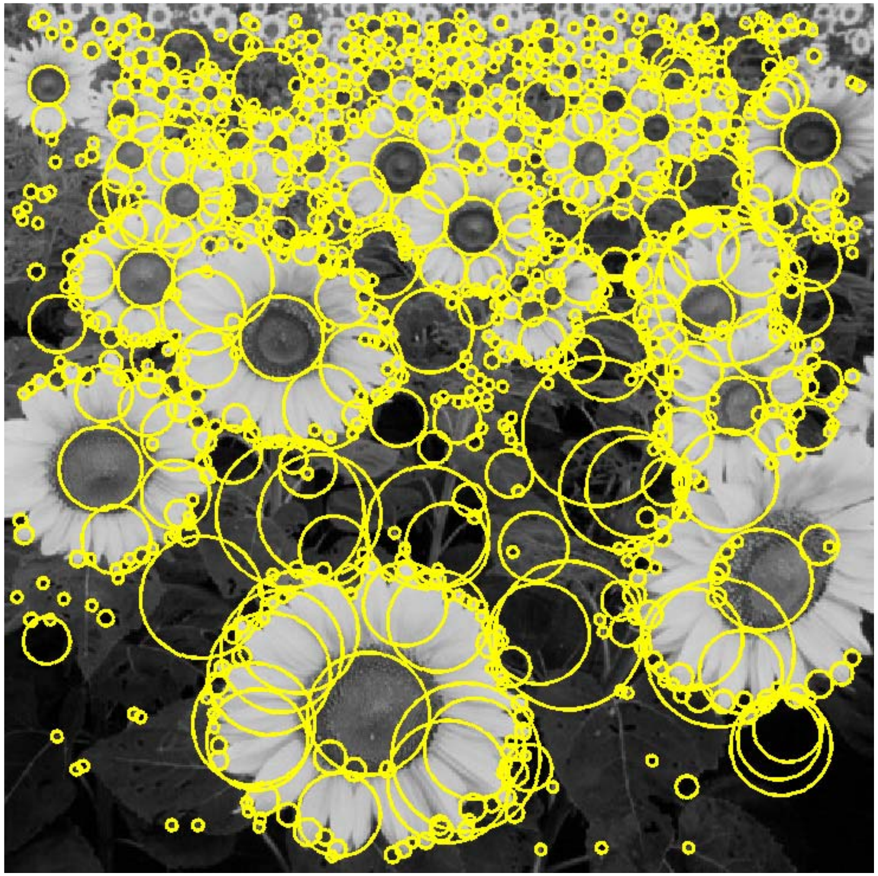
Feature extraction: Corners and blobs



Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?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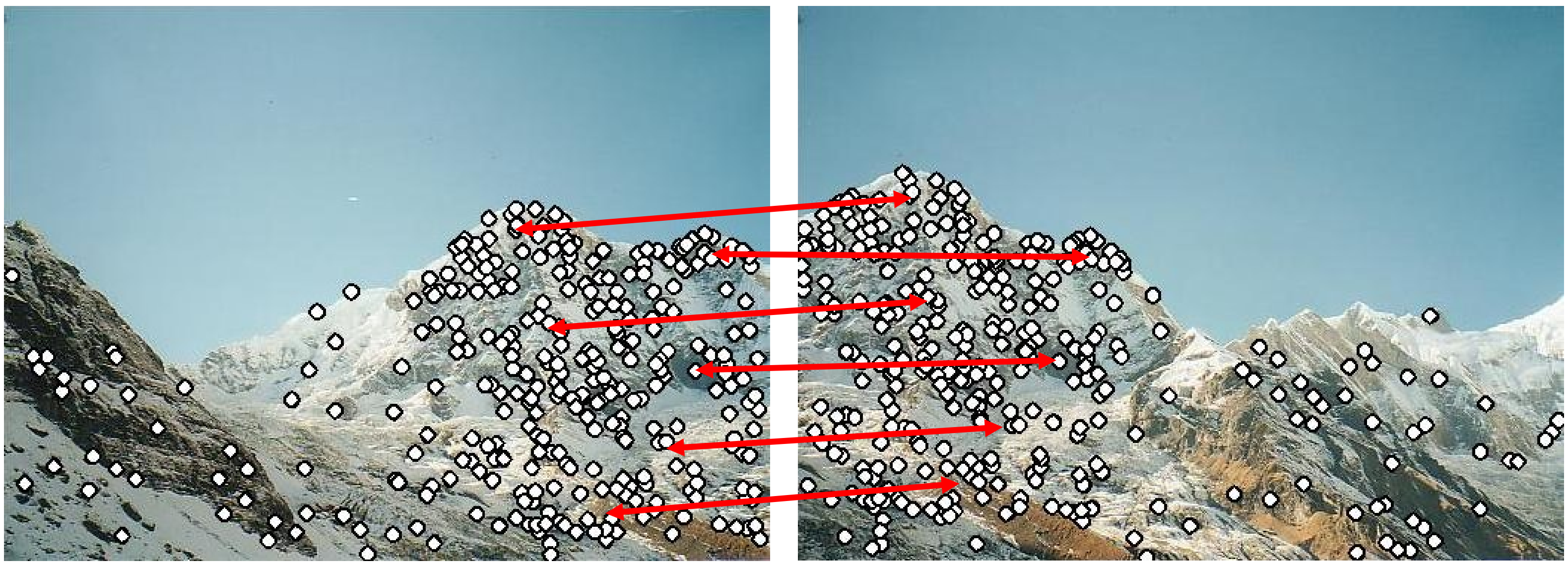


Step 1: extract features

Step 2: match features

Why extract features?

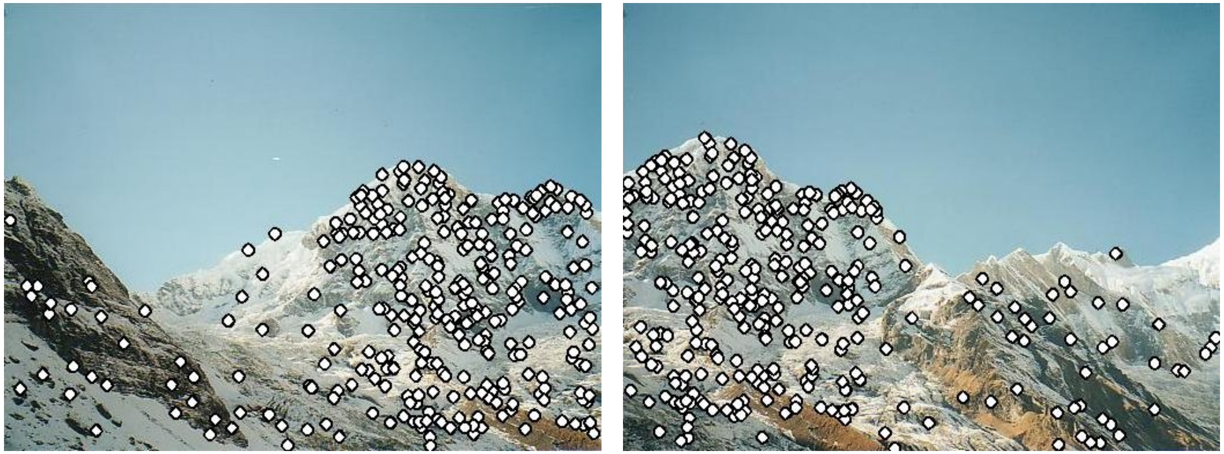
- Motivation: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract features

Step 2: match features

Step 3: align images

Characteristics of good features



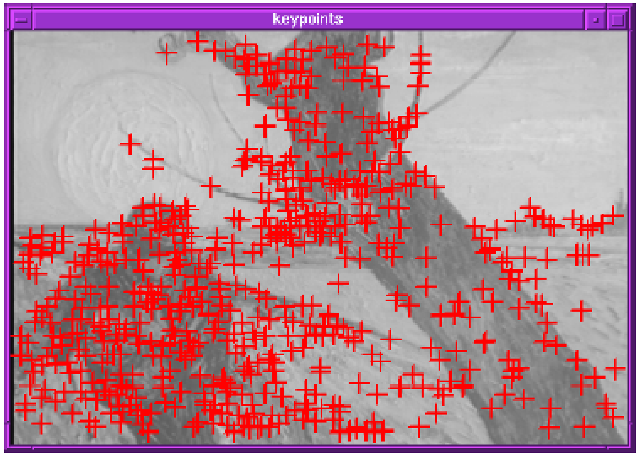
- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

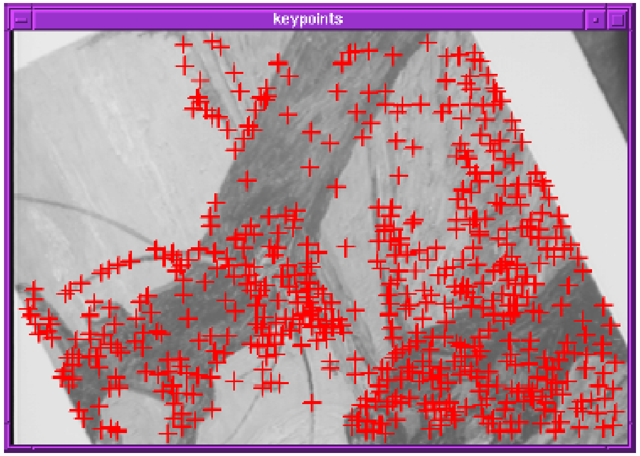
Applications

Feature points are used for:

- Motion tracking
- Image alignment
- 3D reconstruction
- Object recognition
- Indexing and database retrieval
- Robot navigation

Finding Corners



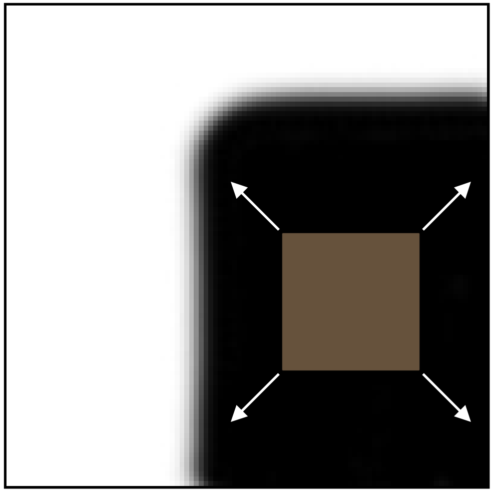


- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

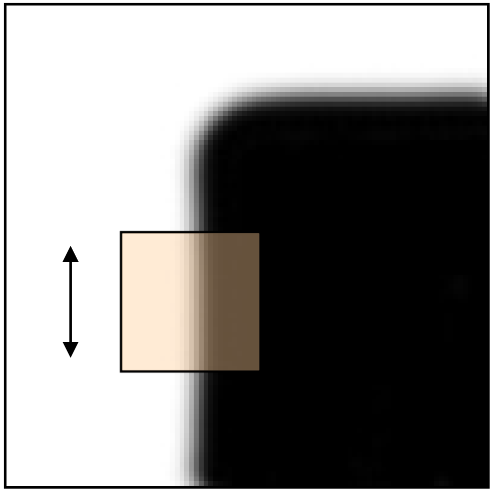
C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147--151.

The Basic Idea

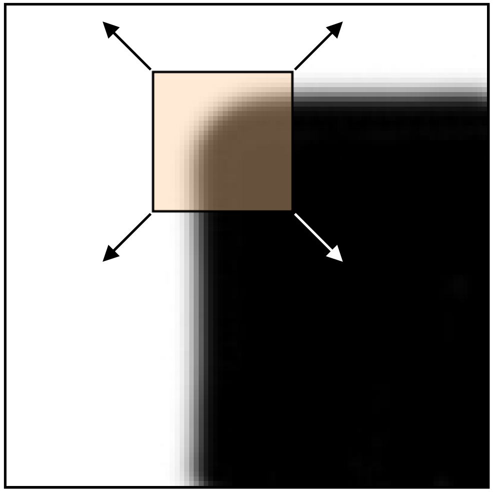
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

Source: A. Efros

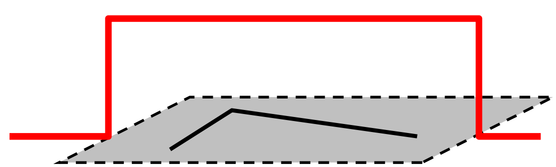
Harris Detector: Mathematics

Change in appearance for the shift [*u,v*]:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

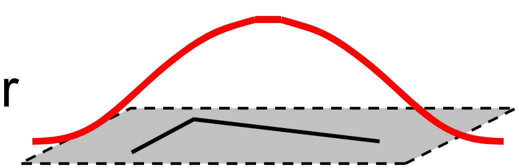
Window function — Shifted intensity — Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Source: R. Szeliski

Harris Detector: Mathematics

Change in appearance for the shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Second-order Taylor expansion of *E*(*u*,*v*) about (0,0) (bilinear approximation for small shifts):

$$E(u,v) \approx E(0,0) + [u \quad v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \quad v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

Harris Detector: Mathematics

The bilinear approximation simplifies to

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

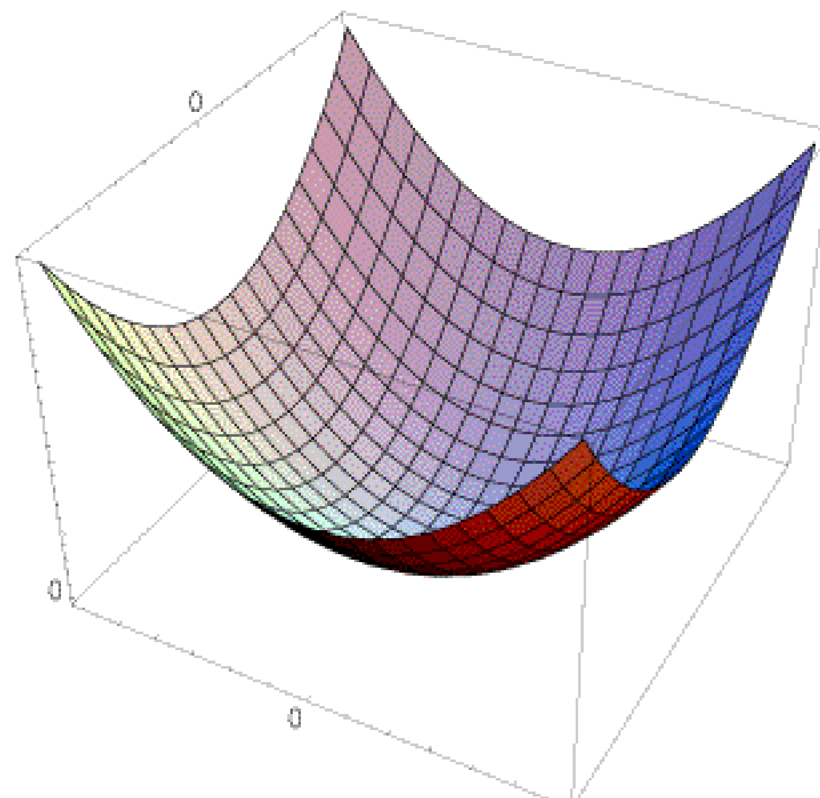
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

First, consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

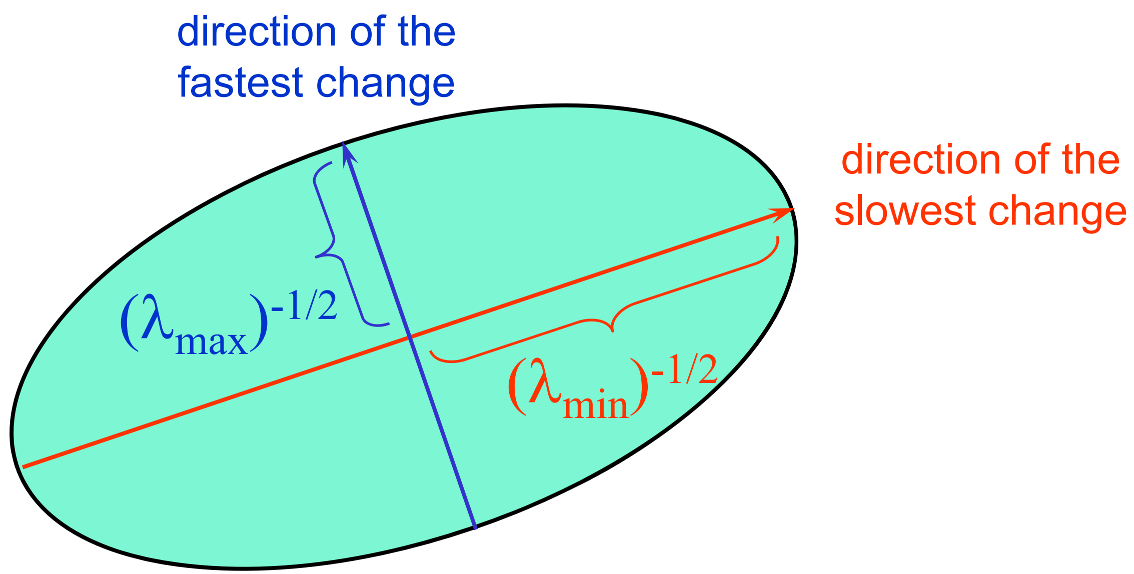
General Case

Since M is symmetric, we have $M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

We can visualize *M* as an ellipse with axis lengths determined by the eigenvalues and orientation determined by *R*

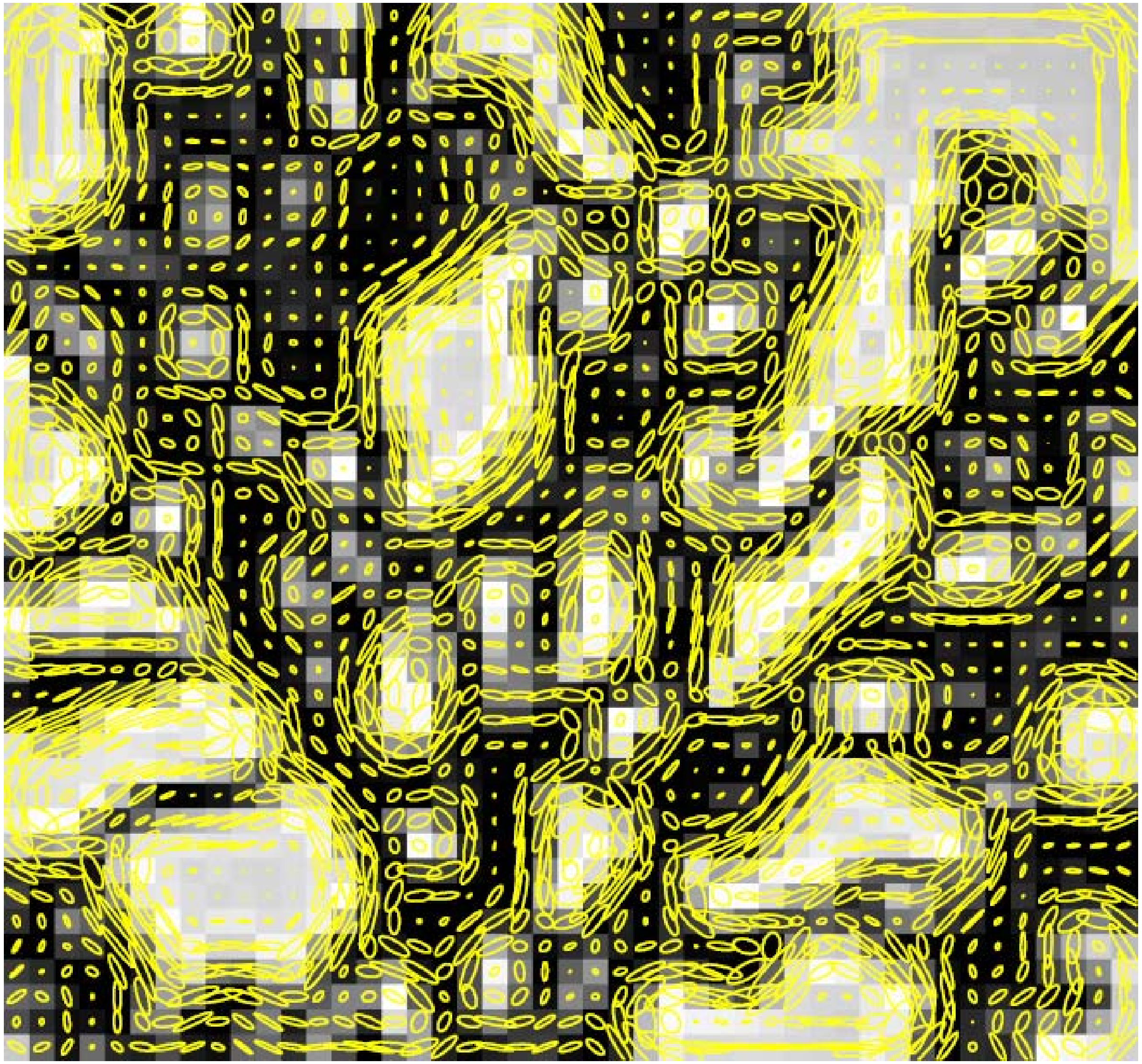
Ellipse equation:

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



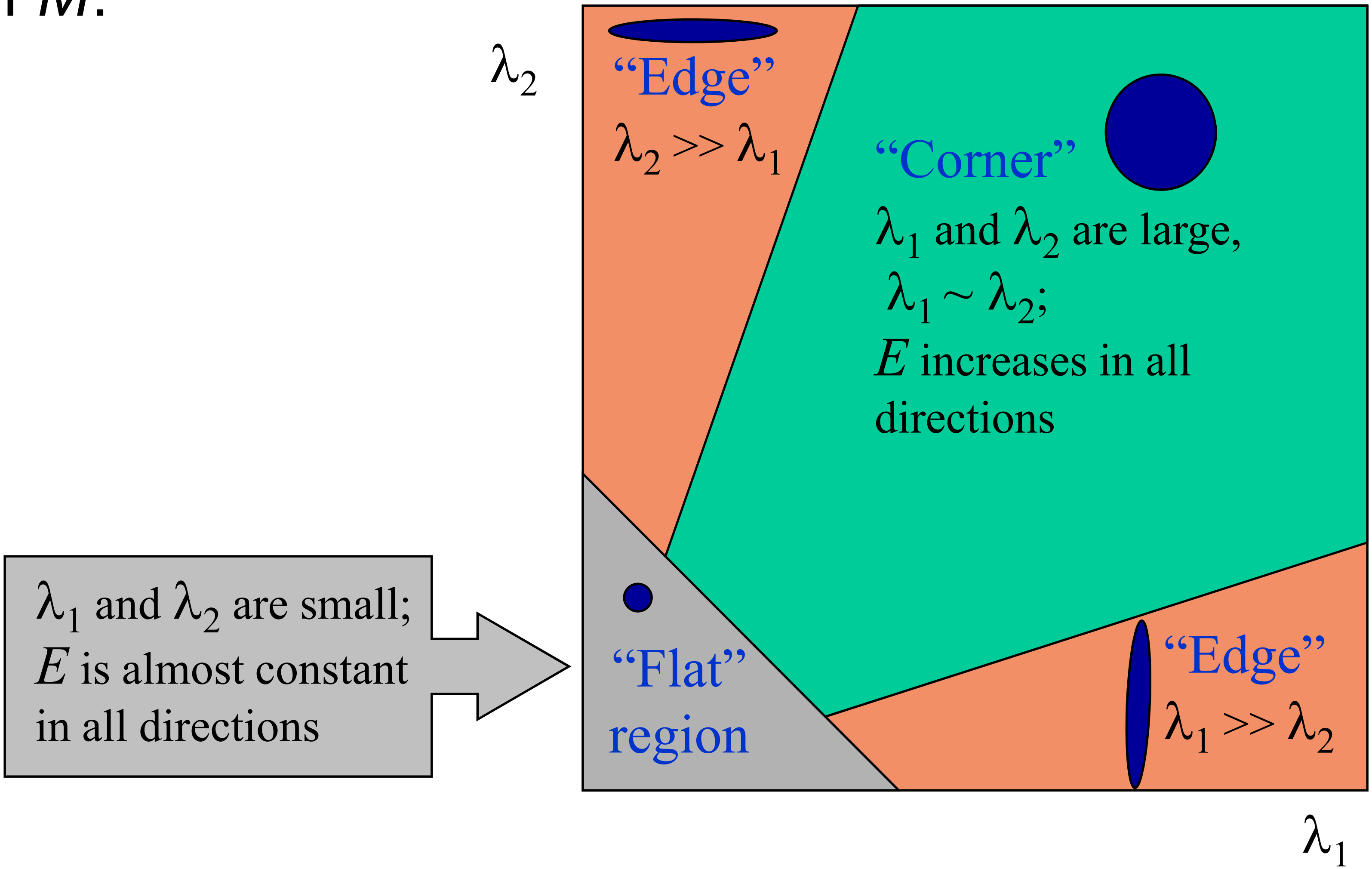
Visualization of second moment matrices

Visualization of second moment matrices



Interpreting the eigenvalues

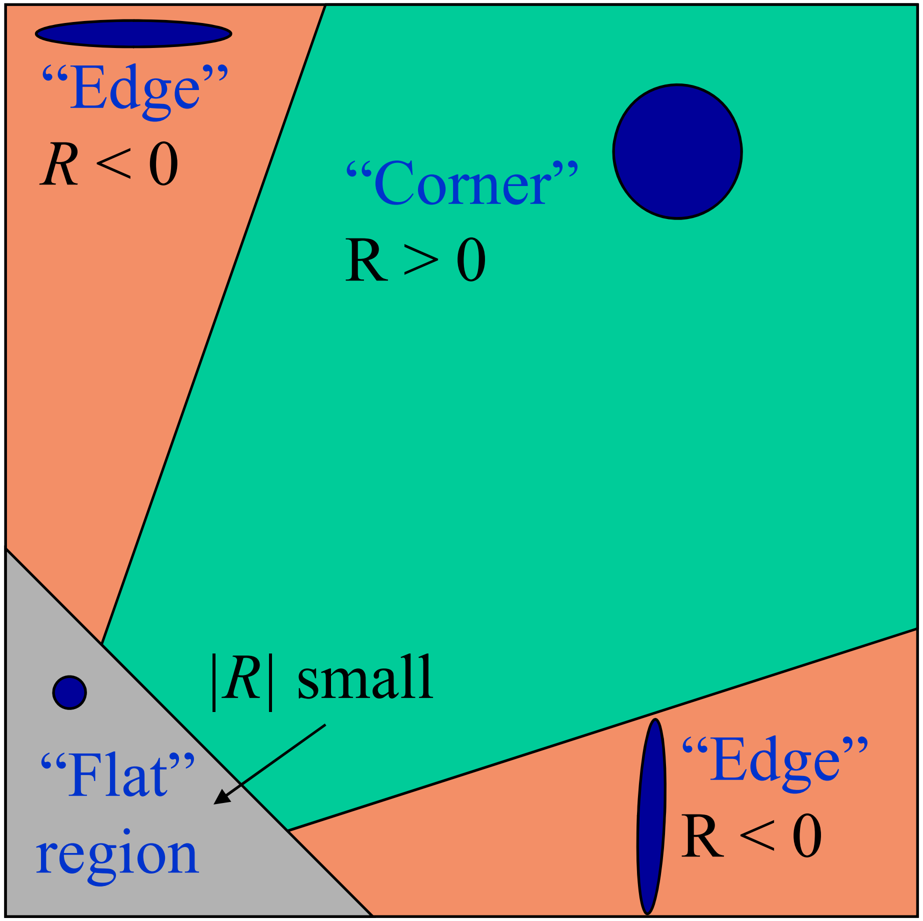
Classification of image points using eigenvalues of *M*:



Corner response function

$$R = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



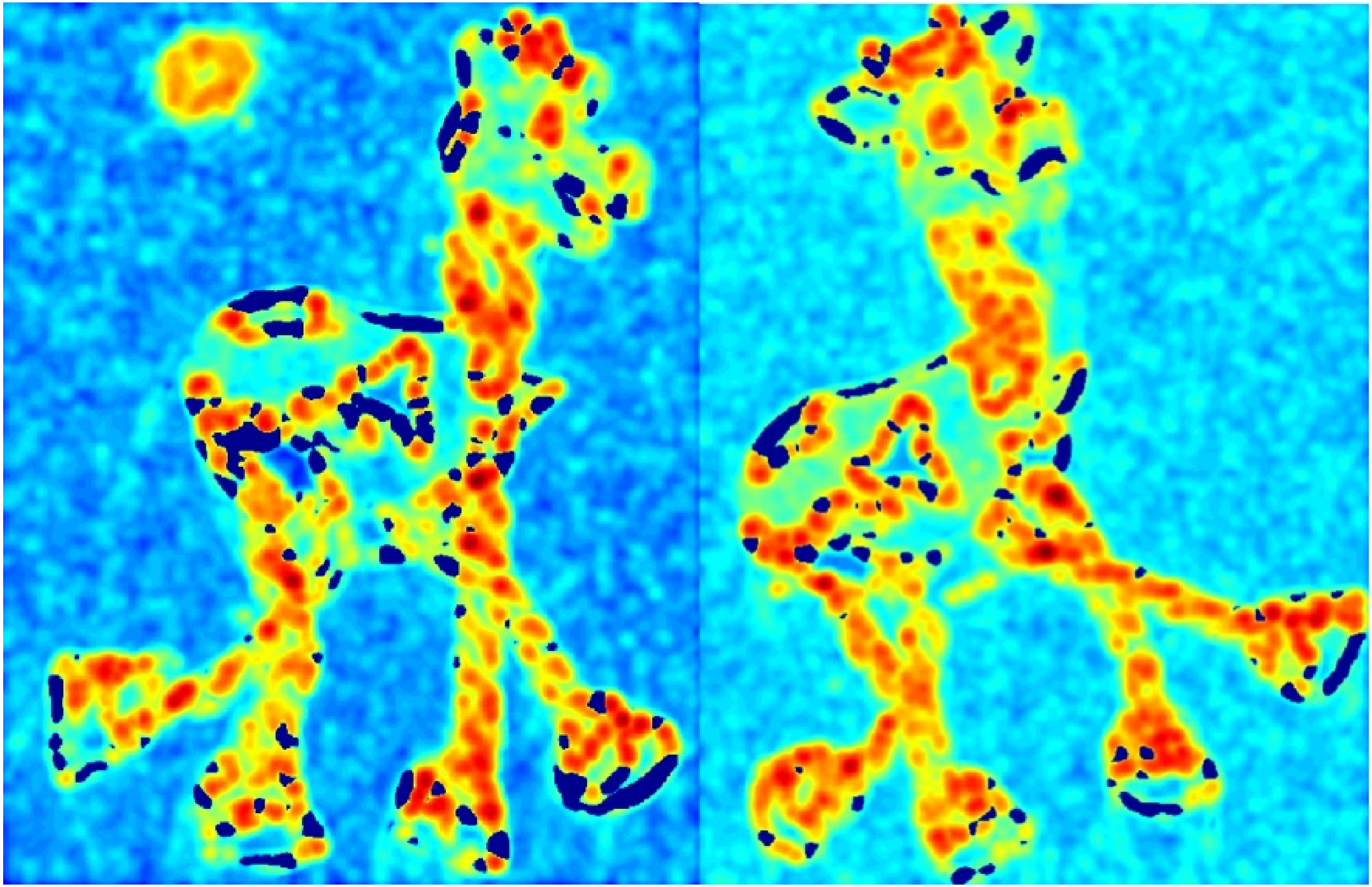
Harris detector: Steps

1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R
4. Threshold R
5. Find local maxima of response function (nonmaximum suppression)

