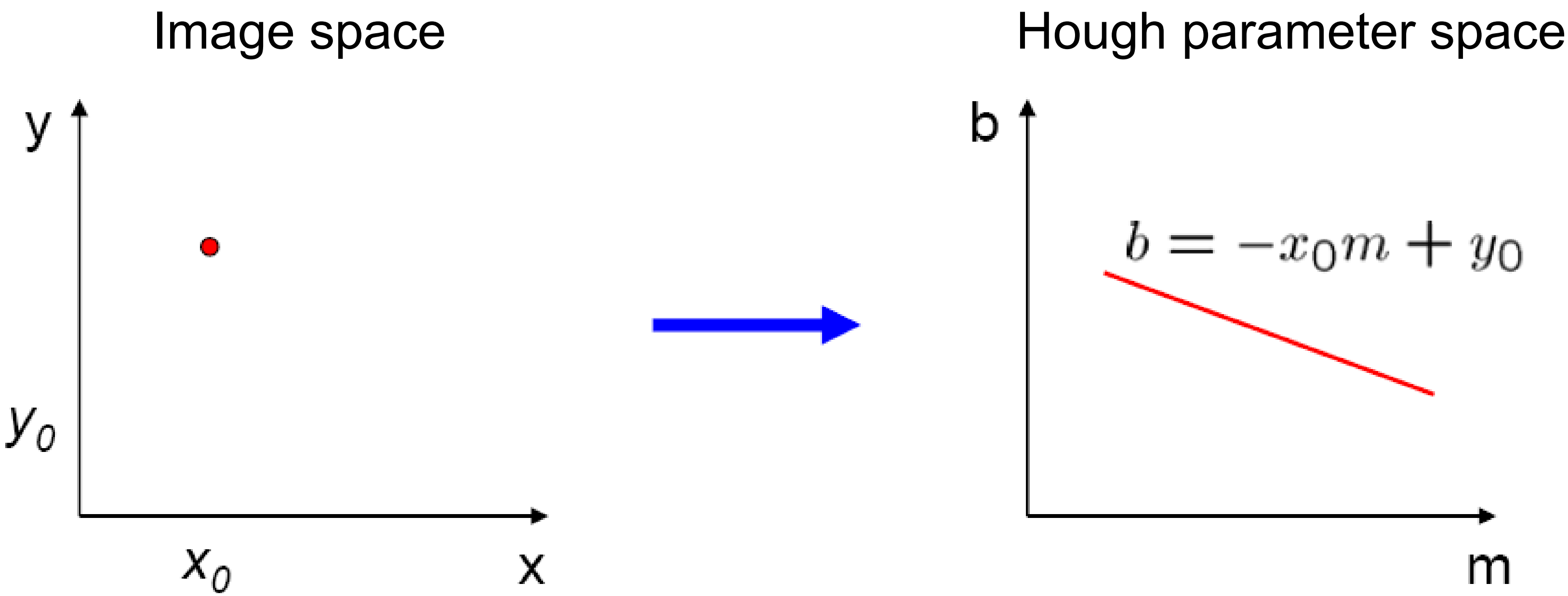
Parameter space representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?



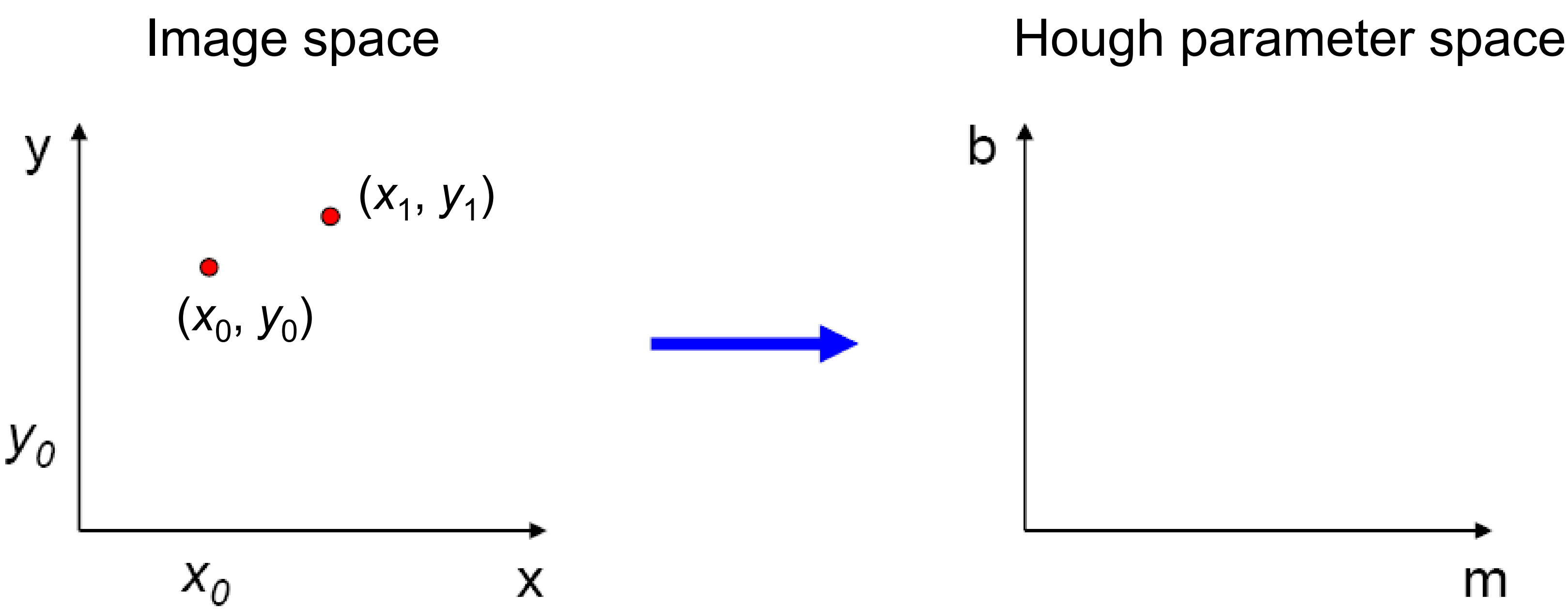
Parameter space representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?
  - Answer: the solutions of $b = -x_0m + y_0$
  - This is a line in Hough space



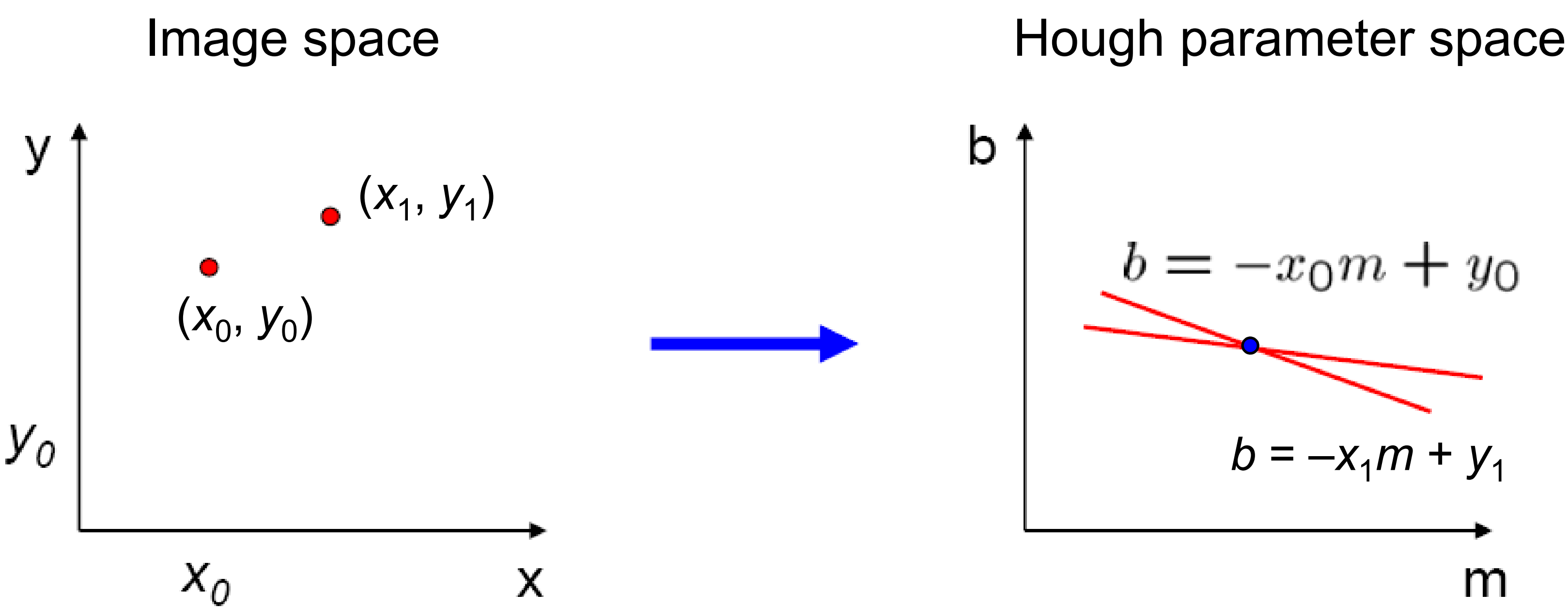
Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?



Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?
  - It is the intersection of the lines $b = -x_0m + y_0$ and $b = -x_1m + y_1$

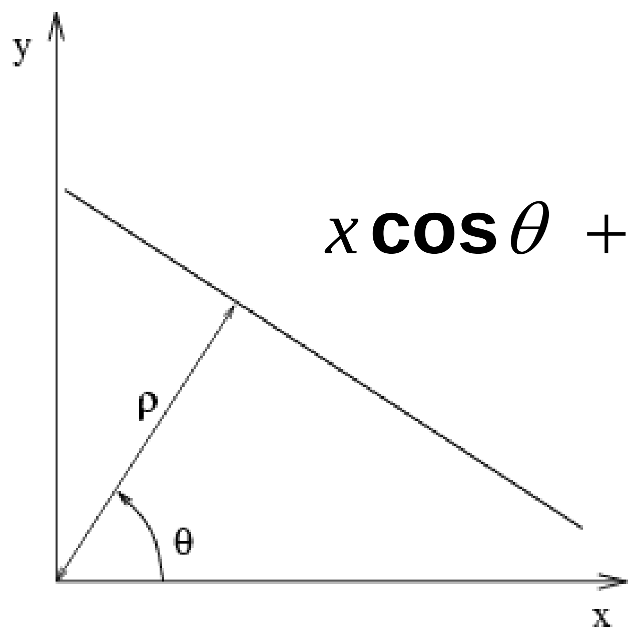


Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m

Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m
- Alternative: polar representation