Harris Detector: Steps

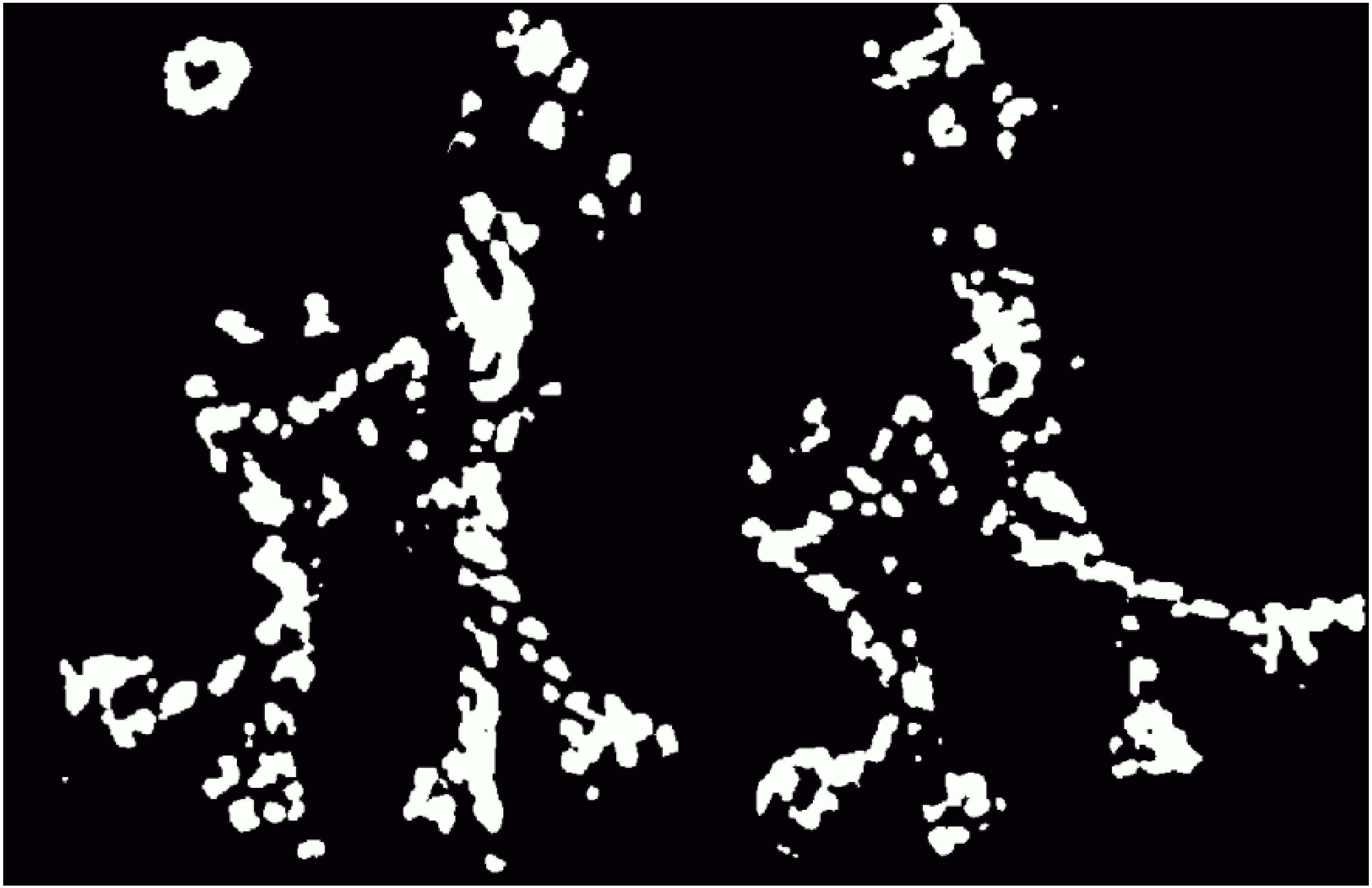
Harris Detector: Steps

Compute corner response R



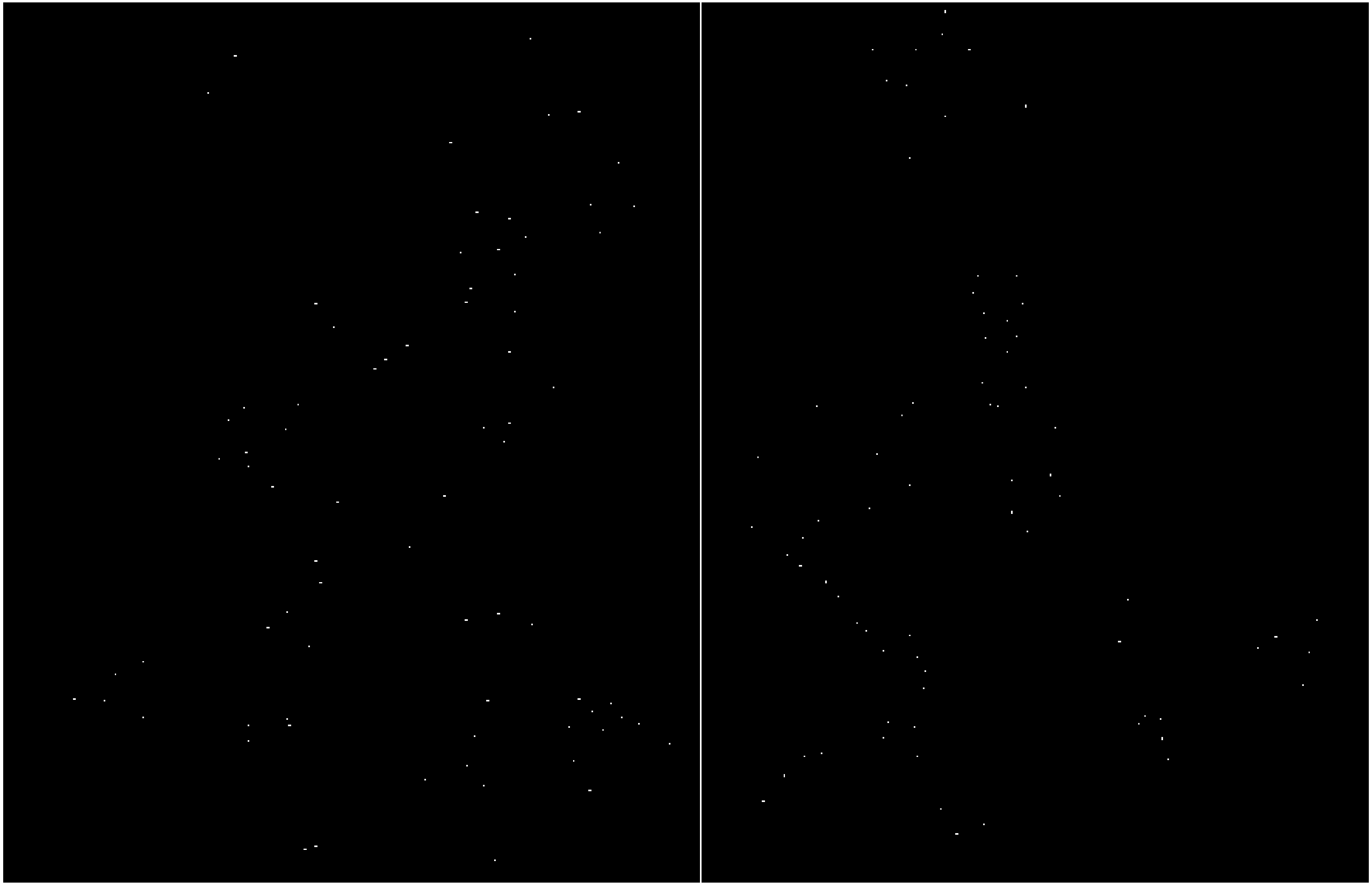
Harris Detector: Steps

Find points with large corner response: R>threshold



Harris Detector: Steps

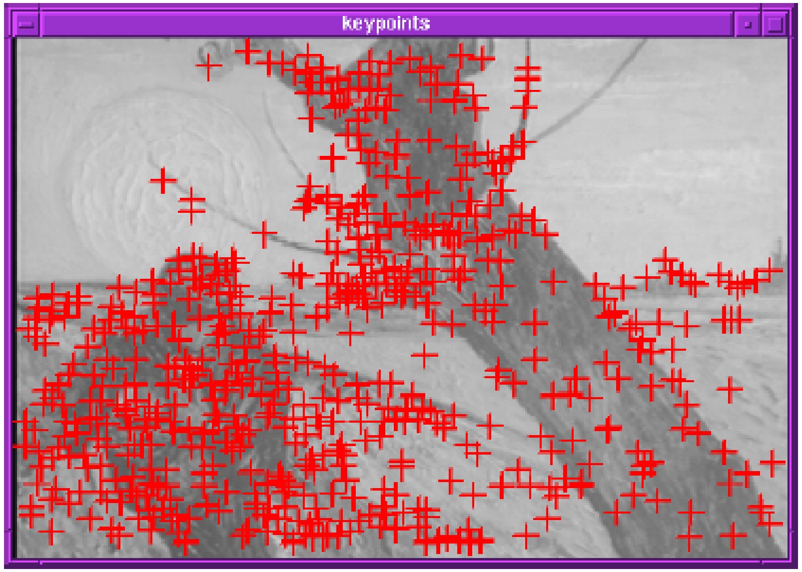
Take only the points of local maxima of R

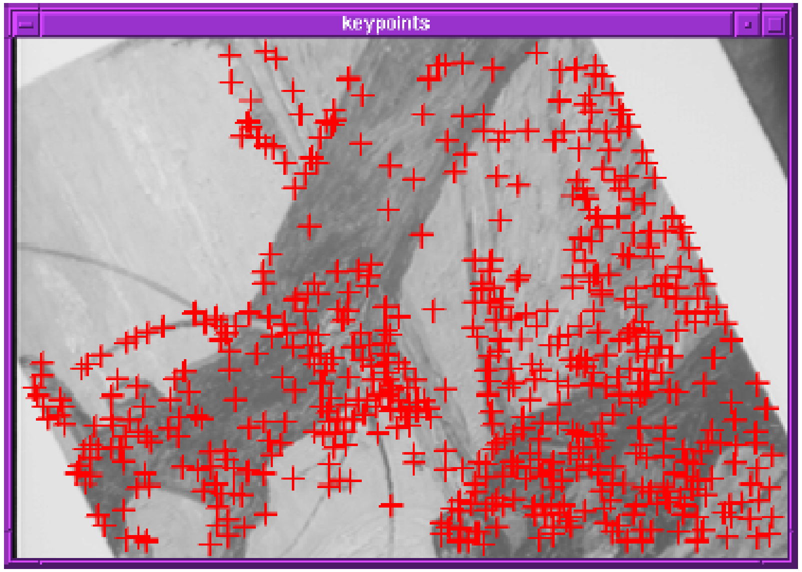


Harris Detector: Steps

Invariance

- We want features to be detected despite geometric or photometric changes in the image: if we have two transformed versions of the same image, features should be detected in corresponding locations

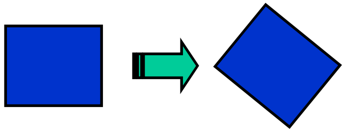




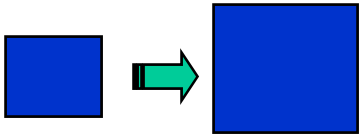
Models of Image Change

Geometric

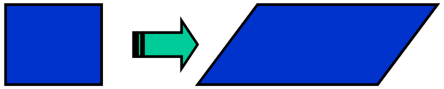
- **Rotation**



- **Scale**

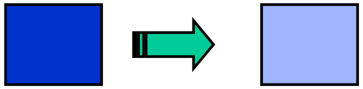


- **Affine**
  valid for: orthographic camera, locally planar object



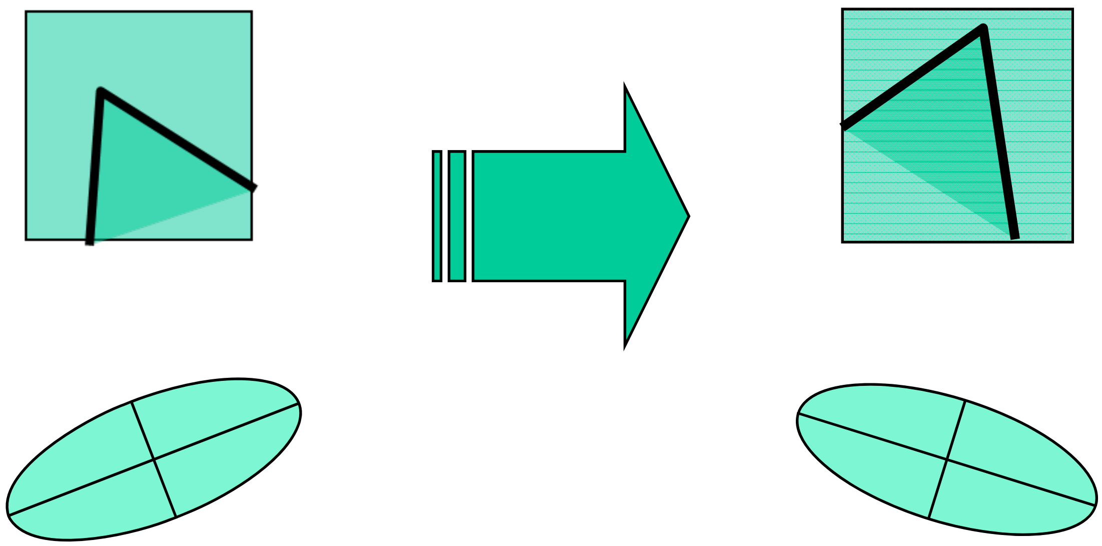
Photometric

- **Affine intensity change** ($I \rightarrow a\,I + b$)



Harris Detector: Invariance Properties

Rotation



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

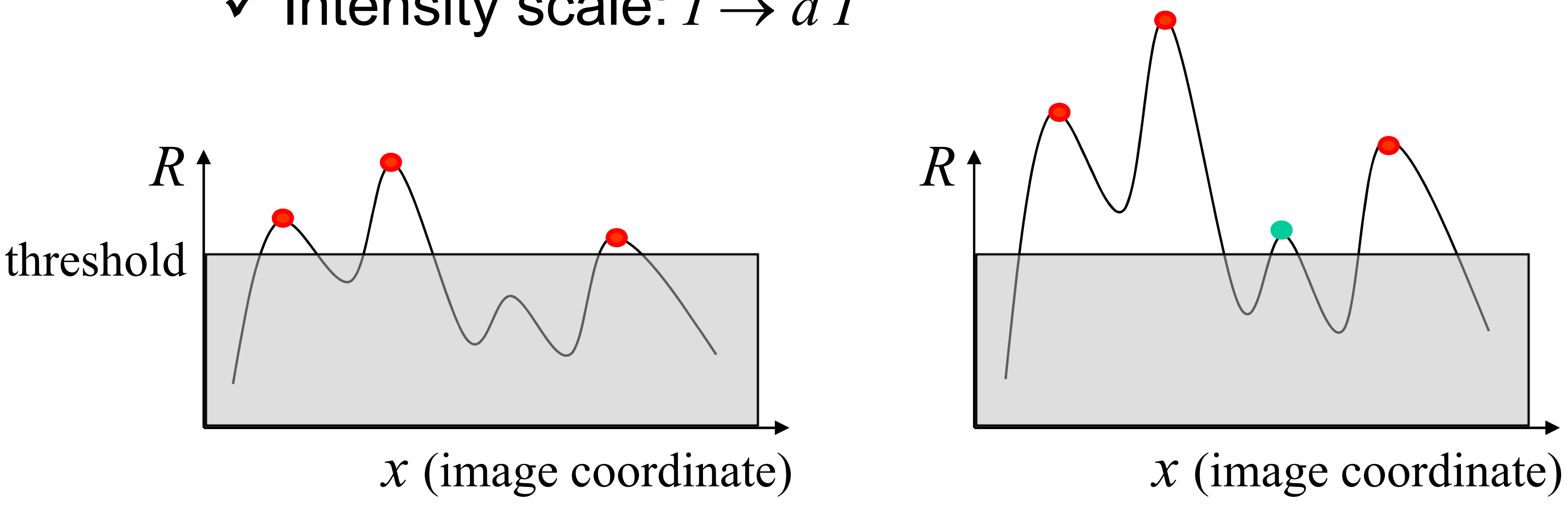
Corner response R is invariant to image rotation

Harris Detector: Invariance Properties

Affine intensity change

✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

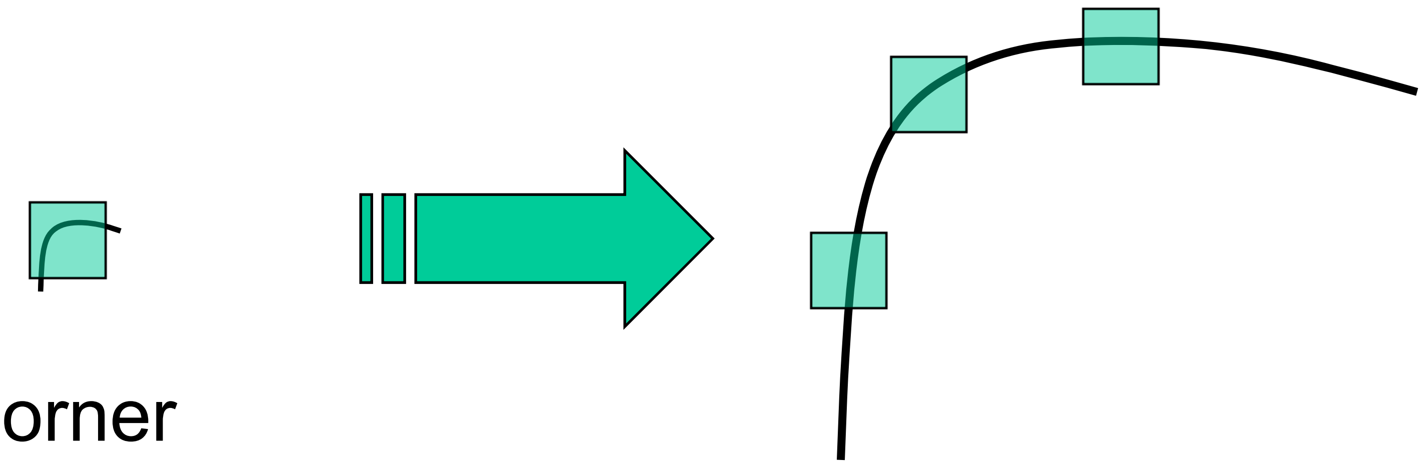
✓ Intensity scale: $I \rightarrow a\, I$



Partially invariant to affine intensity change

Harris Detector: Invariance Properties

Scaling

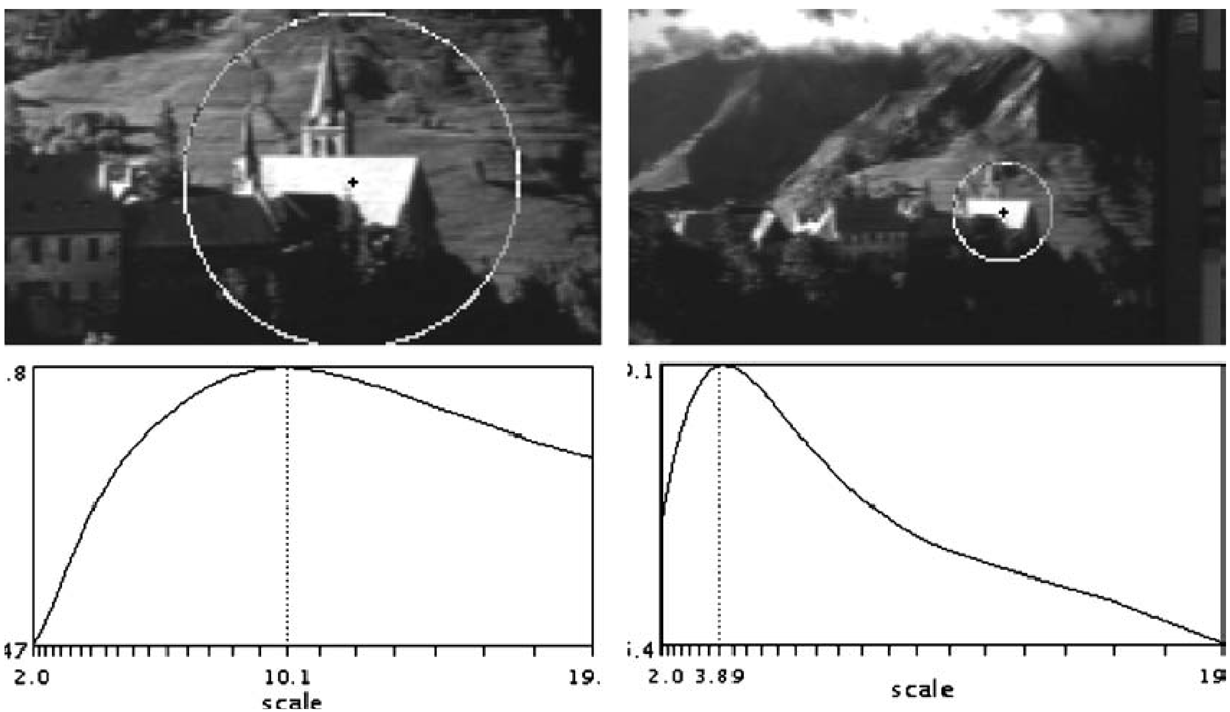


All points will be classified as edges

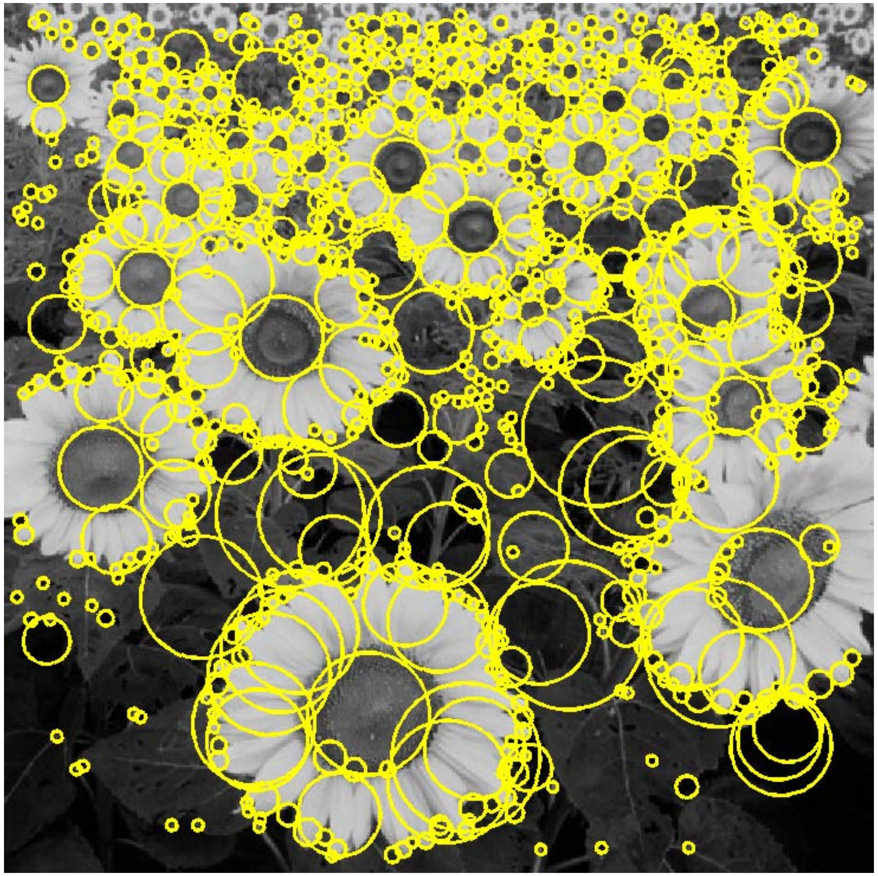
Not invariant to scaling

Scale-invariant feature detection

- Goal: independently detect corresponding regions in scaled versions of the same image
- Need *scale selection* mechanism for finding characteristic region size that is *covariant* with the image transformation