$$x\,\mathbf{cos}\,\theta + y\,\mathbf{sin}\,\theta = \rho$$

Each point will add a sinusoid in the (θ,ρ) parameter space

Algorithm outline

- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image

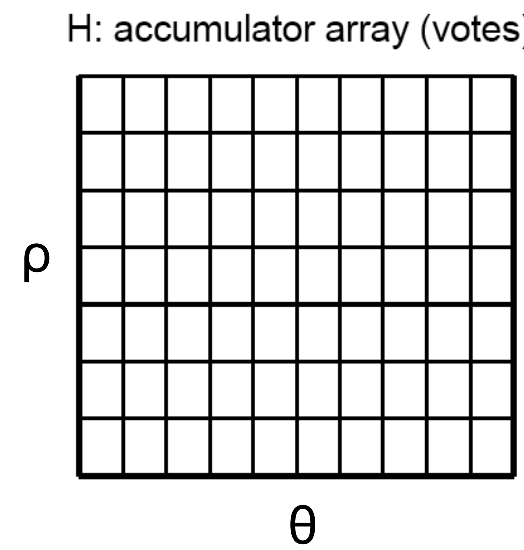
    For θ = 0 to 180

        $\rho = x \cos \theta + y \sin \theta$

        $H(\theta, \rho) = H(\theta, \rho) + 1$

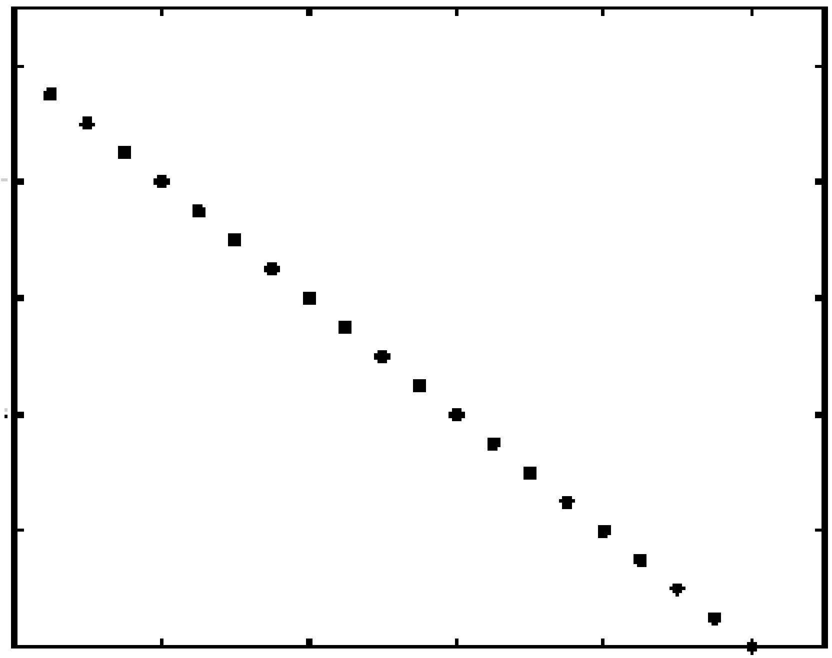
    end

  end

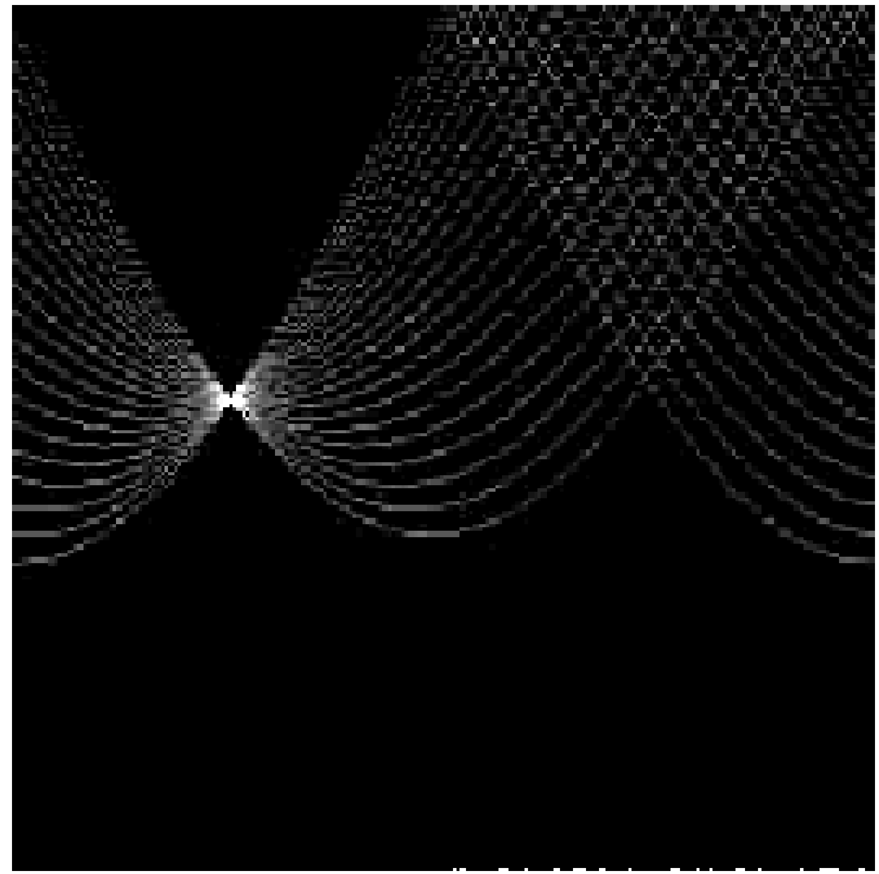


- Find the value(s) of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
    - The detected line in the image is given by $\rho = x \cos \theta + y \sin \theta$