Scale-invariant features: Blobs

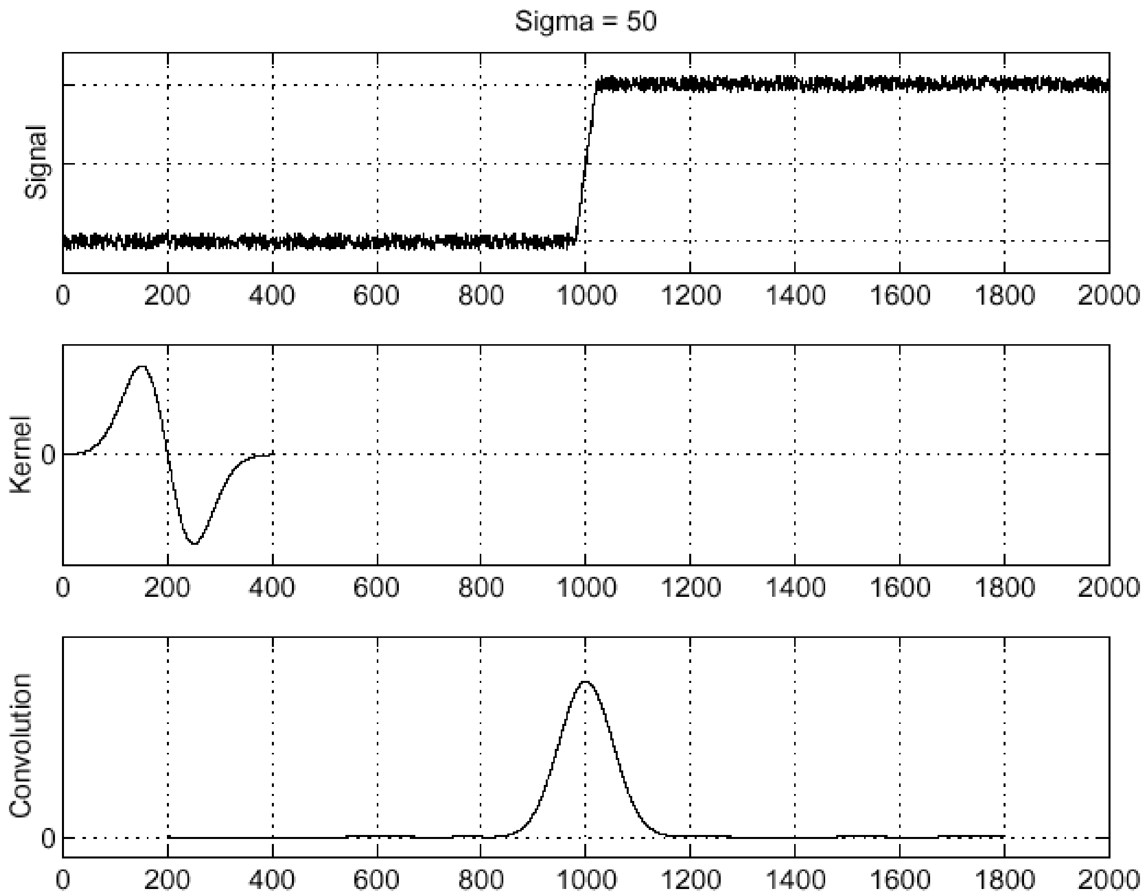


Recall: Edge detection

f

$\frac{d}{dx}g$

$f * \frac{d}{dx}g$



Edge

Derivative of Gaussian

Edge = maximum of derivative

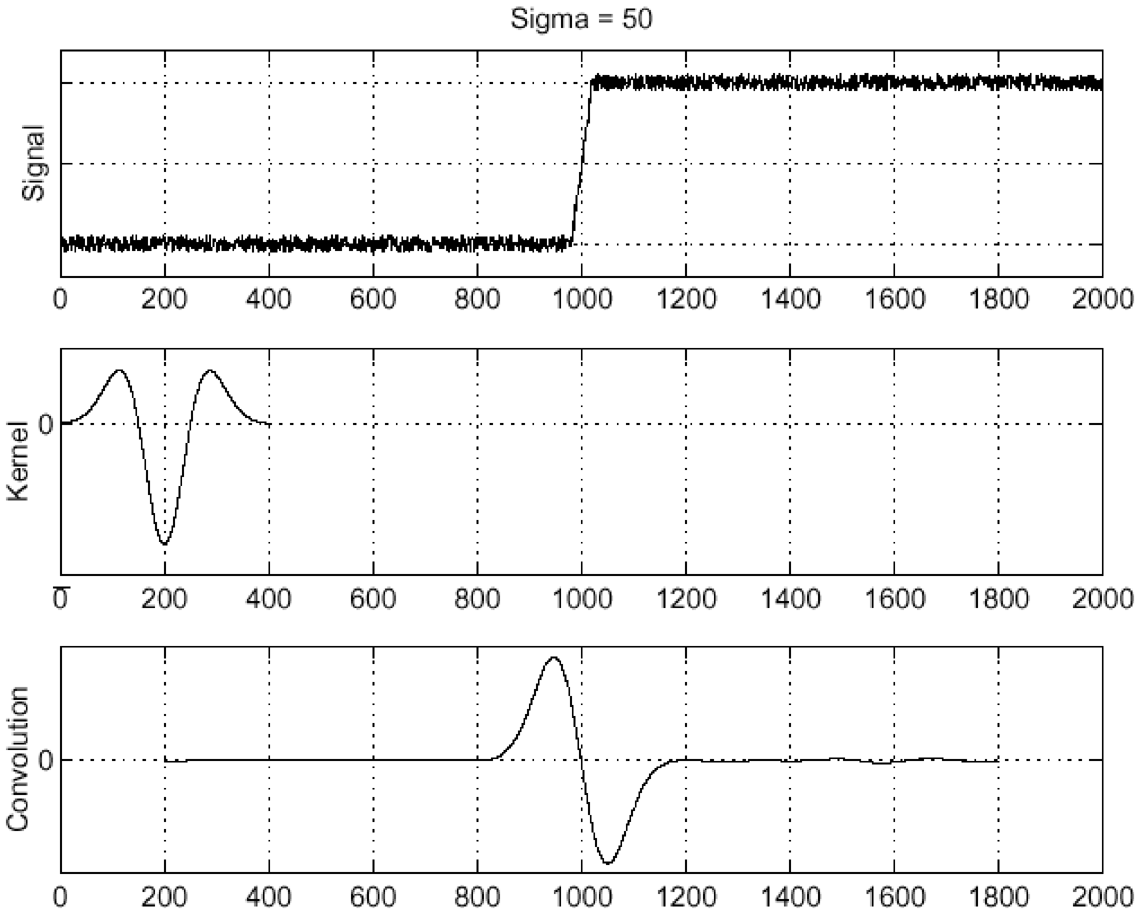
Source: S. Seitz

Edge detection, Take 2

f

$$\frac{d^2}{dx^2} g$$

$$f * \frac{d^2}{dx^2} g$$



Edge

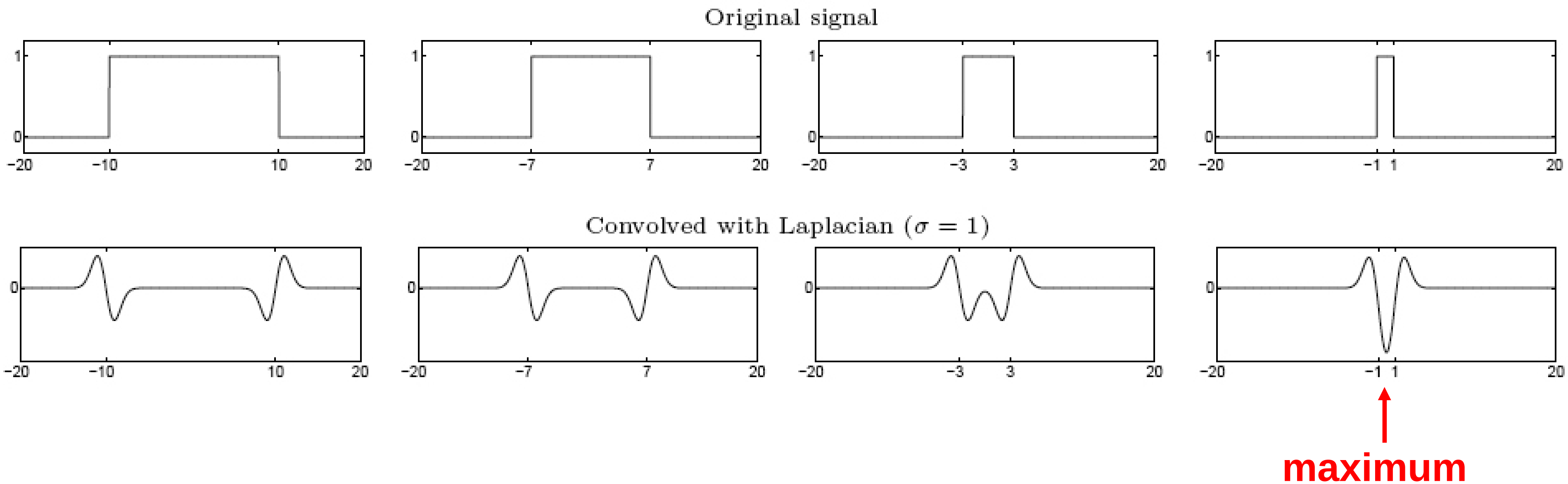
Second derivative of Gaussian (Laplacian)

Edge = zero crossing of second derivative

Source: S. Seitz

From edges to blobs

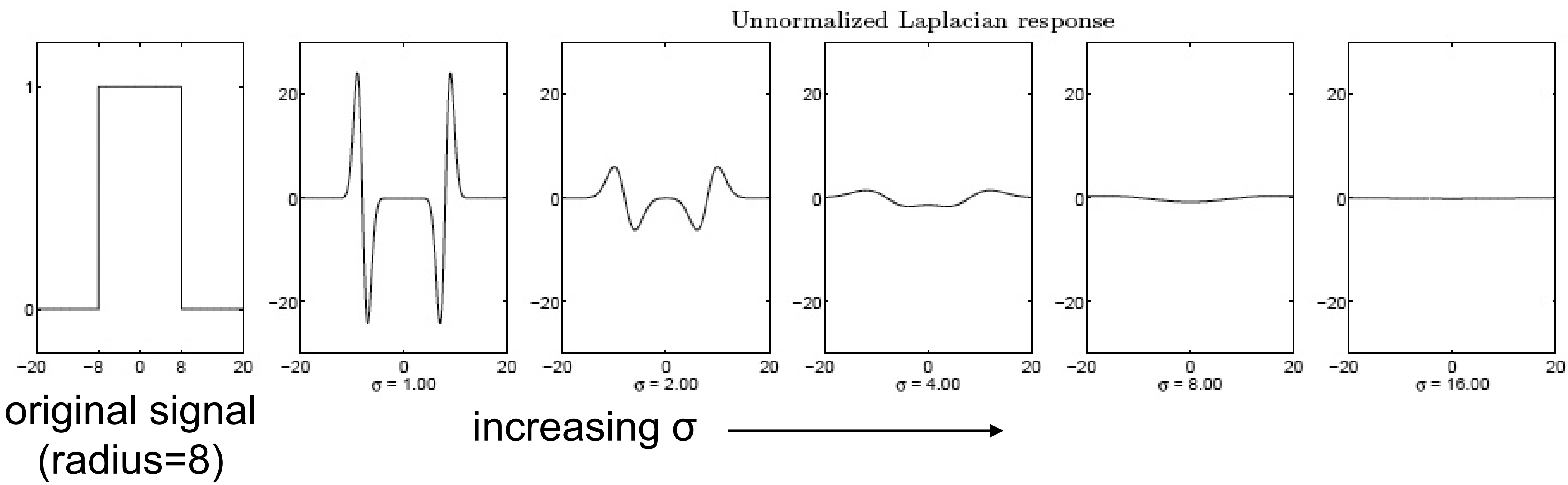
- Edge = ripple
- Blob = superposition of two ripples



Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

Scale selection

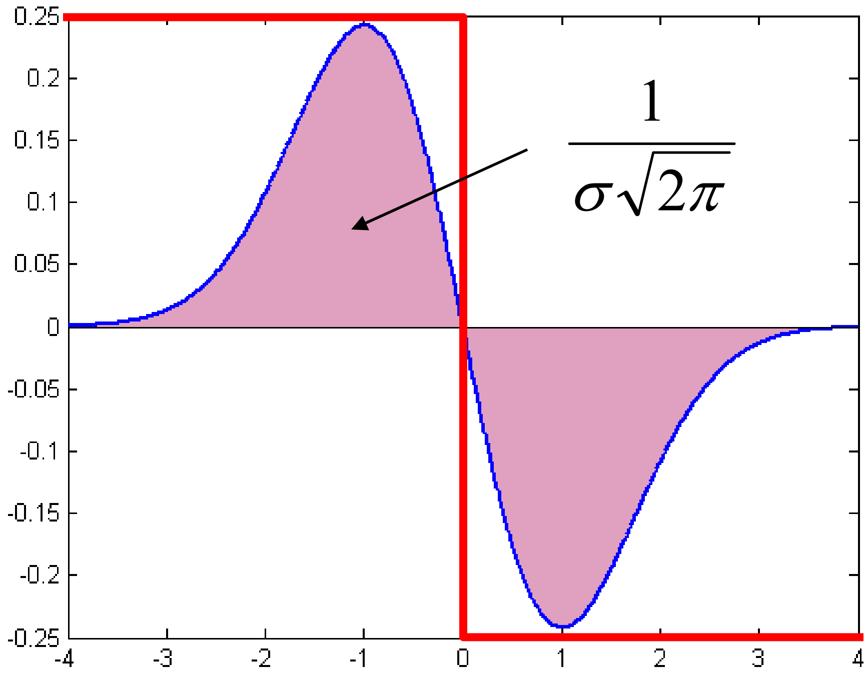
- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



Why does this happen?

Scale normalization

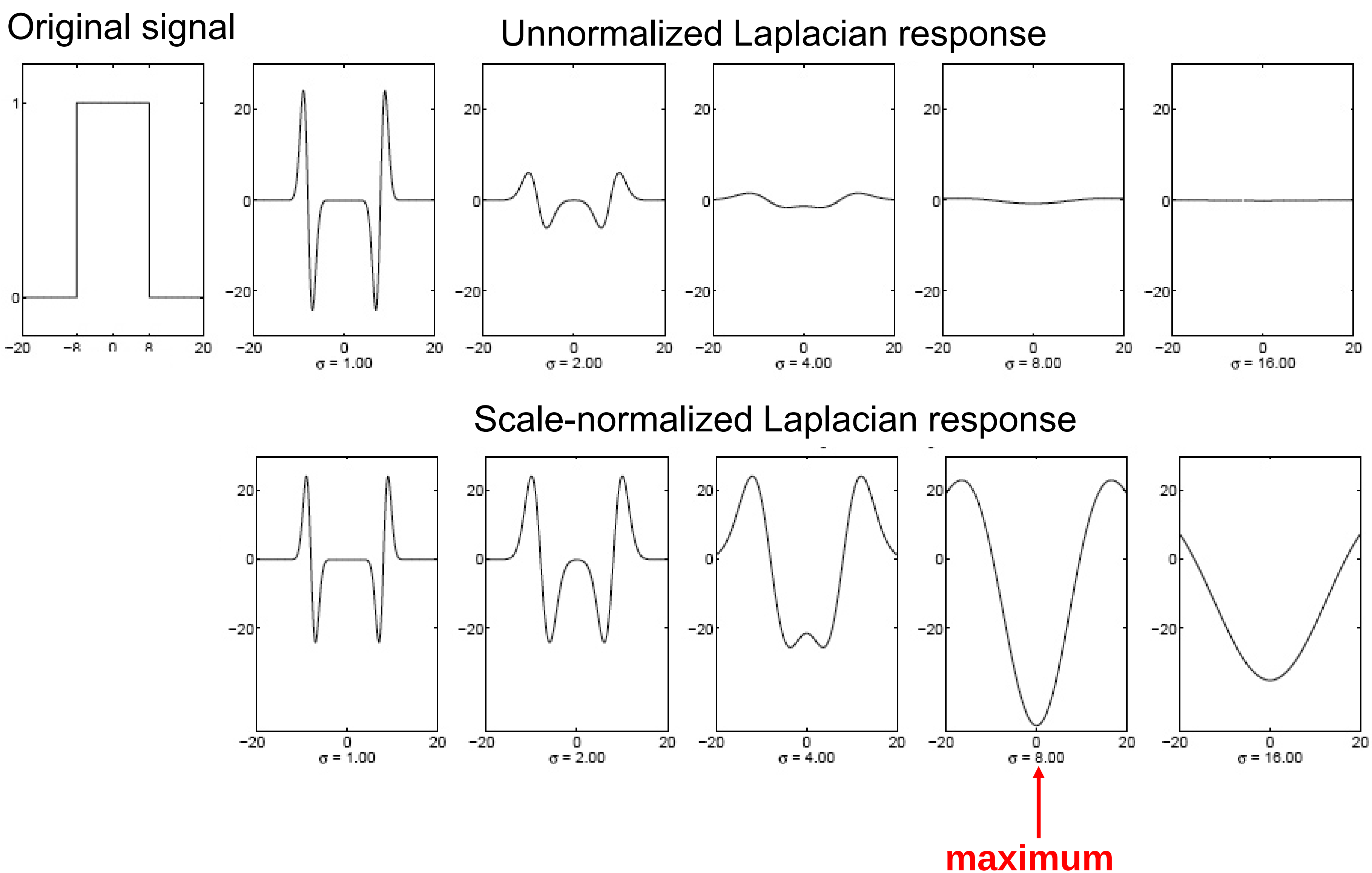
- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



Scale normalization

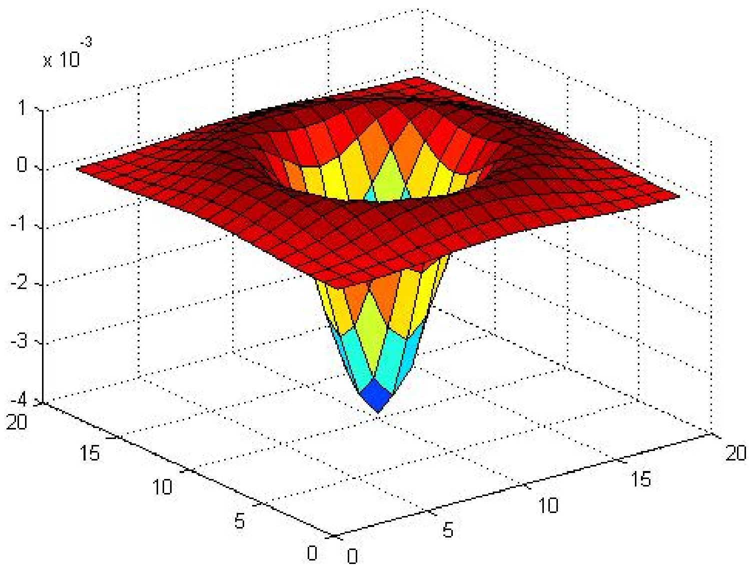
- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

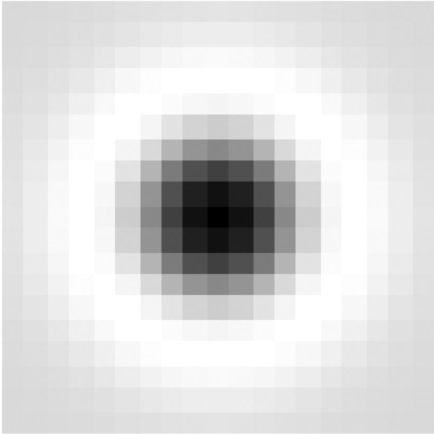
Effect of scale normalization



Blob detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

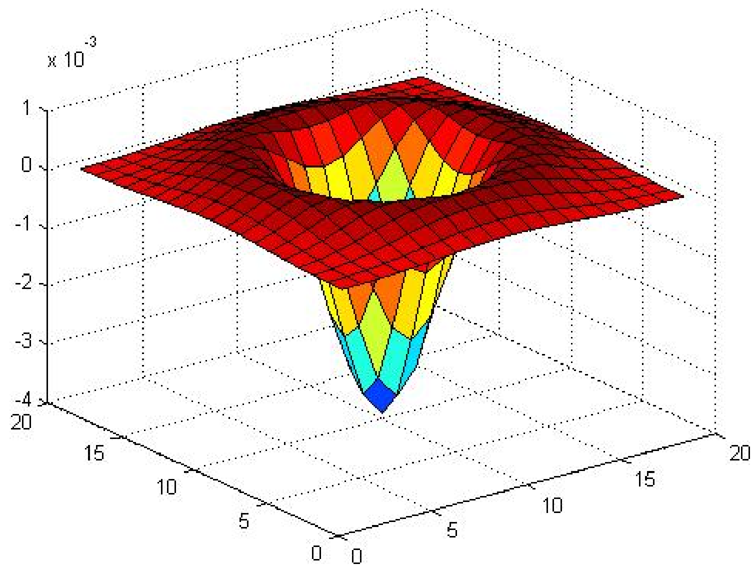


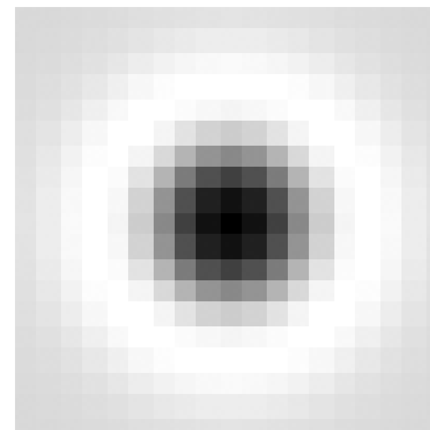


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Blob detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