Basic illustration



features

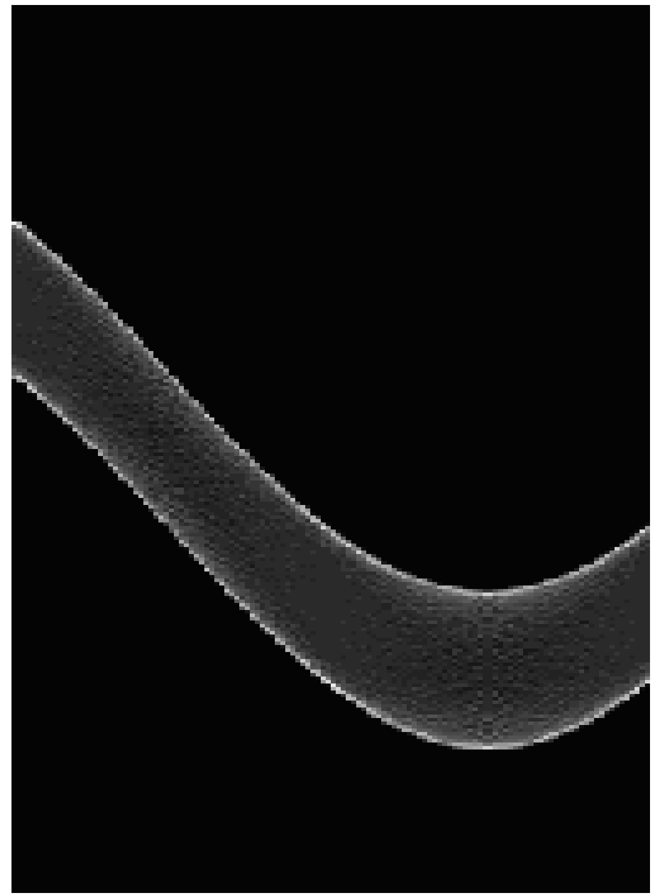


votes

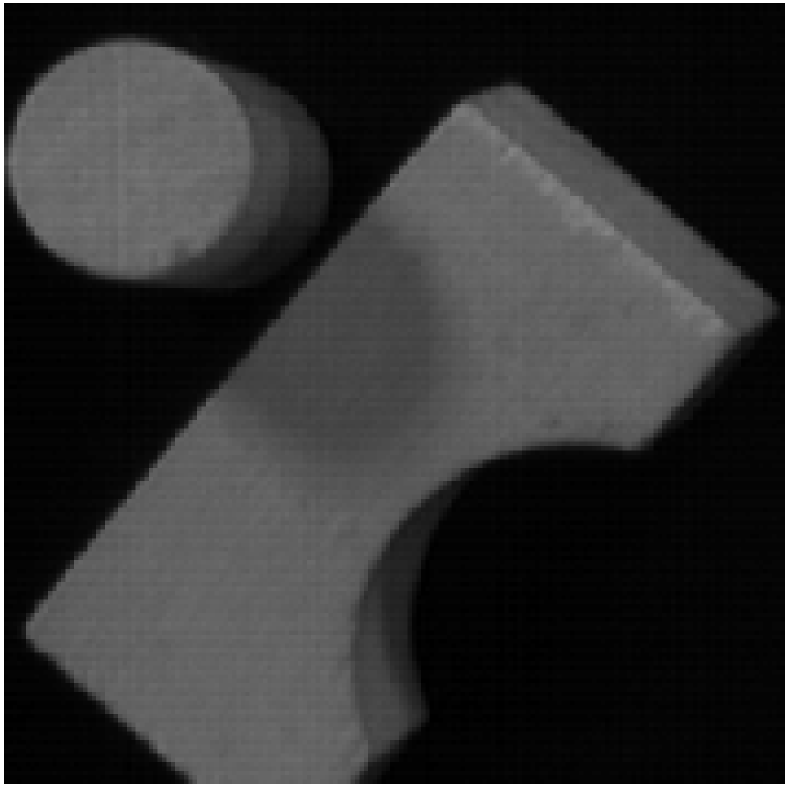
Other shapes

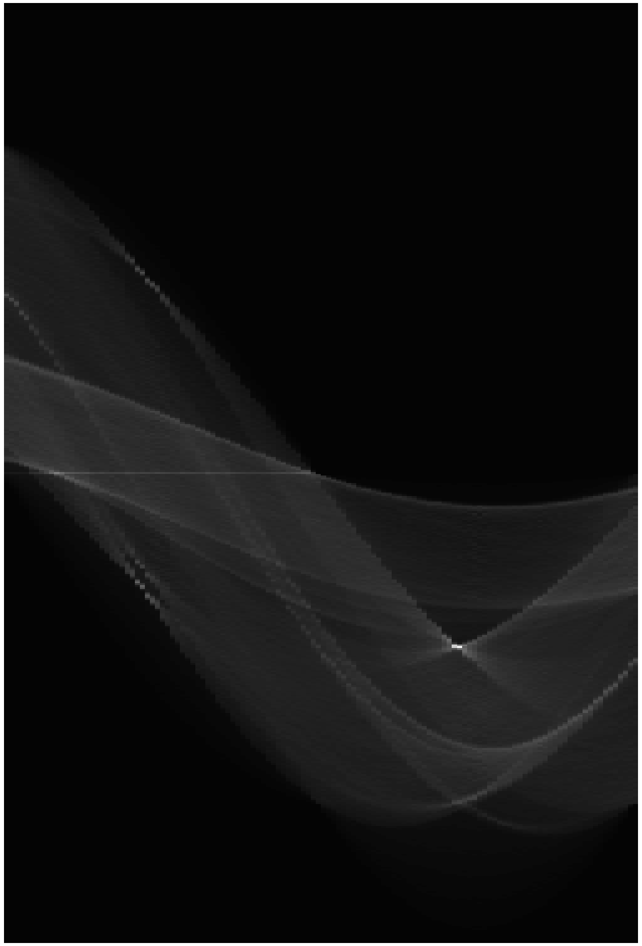
Square

Circle

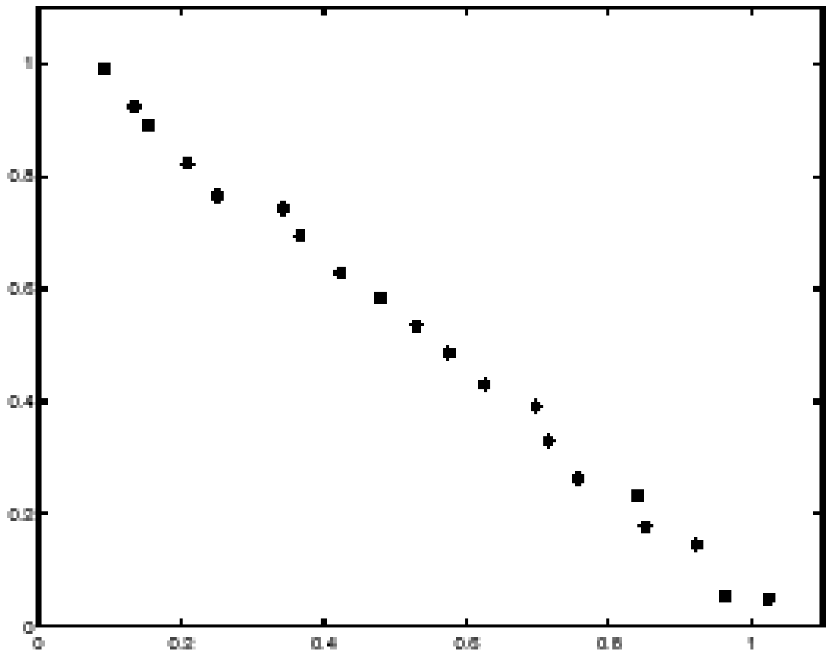


Several lines



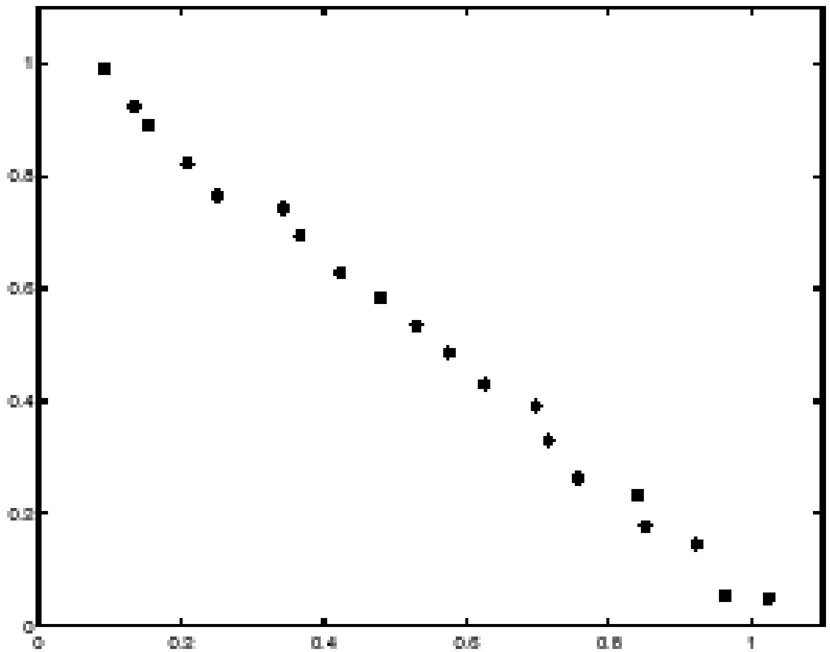


Effect of noise

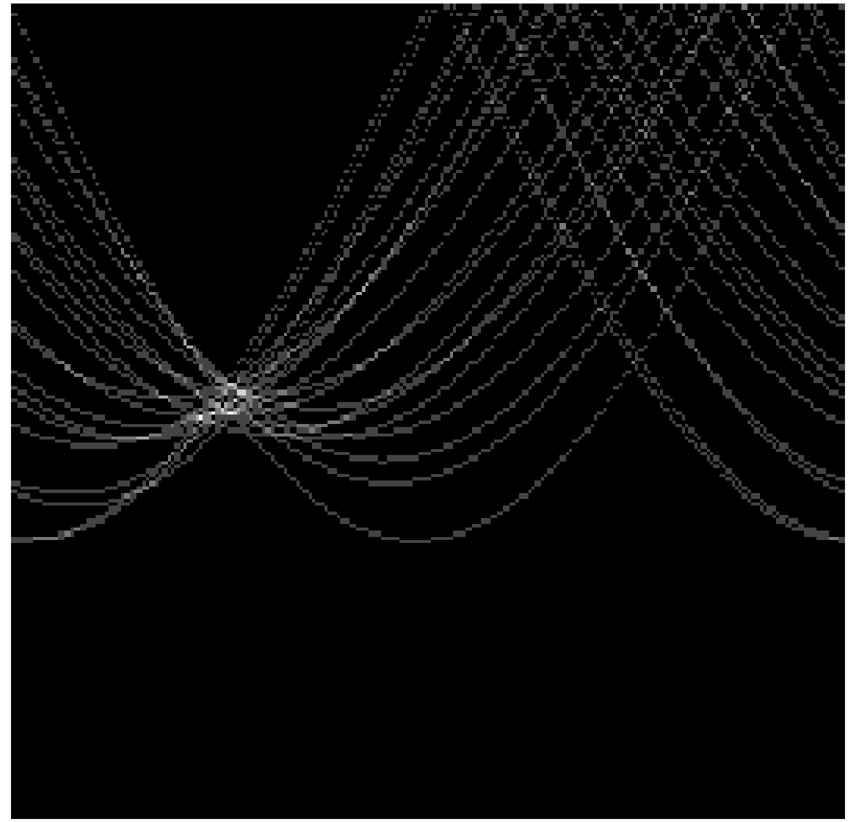


features

Effect of noise



features

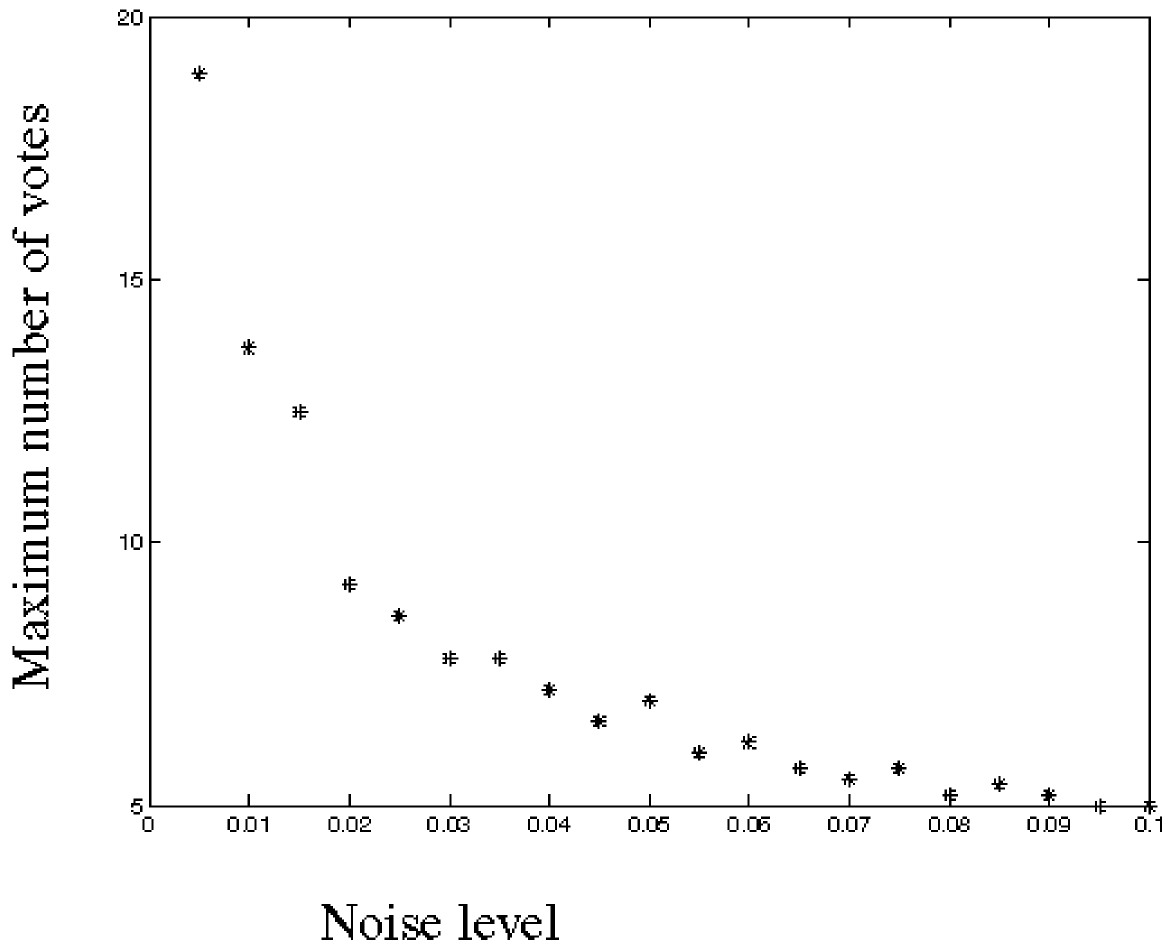


votes

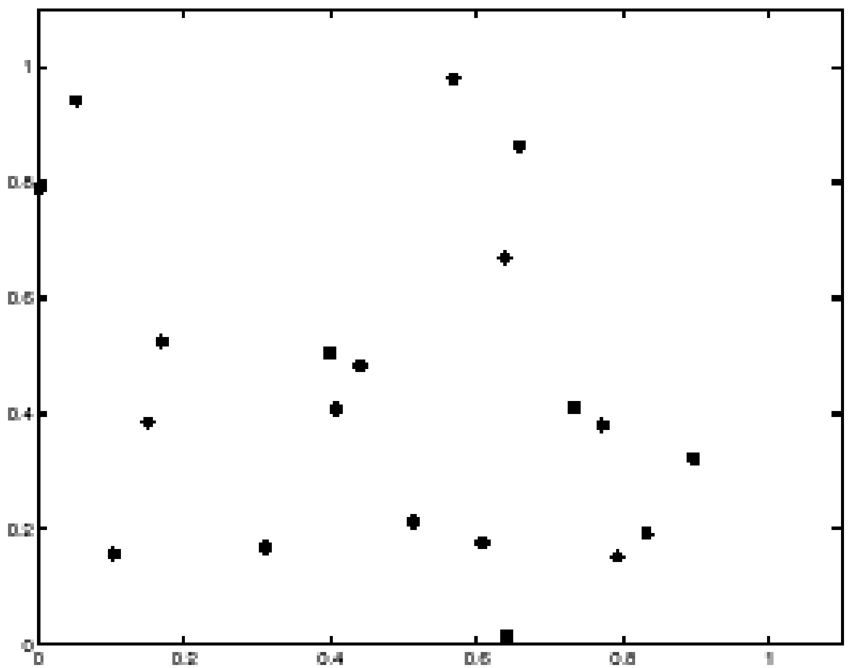
Peak gets fuzzy and hard to locate

Effect of noise

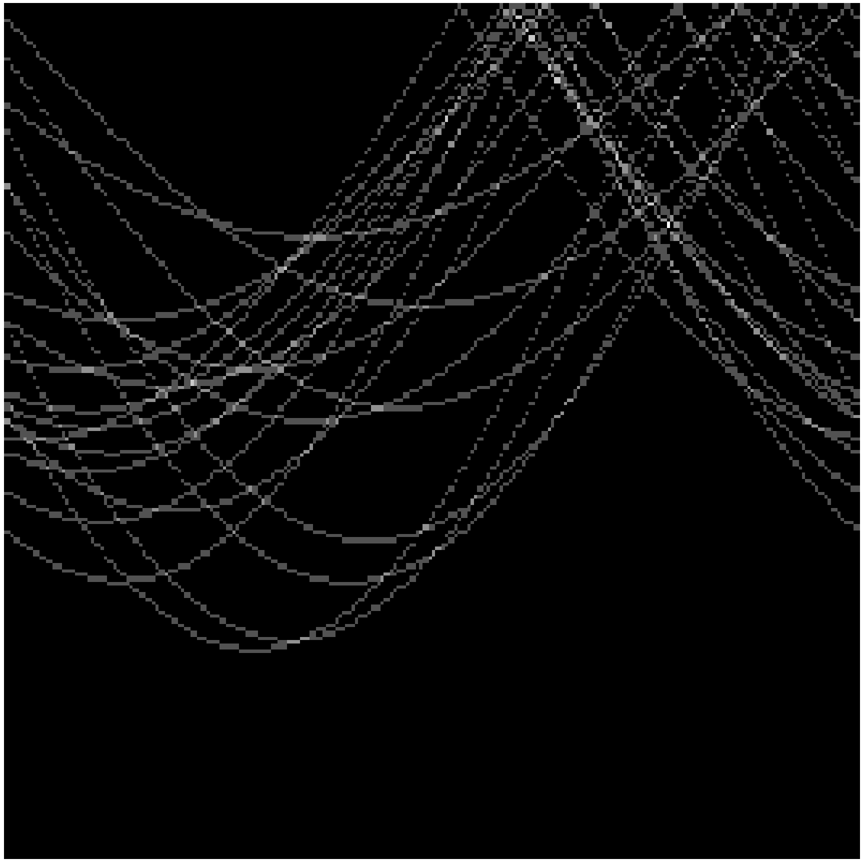
- Number of votes for a line of 20 points with increasing noise:



Random points



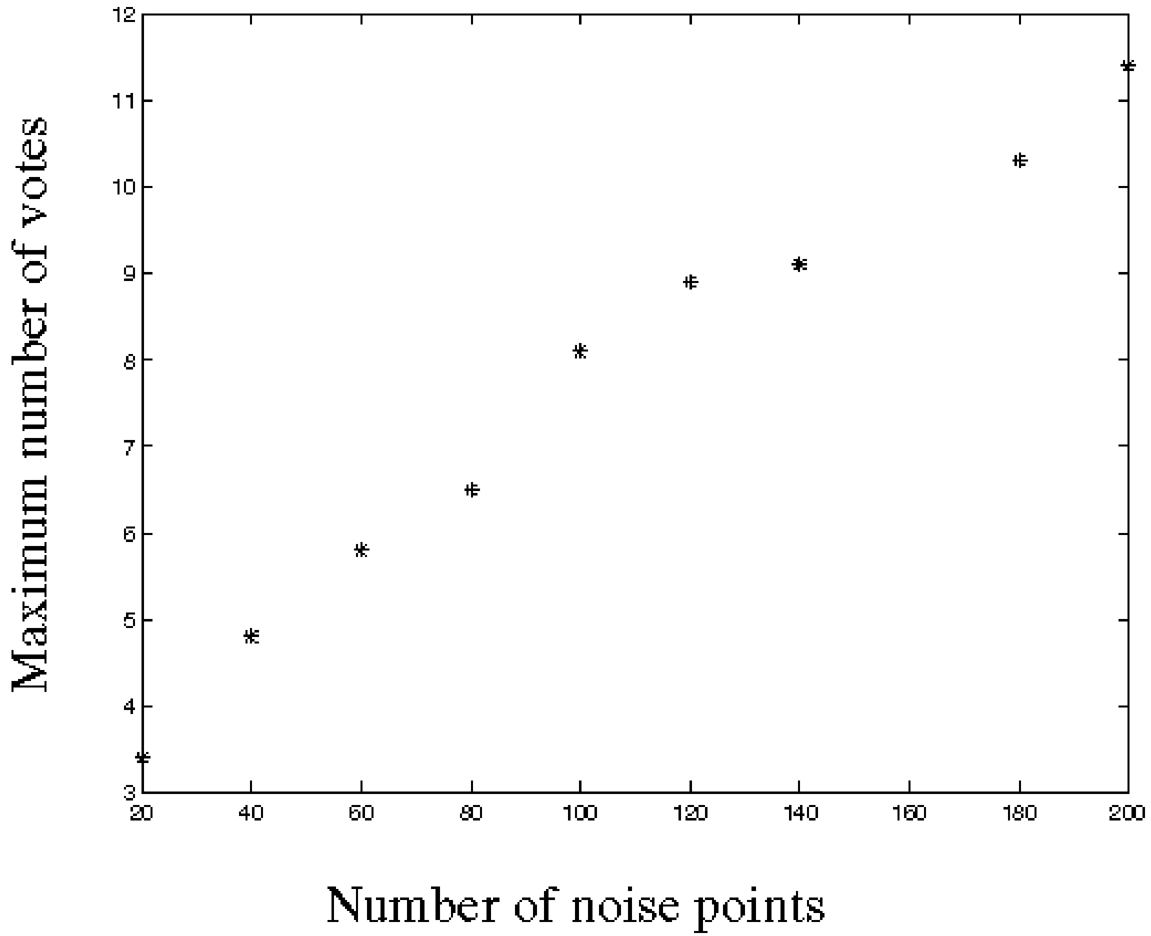
features


votes

Uniform noise can lead to spurious peaks in the array

Random points

- As the level of uniform noise increases, the maximum number of votes increases too:



Practical details

- Try to get rid of irrelevant features
  - Take only edge points with significant gradient magnitude
- Choose a good grid / discretization
  - Too coarse: large votes obtained when too many different lines correspond to a single bucket
  - Too fine: miss lines because some points that are not exactly collinear cast votes for different buckets
- Increment neighboring bins (smoothing in accumulator array)
- Who belongs to which line?
  - Tag the votes

Hough transform: Pros

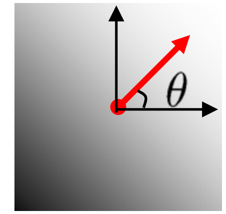
- Can deal with non-locality and occlusion
- Can detect multiple instances of a model in a single pass
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin

Hough transform: Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- It’s hard to pick a good grid size

Extension: Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

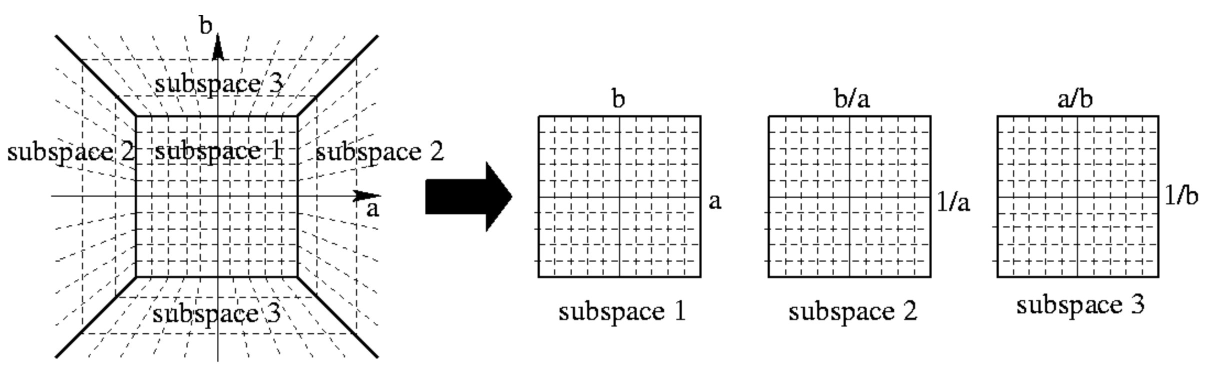
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- Modified Hough transform:

For each edge point (x,y)
  θ = gradient orientation at (x,y)
  ρ = x cos θ + y sin θ
  H(θ, ρ) = H(θ, ρ) + 1
end

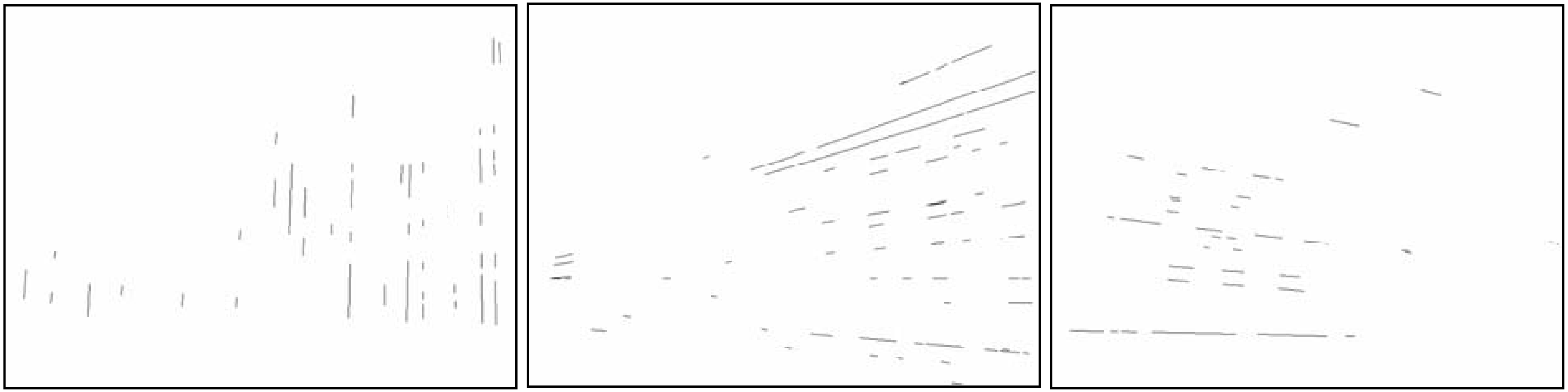
Extension: Cascaded Hough transform

- Let's go back to the original (m,b) parametrization
- A line in the image maps to a pencil of lines in the Hough space
- What do we get with parallel lines or a pencil of lines?
  - Collinear peaks in the Hough space!
- So we can apply a Hough transform to the output of the first Hough transform to find vanishing points
- Issue: dealing with unbounded parameter space



T. Tuytelaars, M. Proesmans, L. Van Gool ***"The cascaded Hough transform,"*** ***ICIP, vol. II, pp. 736-739, 1997.***

Cascaded Hough transform

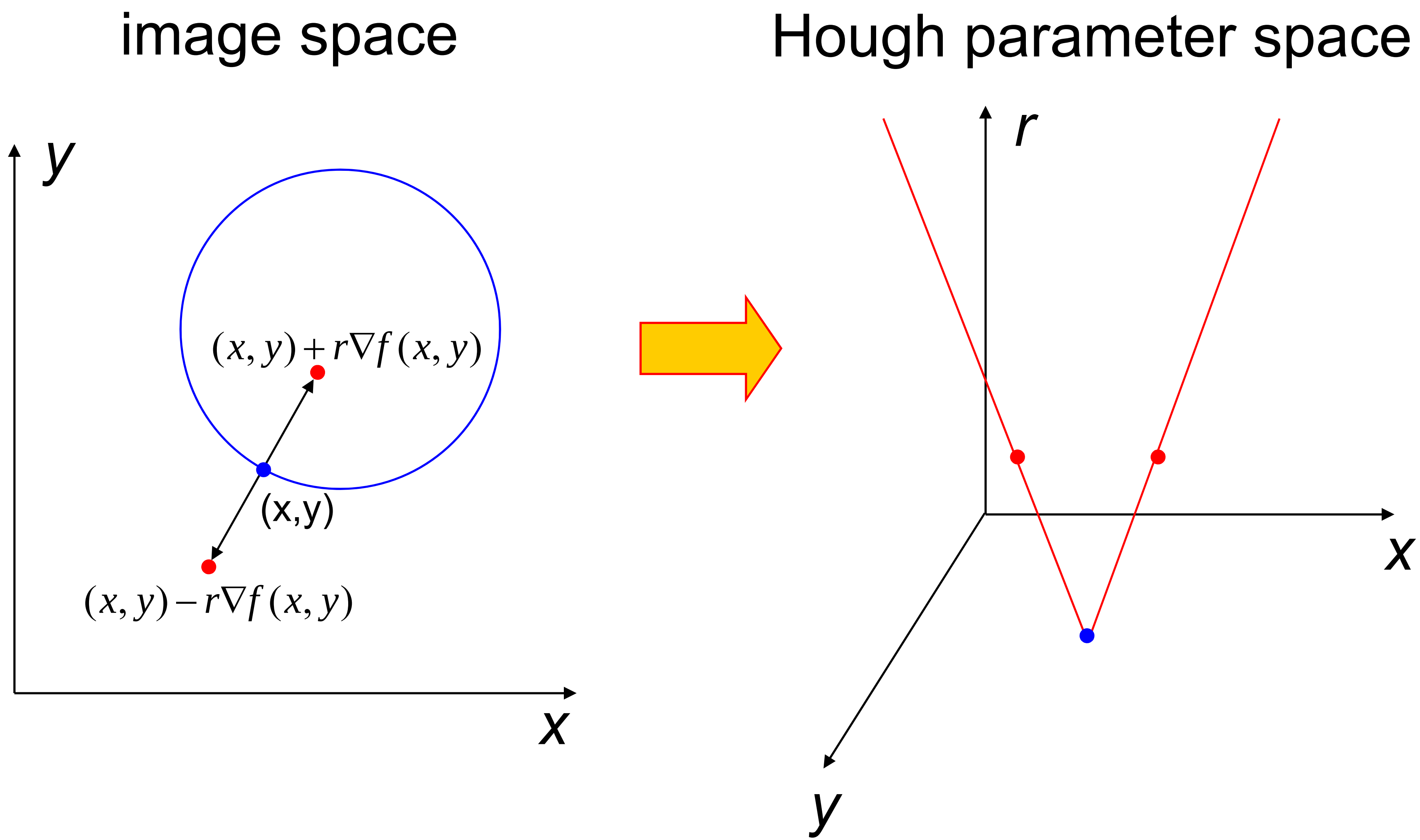


T. Tuytelaars, M. Proesmans, L. Van Gool **_"The cascaded Hough transform,"_** ***ICIP, vol. II, pp. 736-739, 1997.***

Hough transform for circles

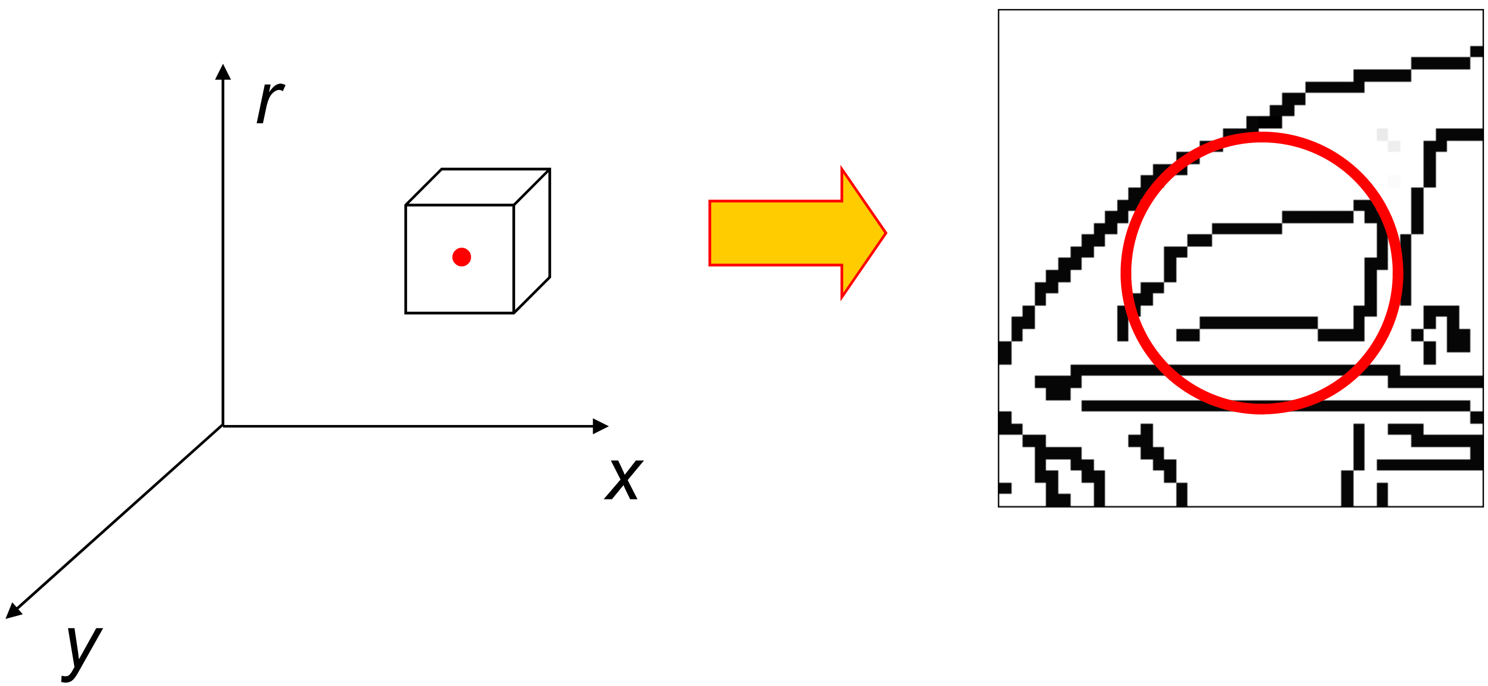
- How many dimensions will the parameter space have?
- Given an oriented edge point, what are all possible bins that it can vote for?

Hough transform for circles



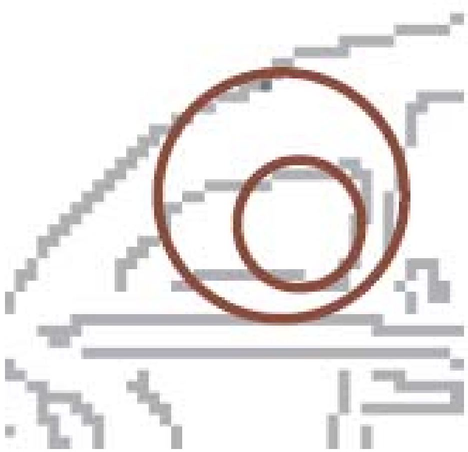
Hough transform for circles

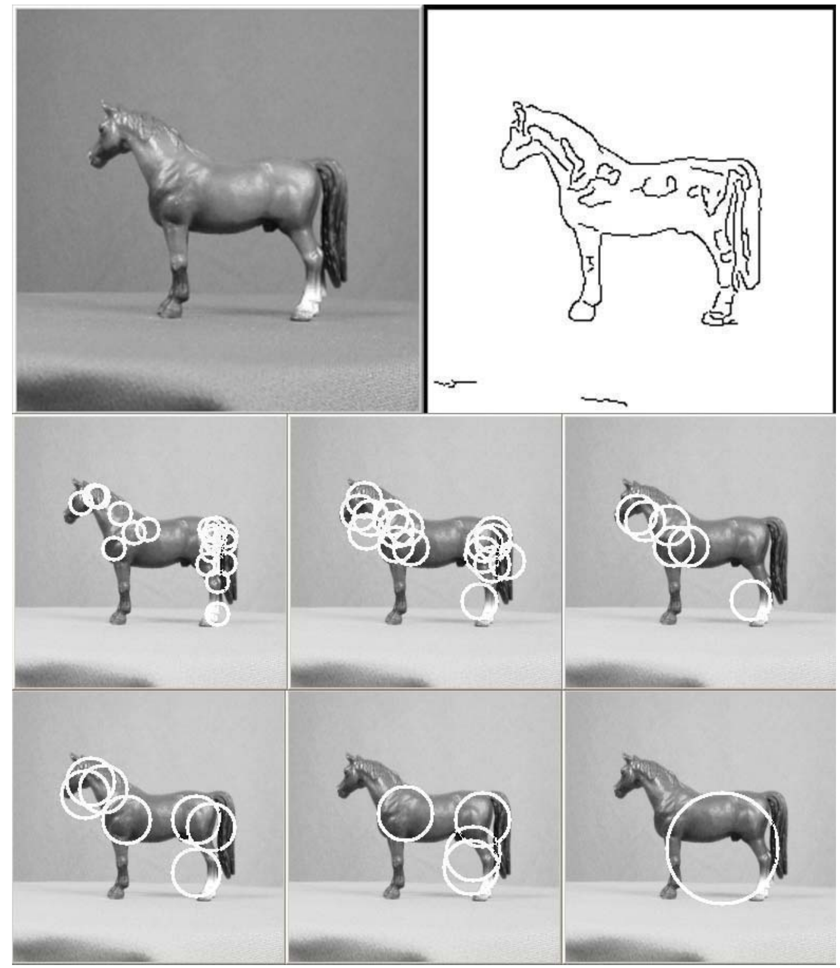
- Conceptually equivalent procedure: for each (x,y,r), draw the corresponding circle in the image and compute its "support"



What is more efficient: going from the image space to the parameter space or vice versa?