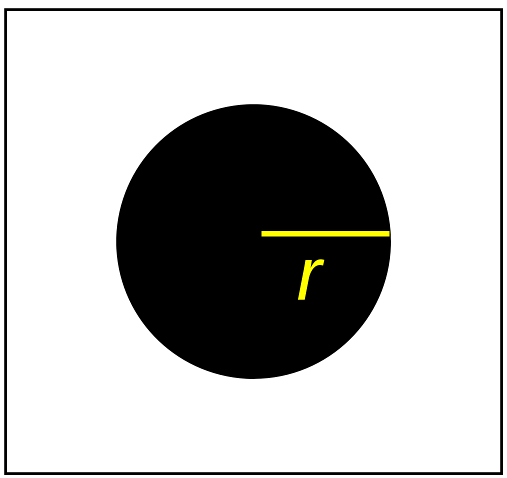




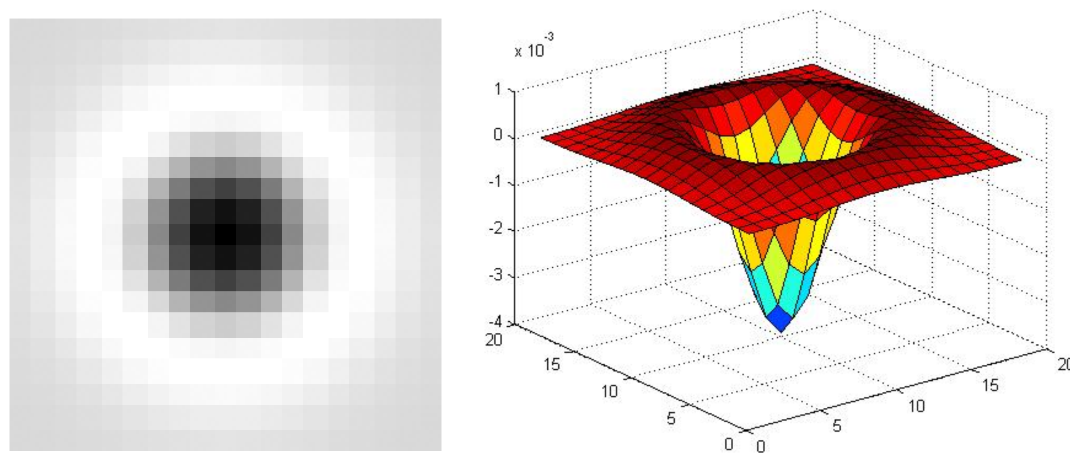
Scale-normalized: $$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

Scale selection

- At what scale does the Laplacian achieve a maximum response for a binary circle of radius r?



image



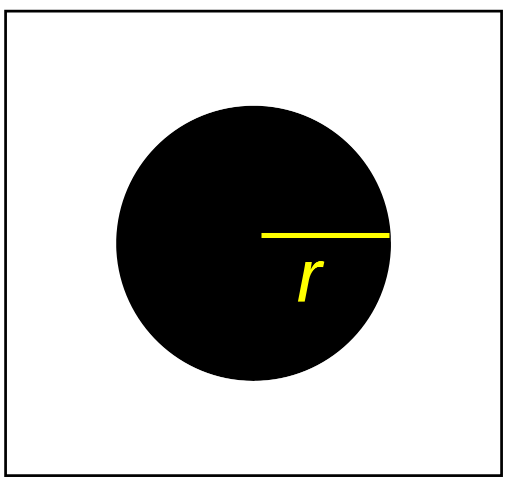
Laplacian

Scale selection

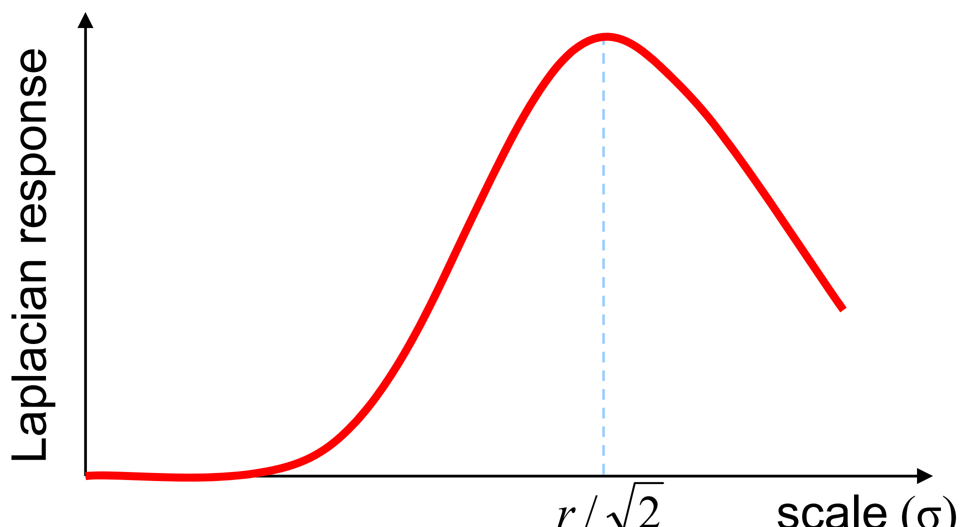
- The 2D Laplacian is given by

$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2} \quad \text{(up to scale)}$$

- Therefore, for a binary circle of radius r, the Laplacian achieves a maximum at $\sigma = r/\sqrt{2}$

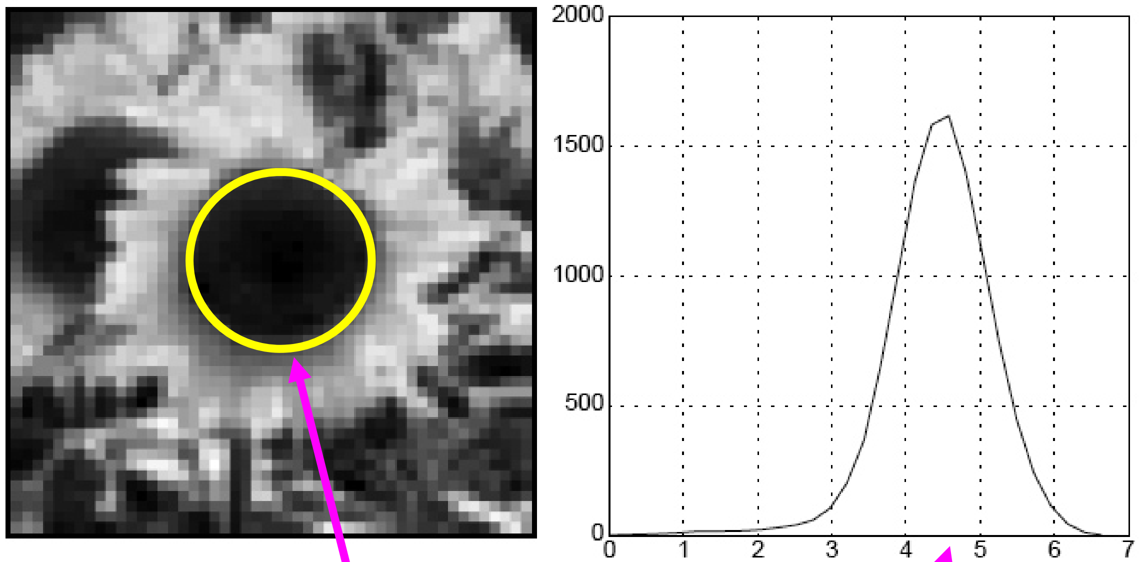


image



Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response

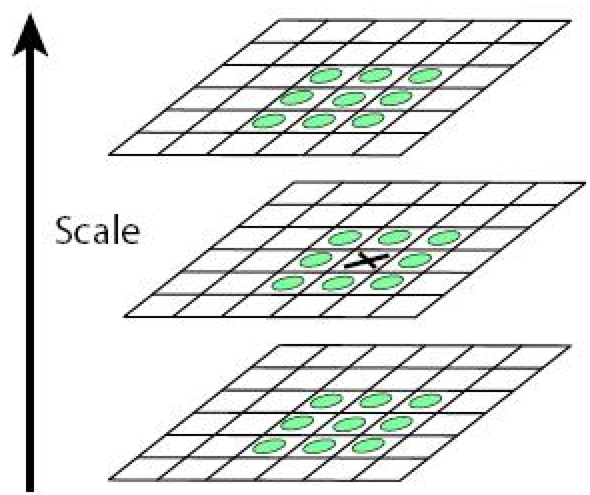


characteristic scale

T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space

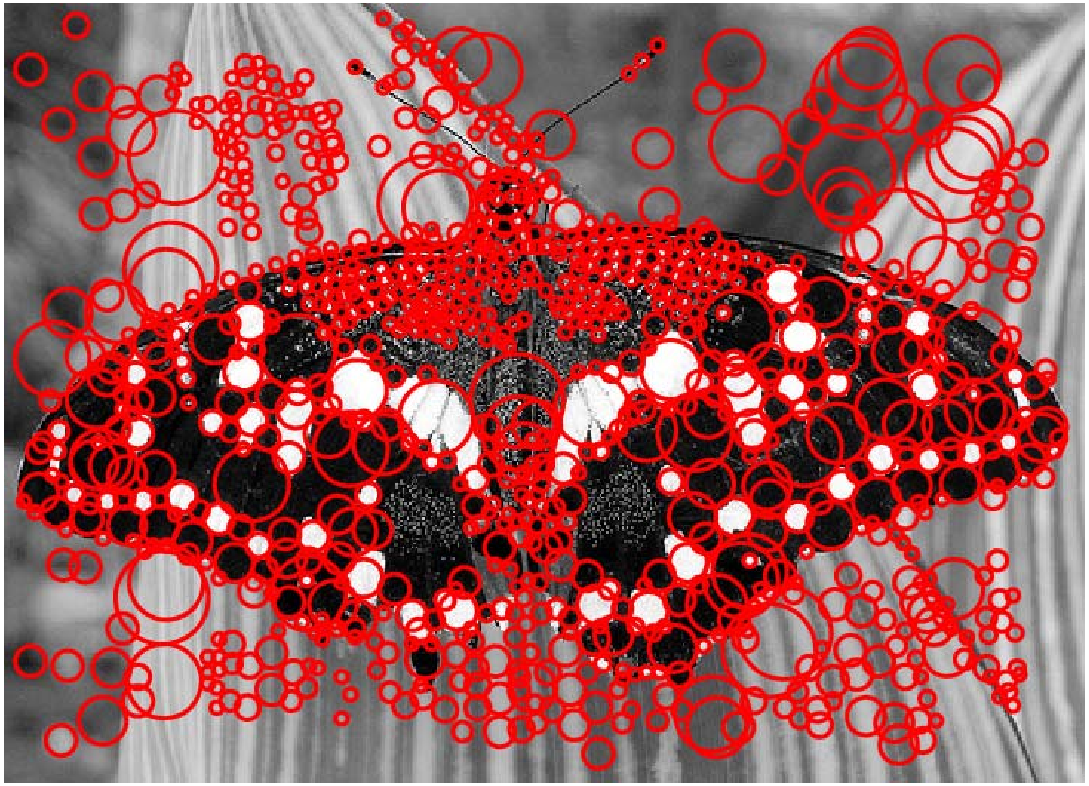


Scale-space blob detector: Example

Scale-space blob detector: Example

sigma = 11.9912

Scale-space blob detector: Example



Efficient implementation

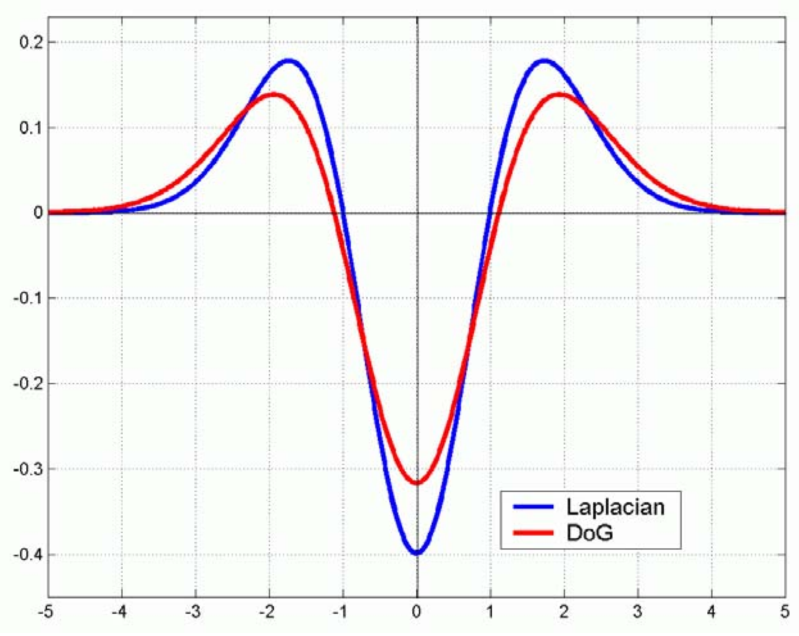
Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

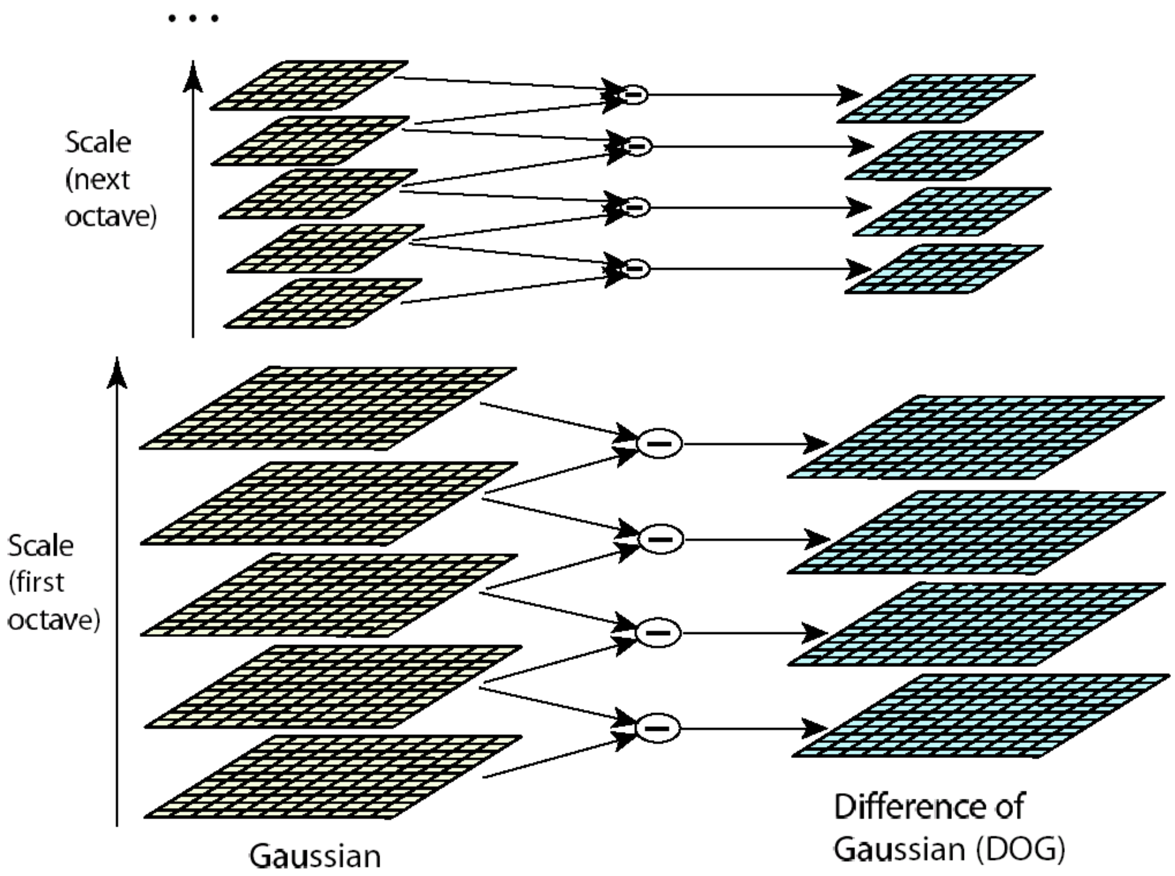
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