Application in recognition





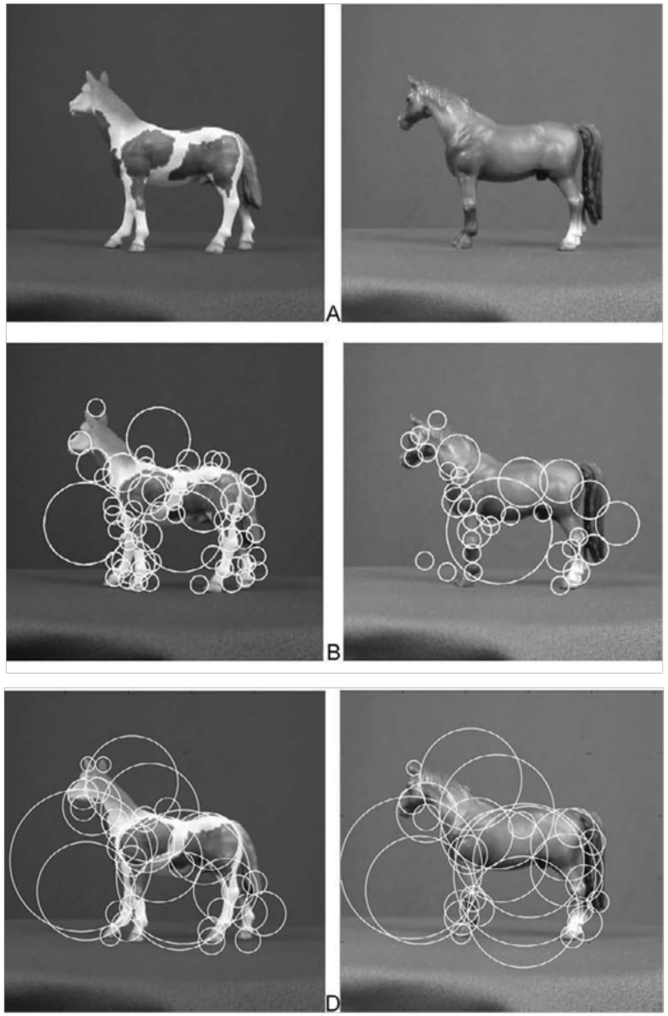
F. Jurie and C. Schmid, *Scale-invariant shape features for recognition of object categories*, CVPR 2004

Hough circles vs. Laplacian blobs

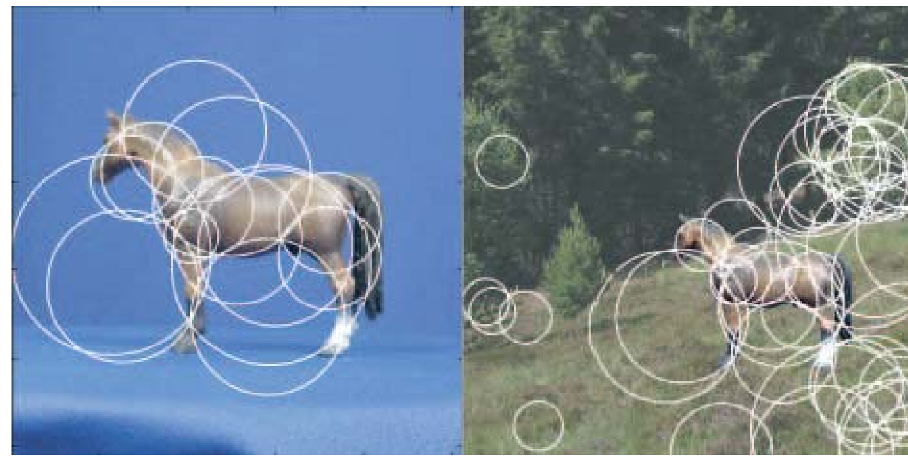
Original images

Laplacian circles

Hough-like circles



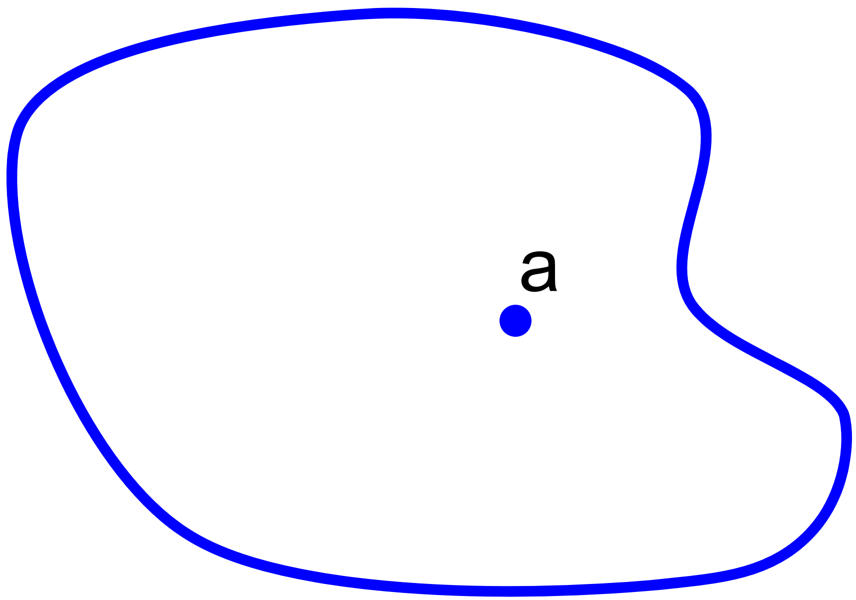
Robustness to scale and clutter



F. Jurie and C. Schmid, *Scale-invariant shape features for recognition of object categories*, CVPR 2004

Generalized Hough transform

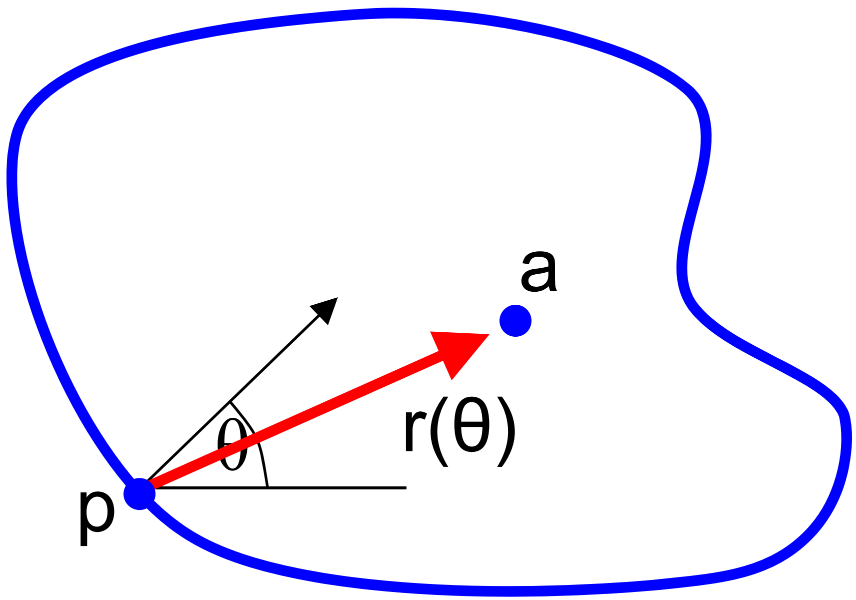
- We want to find a shape defined by its boundary points and a reference point



D. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, Pattern Recognition 13(2), 1981, pp. 111-122.

Generalized Hough transform

- We want to find a shape defined by its boundary points and a reference point
- For every boundary point p, we can compute the displacement vector r = a – p as a function of gradient orientation θ



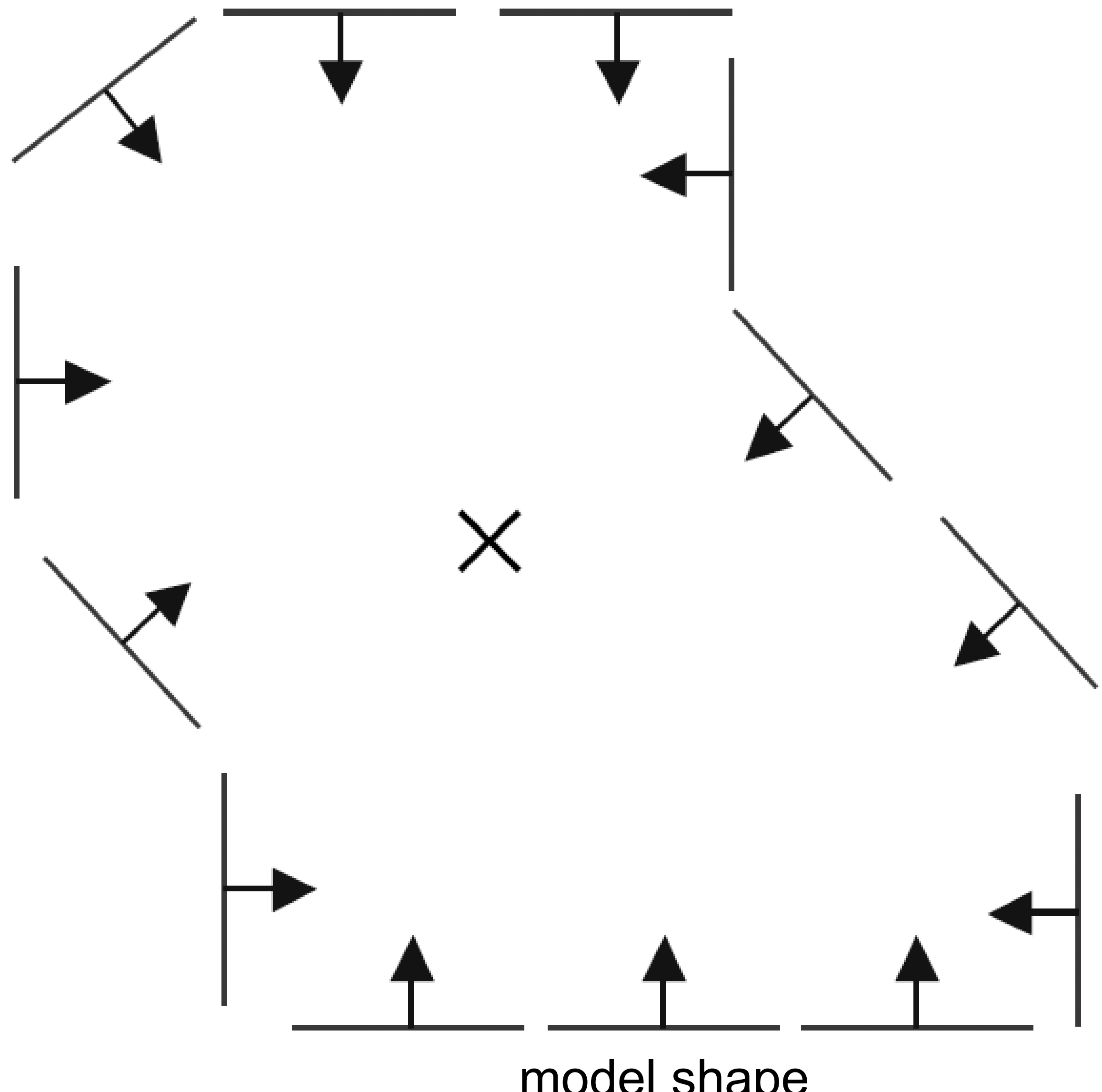
D. Ballard, [Generalizing the Hough Transform to Detect Arbitrary Shapes](), Pattern Recognition 13(2), 1981, pp. 111-122.

Generalized Hough transform

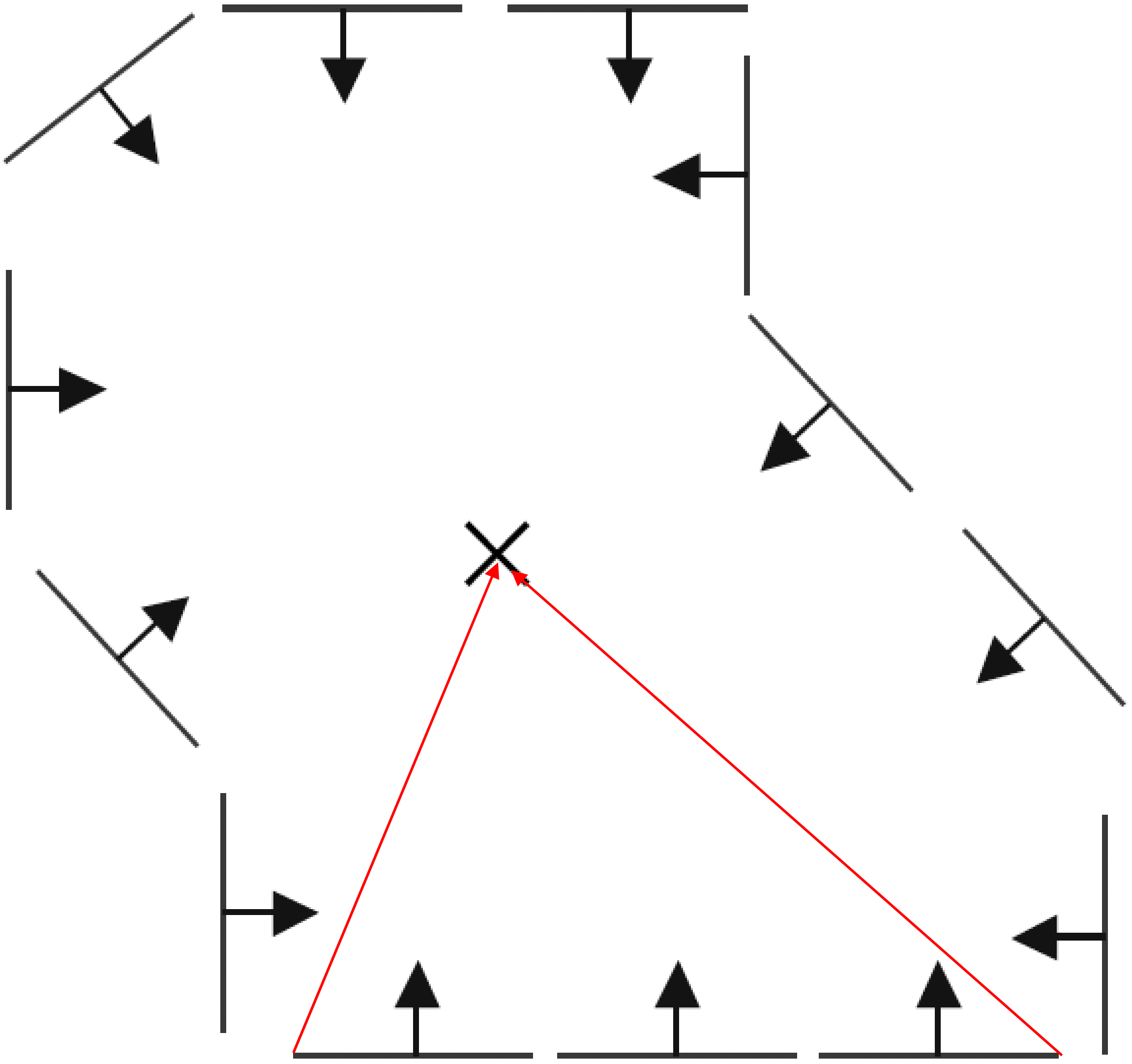
- For model shape: construct a table indexed by θ storing displacement vectors r as function of gradient direction
- Detection: For each edge point *p* with gradient orientation θ:
  - Retrieve all *r* indexed with θ
  - For each $r(\theta)$, put a vote in the Hough space at $p + r(\theta)$
- Peak in this Hough space is reference point with most supporting edges
- Assumption: translation is the only transformation here, i.e., orientation and scale are fixed