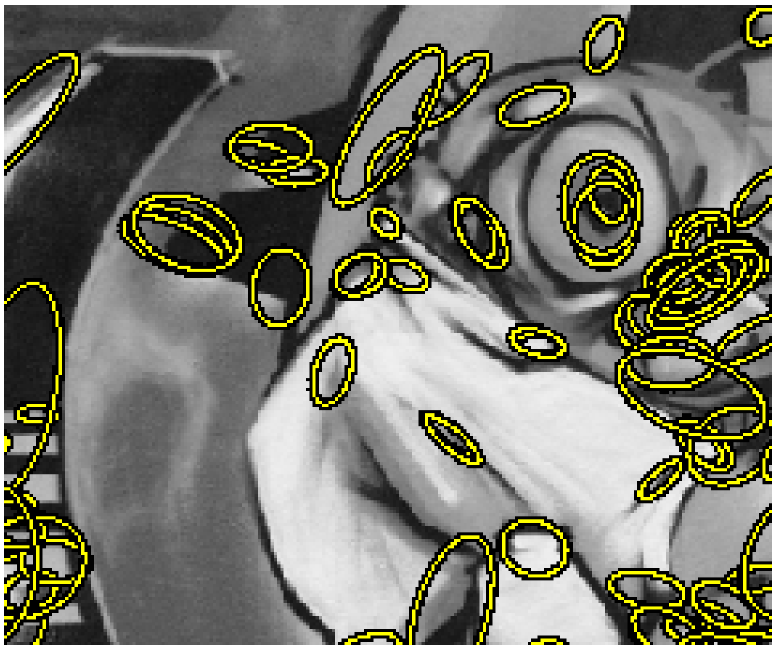


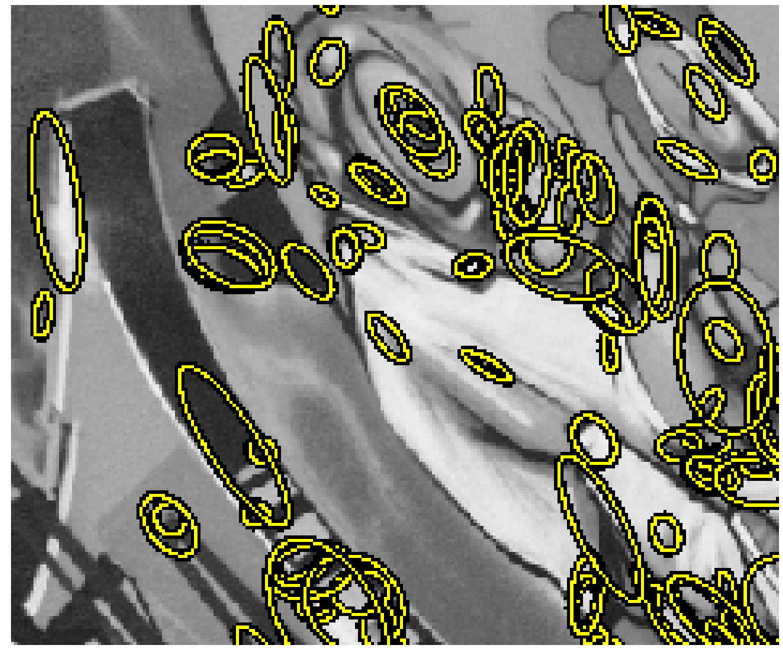
Efficient implementation



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

From scale invariance to affine invariance





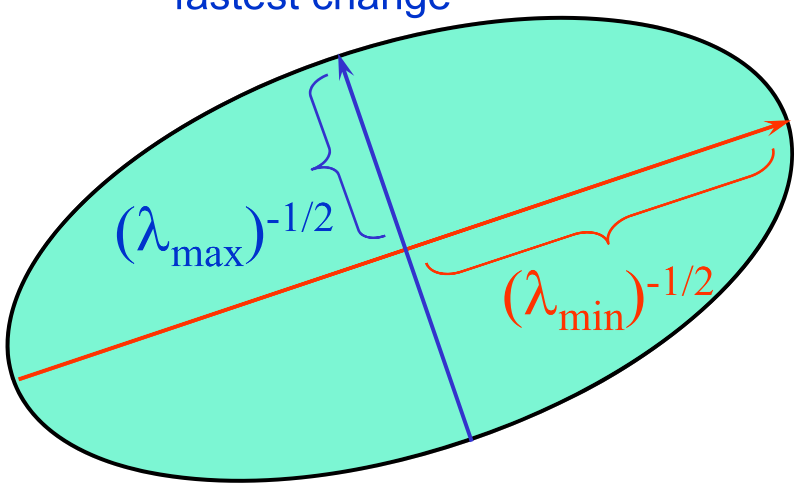
Affine adaptation

Recall: $$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

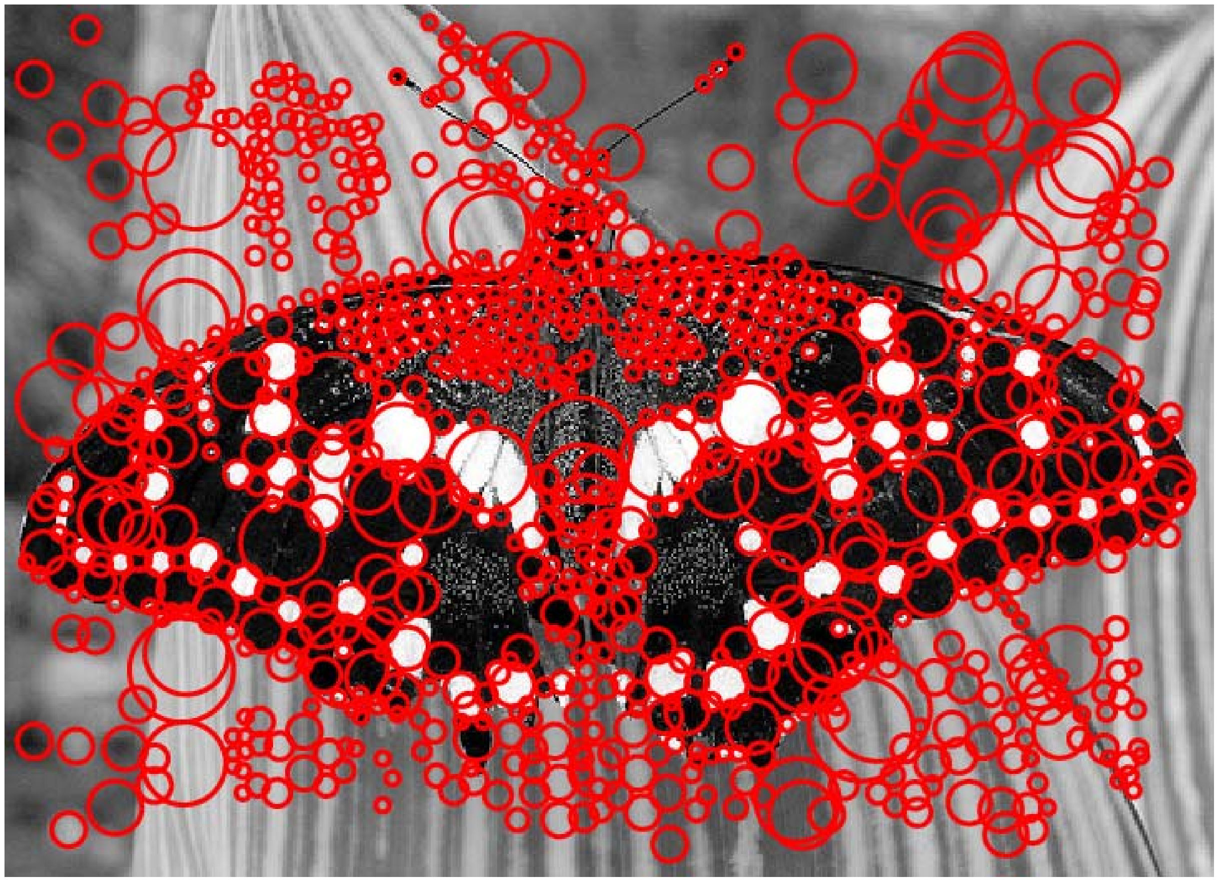
We can visualize *M* as an ellipse with axis lengths determined by the eigenvalues and orientation determined by *R*

Ellipse equation:

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

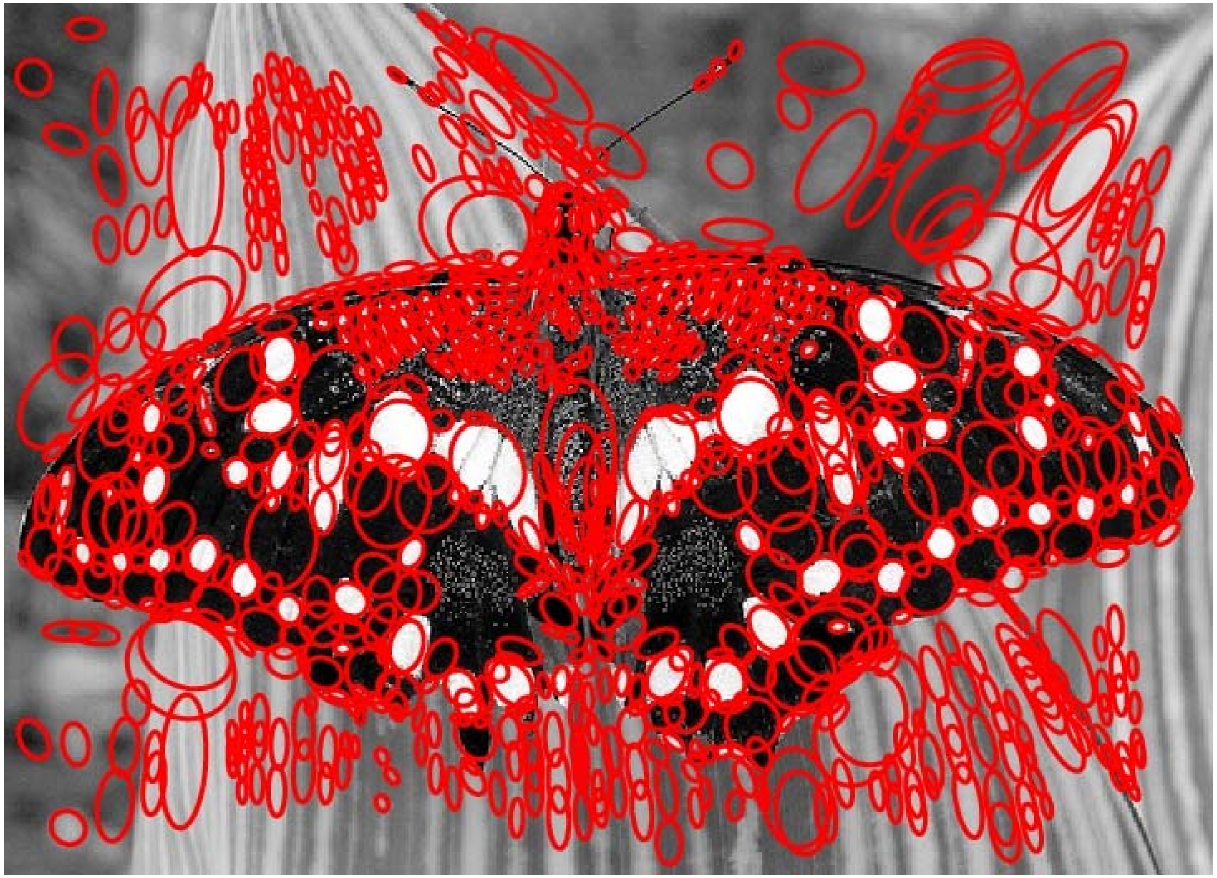


Affine adaptation example



Scale-invariant regions (blobs)

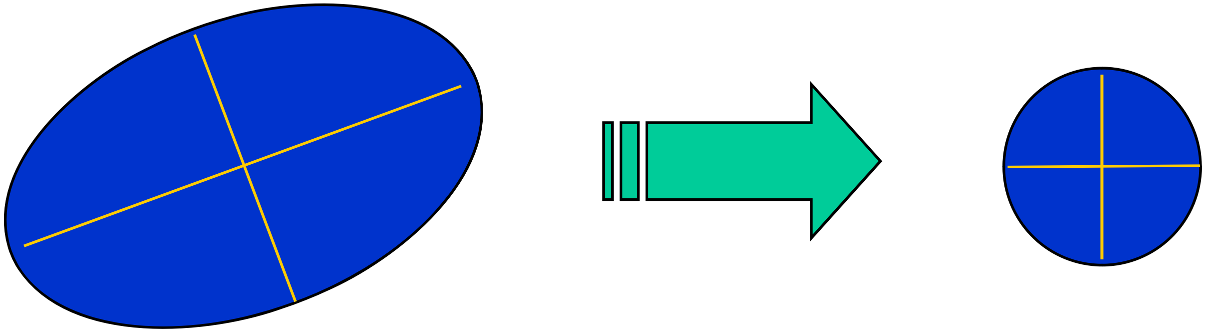
Affine adaptation example



Affine-adapted blobs

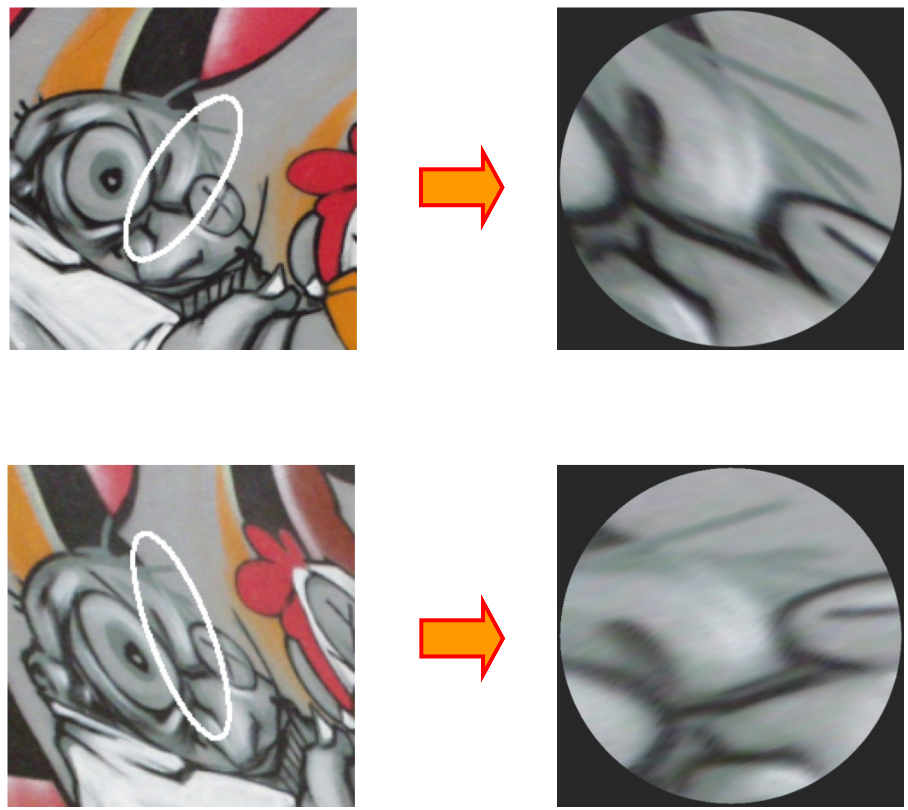
Affine normalization

- The second moment ellipse can be viewed as the “characteristic shape” of a region
- We can normalize the region by transforming the ellipse into a unit circle