Source: K. Grauman

Example

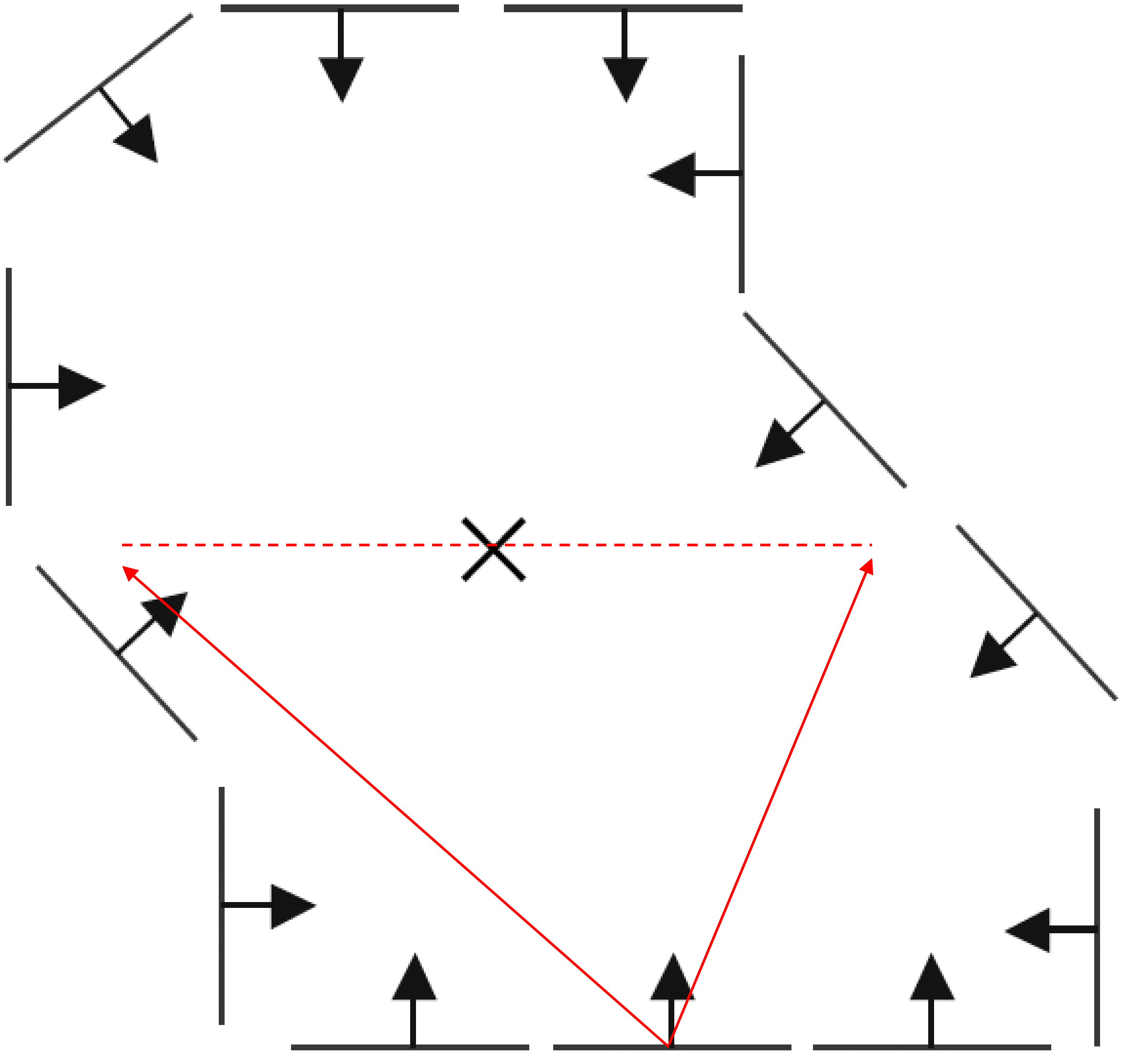


Example

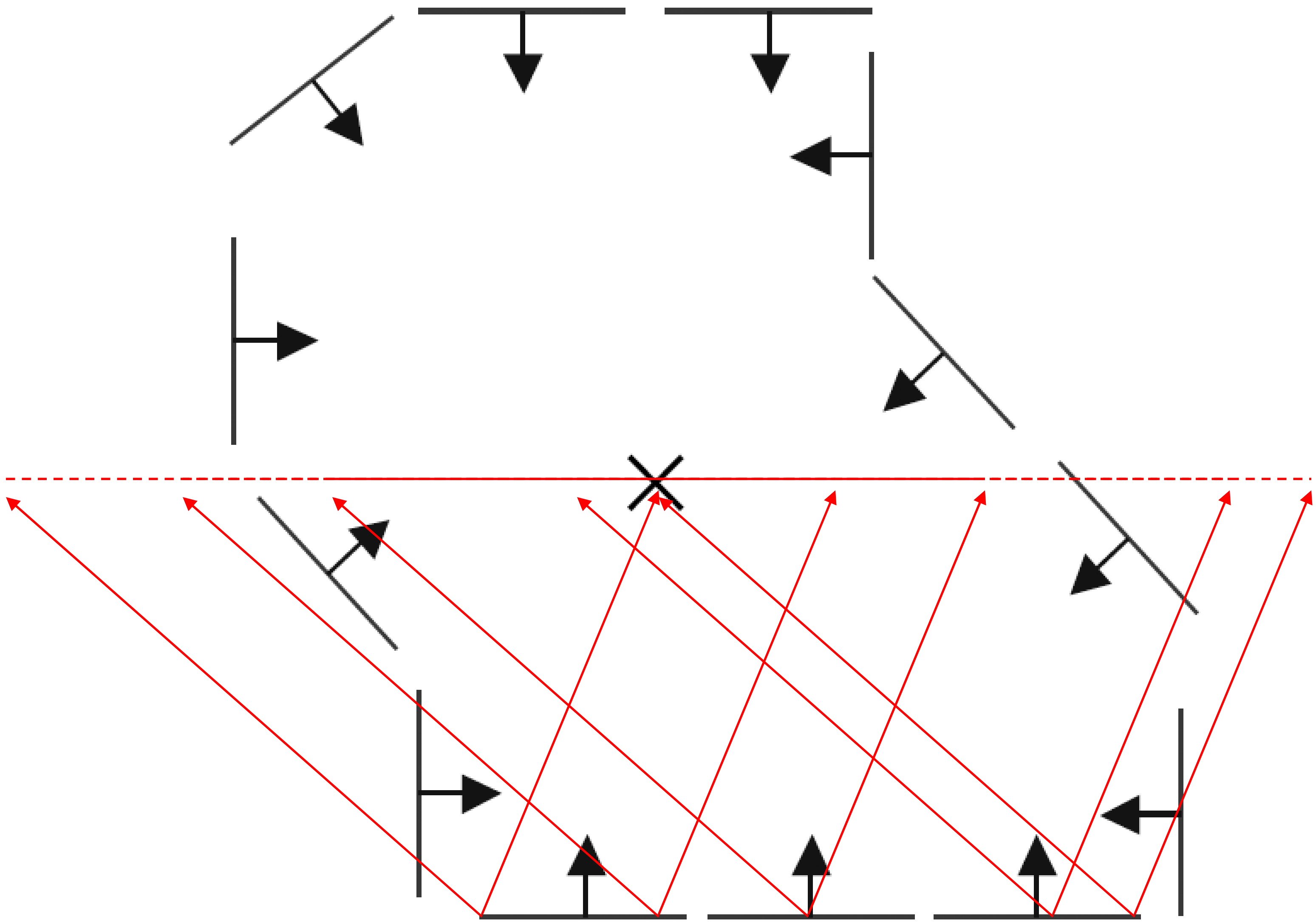


displacement vectors for model points

Example

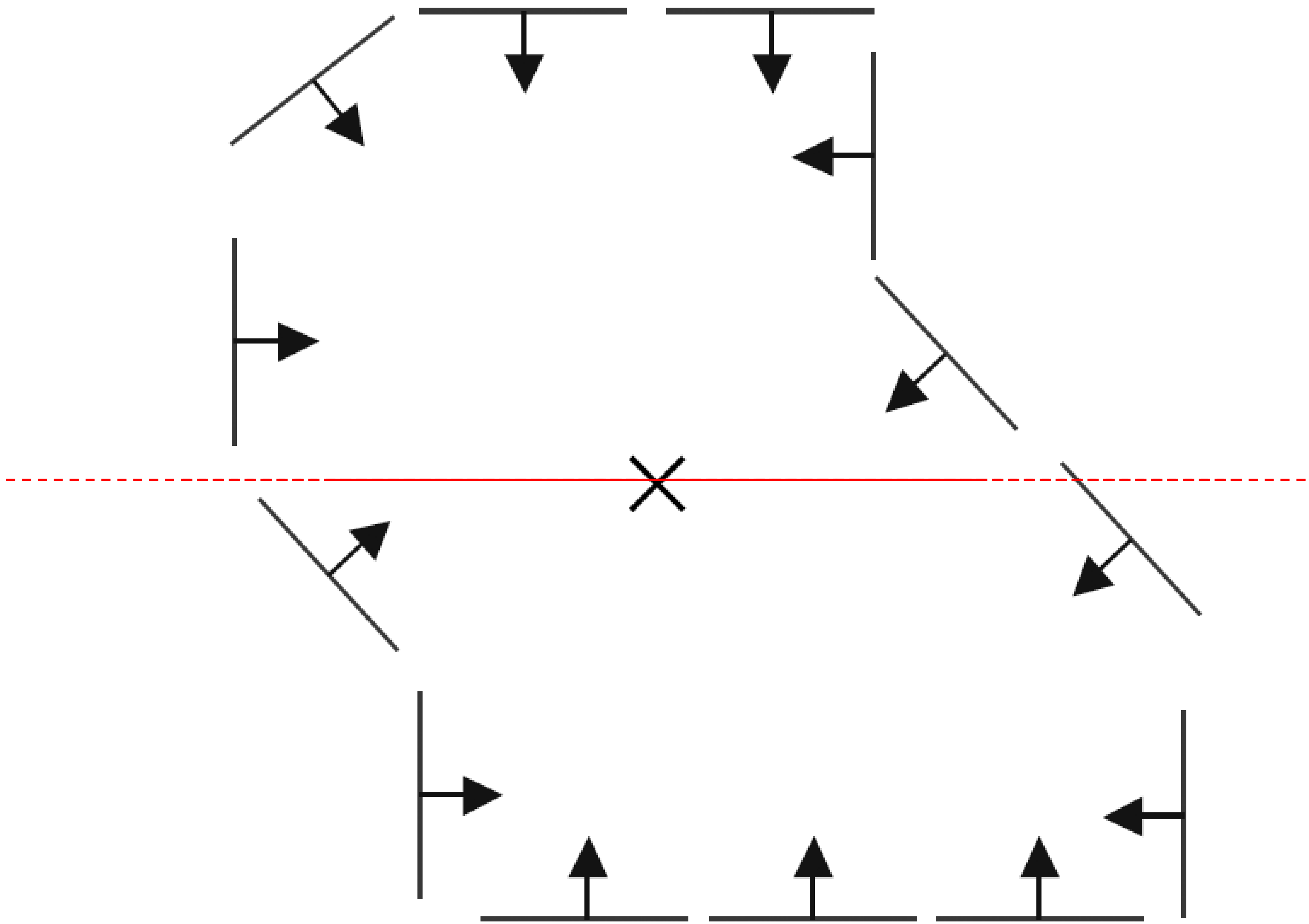


Example



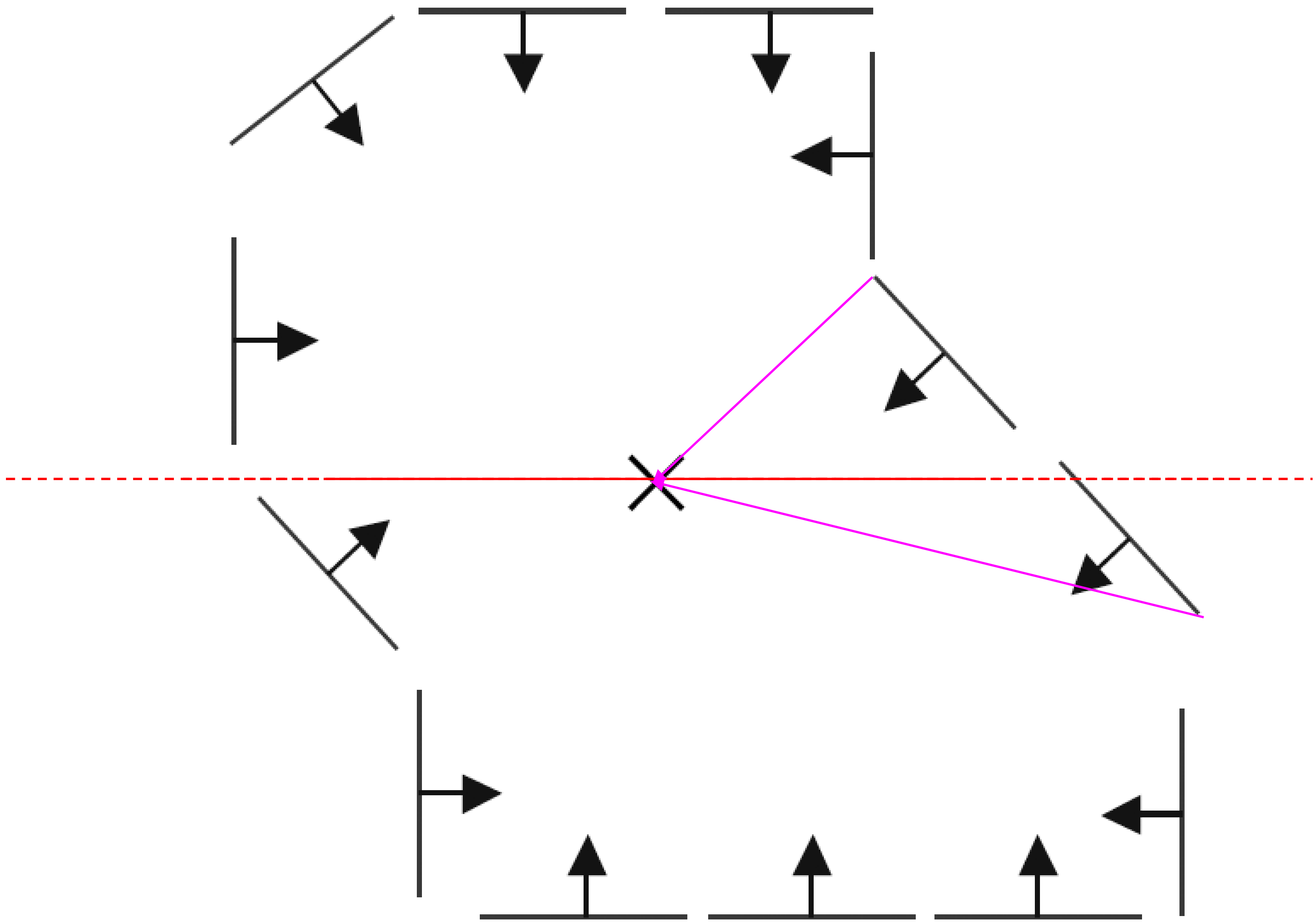
range of voting locations for test point

Example



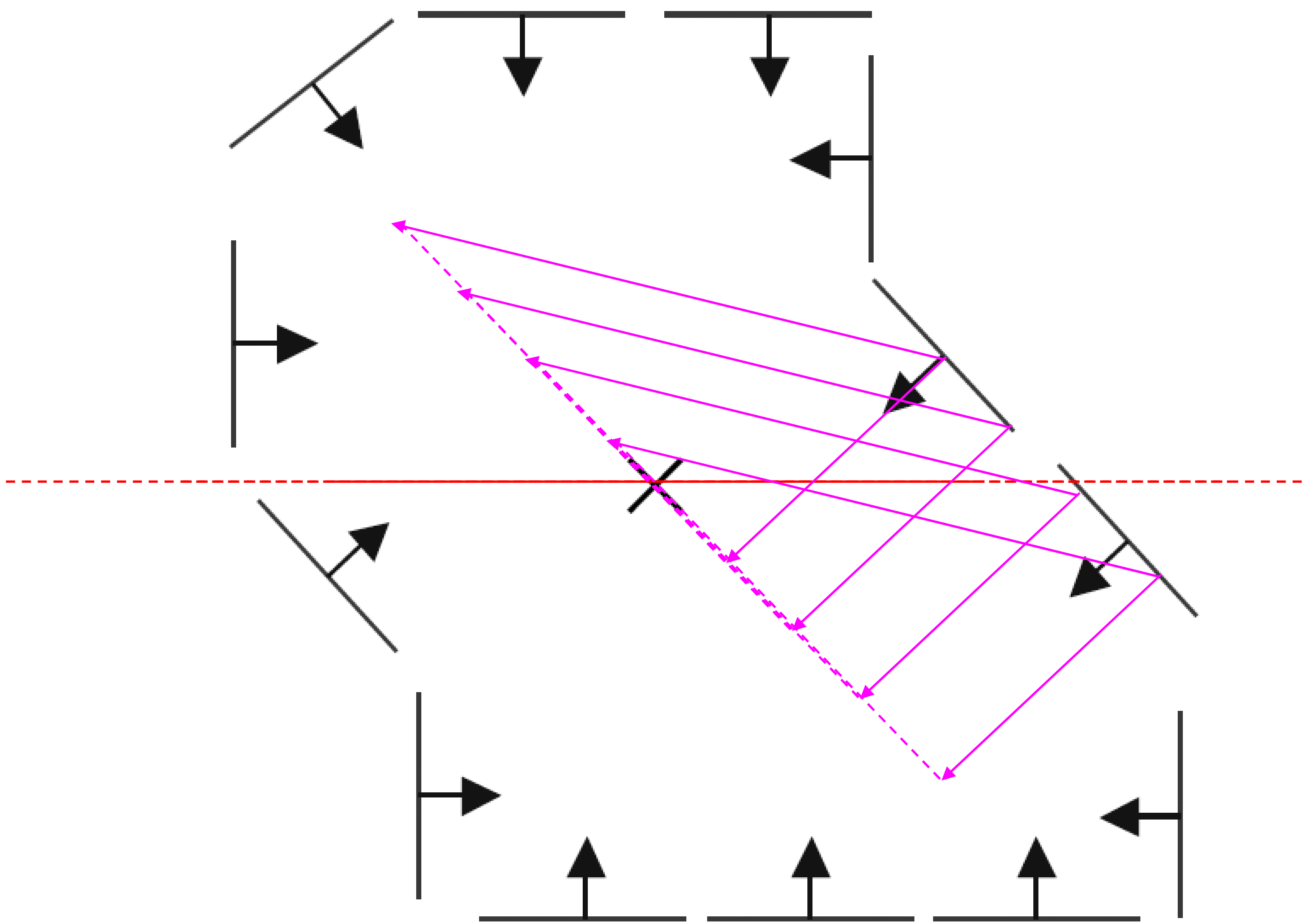
votes for points with $\theta = \uparrow$

Example



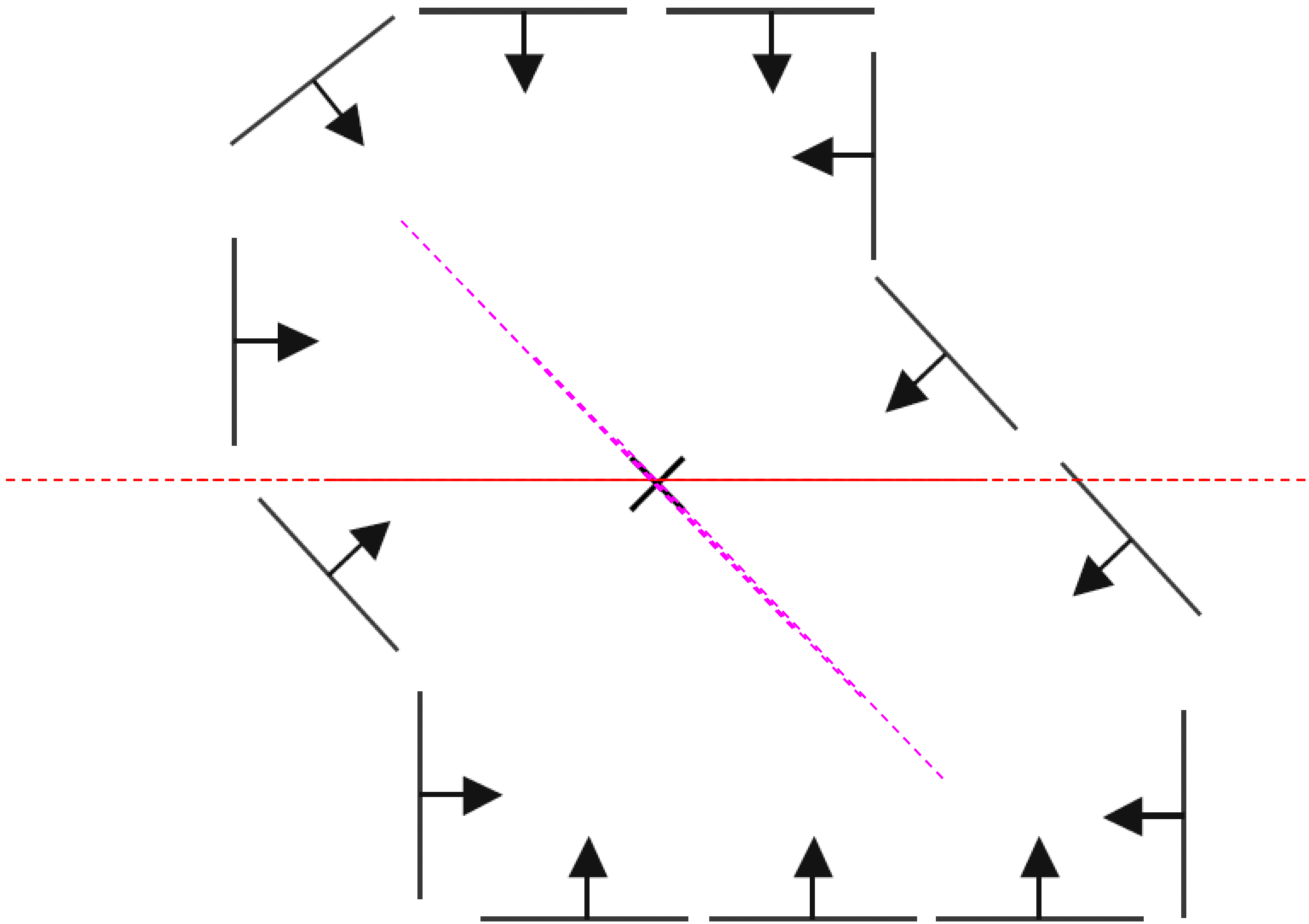
displacement vectors for model points

Example



range of voting locations for test point

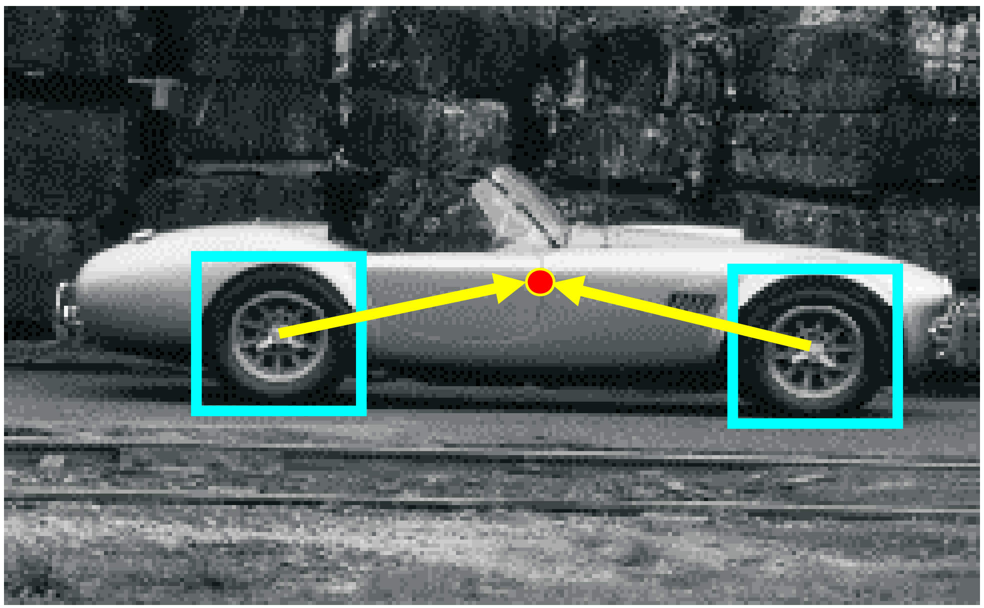
Example



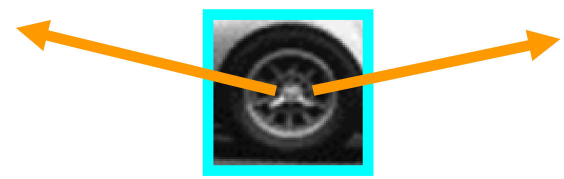
votes for points with $\theta =$ ↙

Application in recognition

- Instead of indexing displacements by gradient orientation, index by “visual codeword”



training image



visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Application in recognition

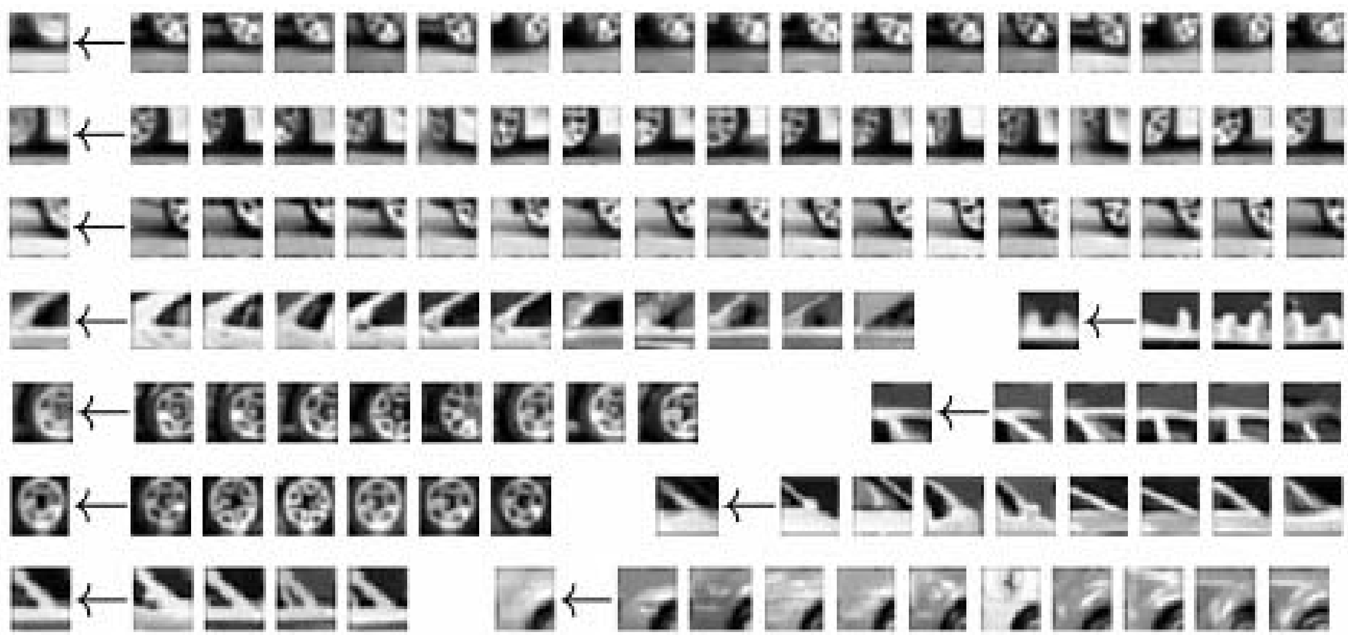
- Instead of indexing displacements by gradient orientation, index by “visual codeword”

test image

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

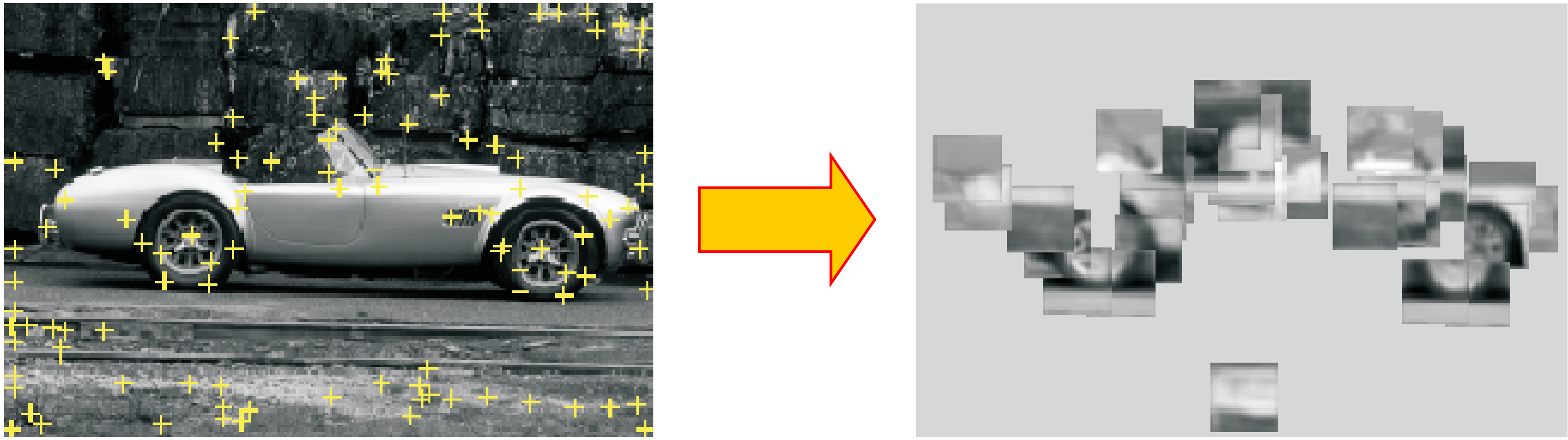
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering (more on this later in the course)



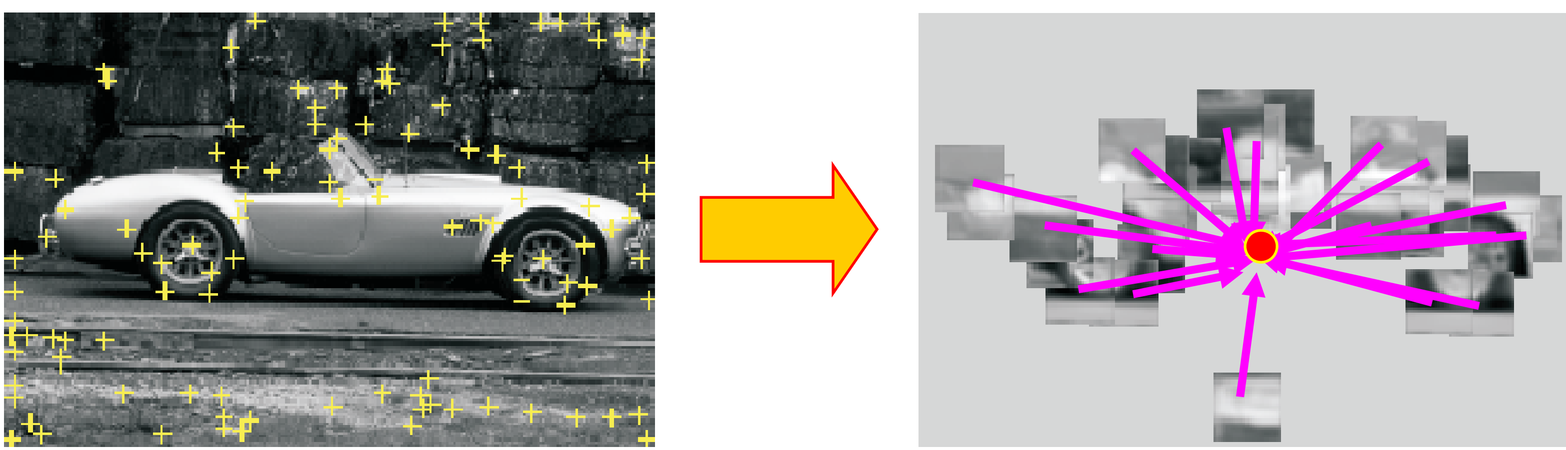
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry



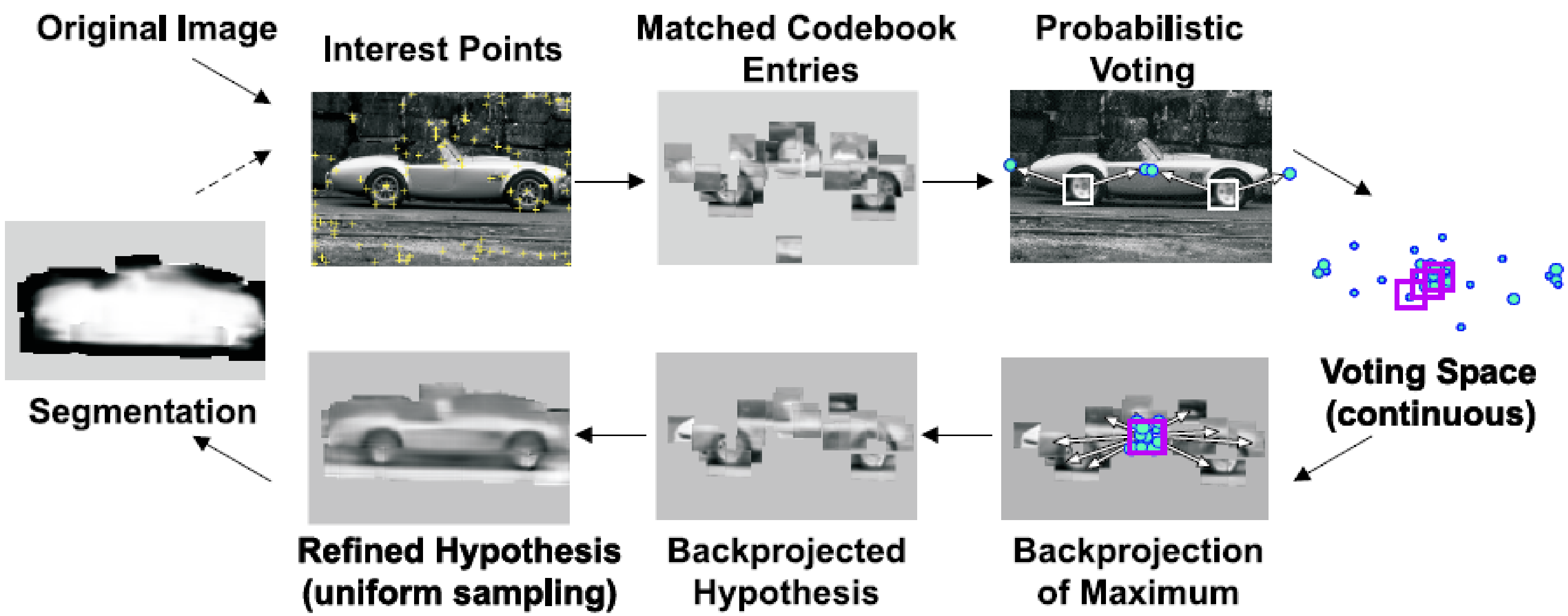
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry
3. For each codebook entry, store all positions it was found, relative to object center



Implicit shape models: Testing

1. Given test image, extract patches, match to codebook entry
2. Cast votes for possible positions of object center
3. Search for maxima in voting space
4. Extract weighted segmentation mask based on stored masks for the codebook occurrences



Implicit shape models: Details

- Supervised training
  - Need reference location and segmentation mask for each training car
- Voting space is continuous, not discrete
  - Clustering algorithm needed to find maxima
- How about dealing with scale changes?
  - Option 1: search a range of scales, as in Hough transform for circles
  - Option 2: use scale-covariant interest points
- Verification stage is very important
  - Once we have a location hypothesis, we can overlay a more detailed template over the image and compare pixel-by-pixel, transfer segmentation masks, etc.