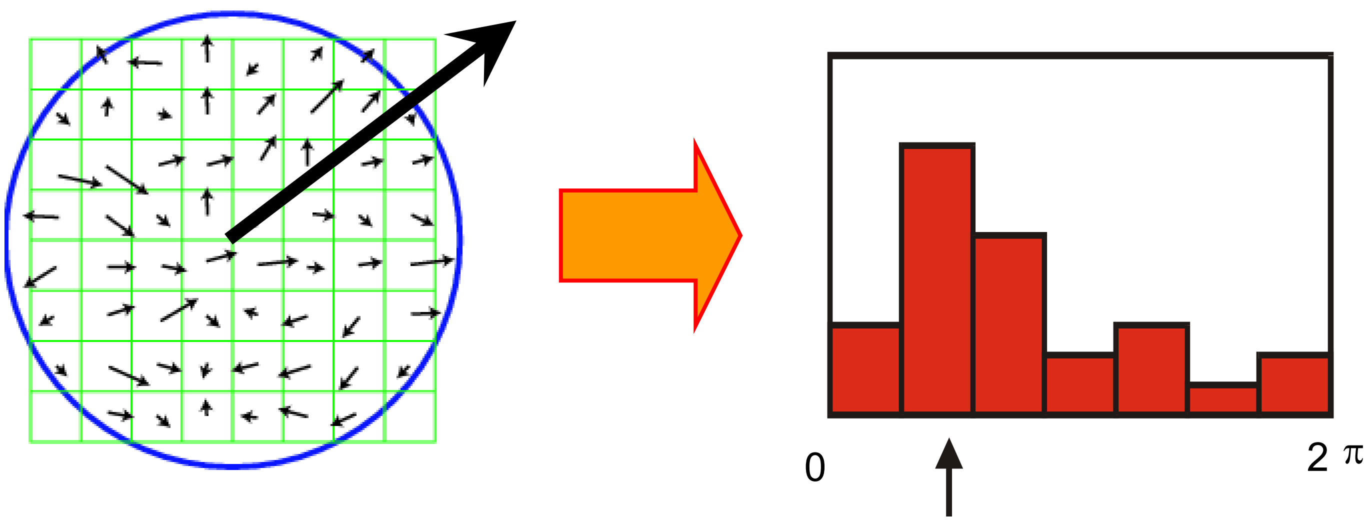
Orientation ambiguity

- There is no unique transformation from an ellipse to a unit circle
  - We can rotate or flip a unit circle, and it still stays a unit circle



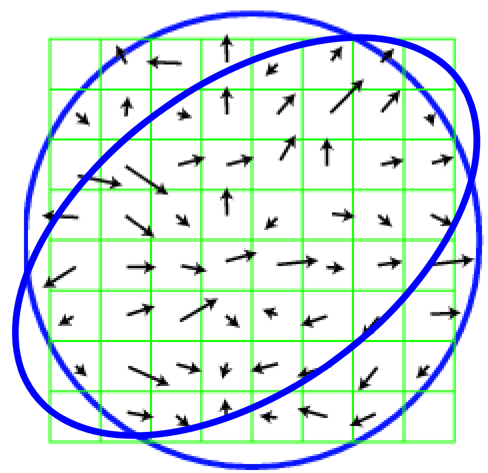
Orientation ambiguity

- There is no unique transformation from an ellipse to a unit circle
  - We can rotate or flip a unit circle, and it still stays a unit circle
- So, to assign a unique orientation to keypoints:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram

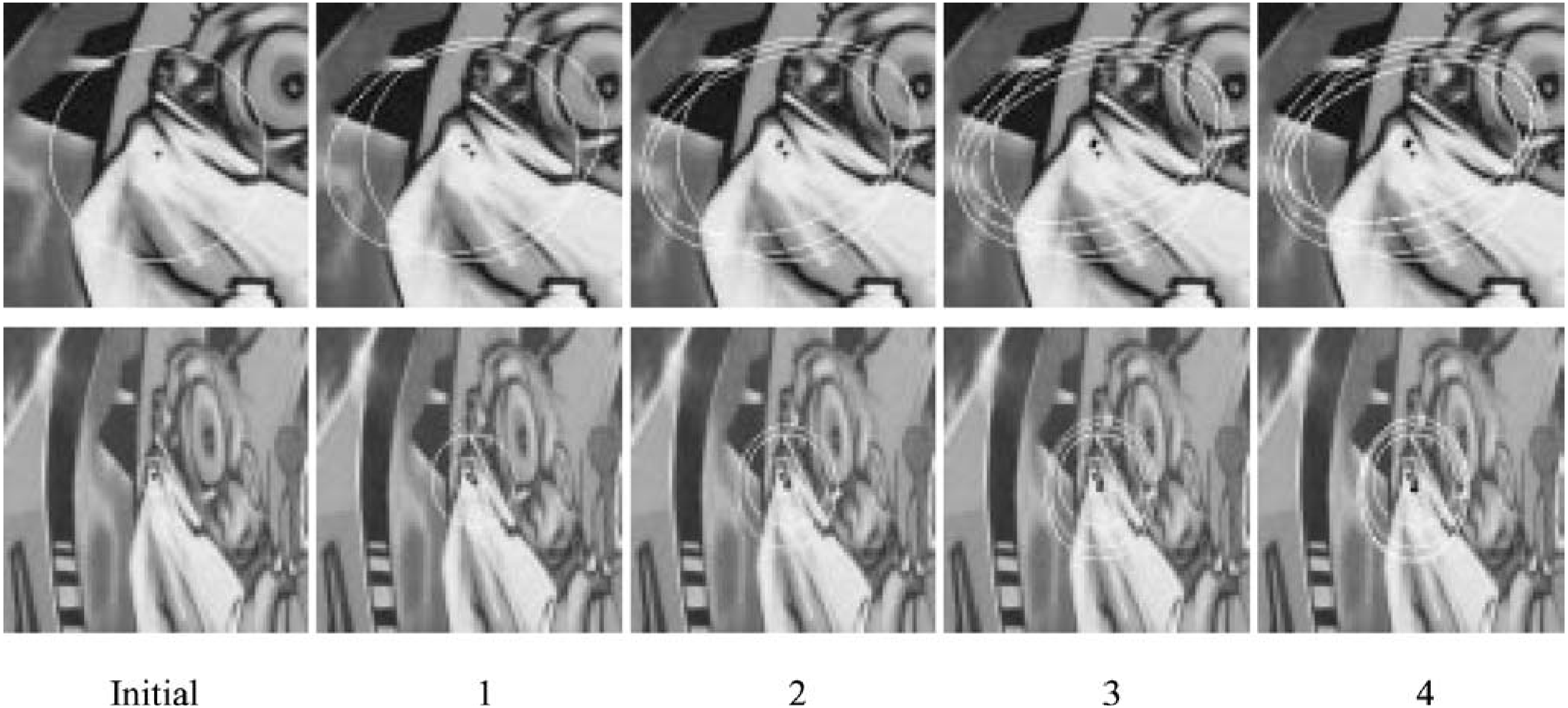


Affine adaptation

- Problem: the second moment "window" determined by weights $w(x,y)$ must match the characteristic shape of the region
- Solution: iterative approach
  - Use a circular window to compute second moment matrix
  - Perform affine adaptation to find an ellipse-shaped window
  - Recompute second moment matrix using new window and iterate



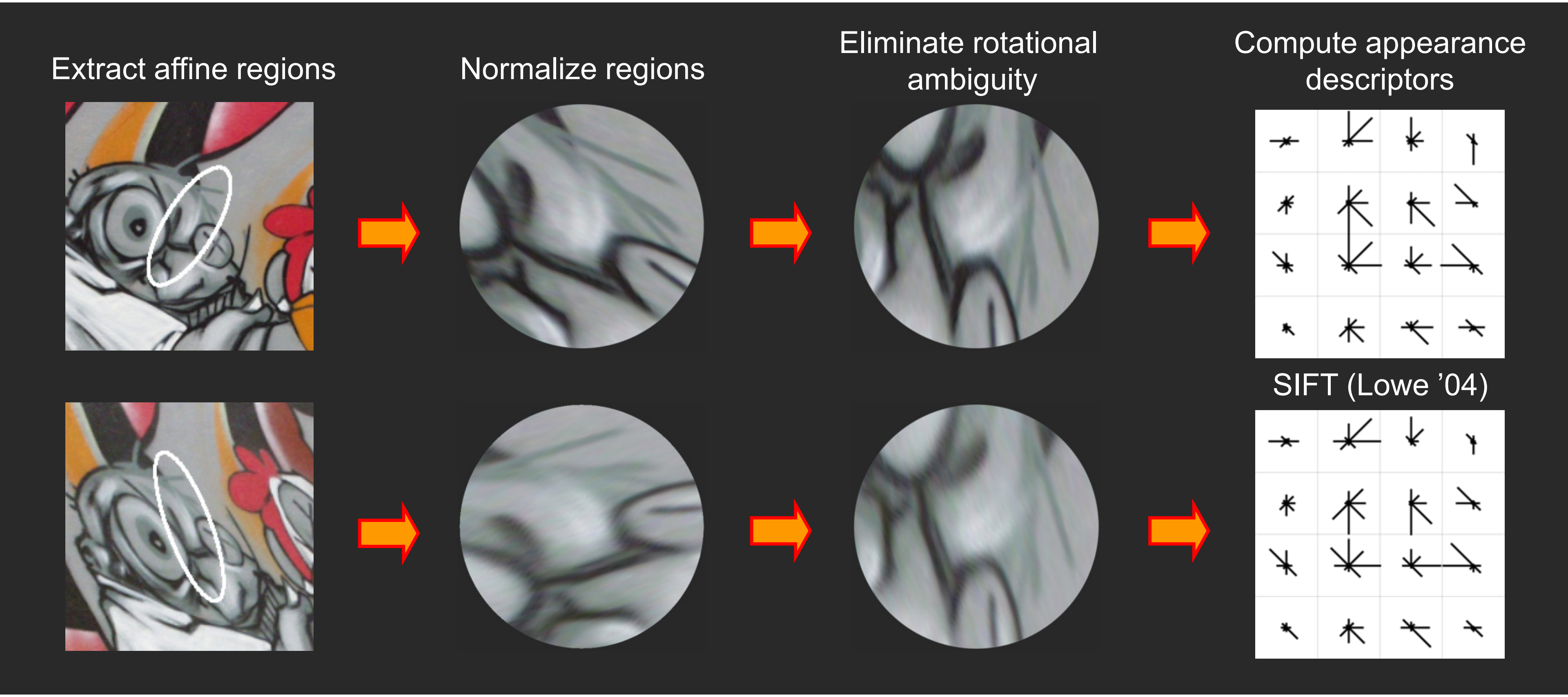
Iterative affine adaptation



K. Mikolajczyk and C. Schmid, [Scale and Affine invariant interest point detectors](), IJCV 60(1):63-86, 2004.

http://www.robots.ox.ac.uk/~vgg/research/affine/

Summary: Feature extraction



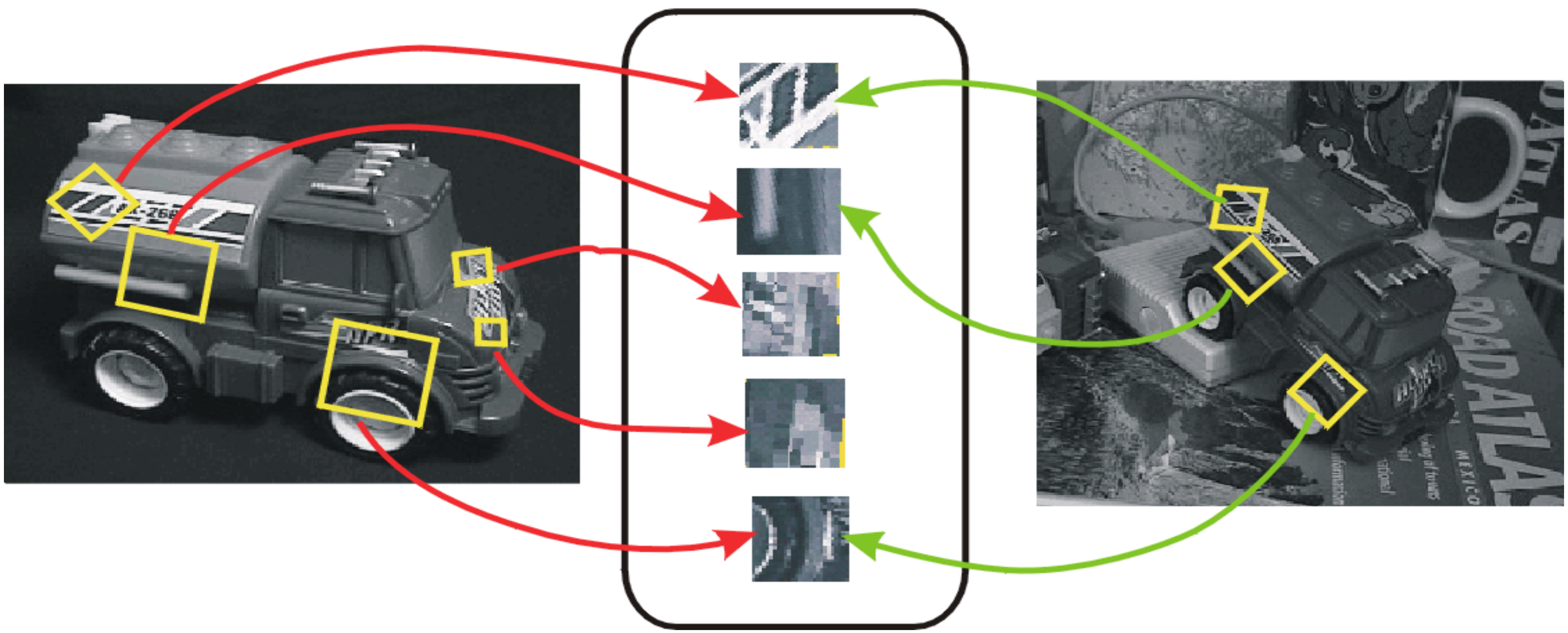
Invariance vs. covariance

Invariance:

- features(transform(image)) = features(image)

Covariance:

- features(transform(image)) = transform(features(image))



Covariant detection => invariant description

Next time: Fitting