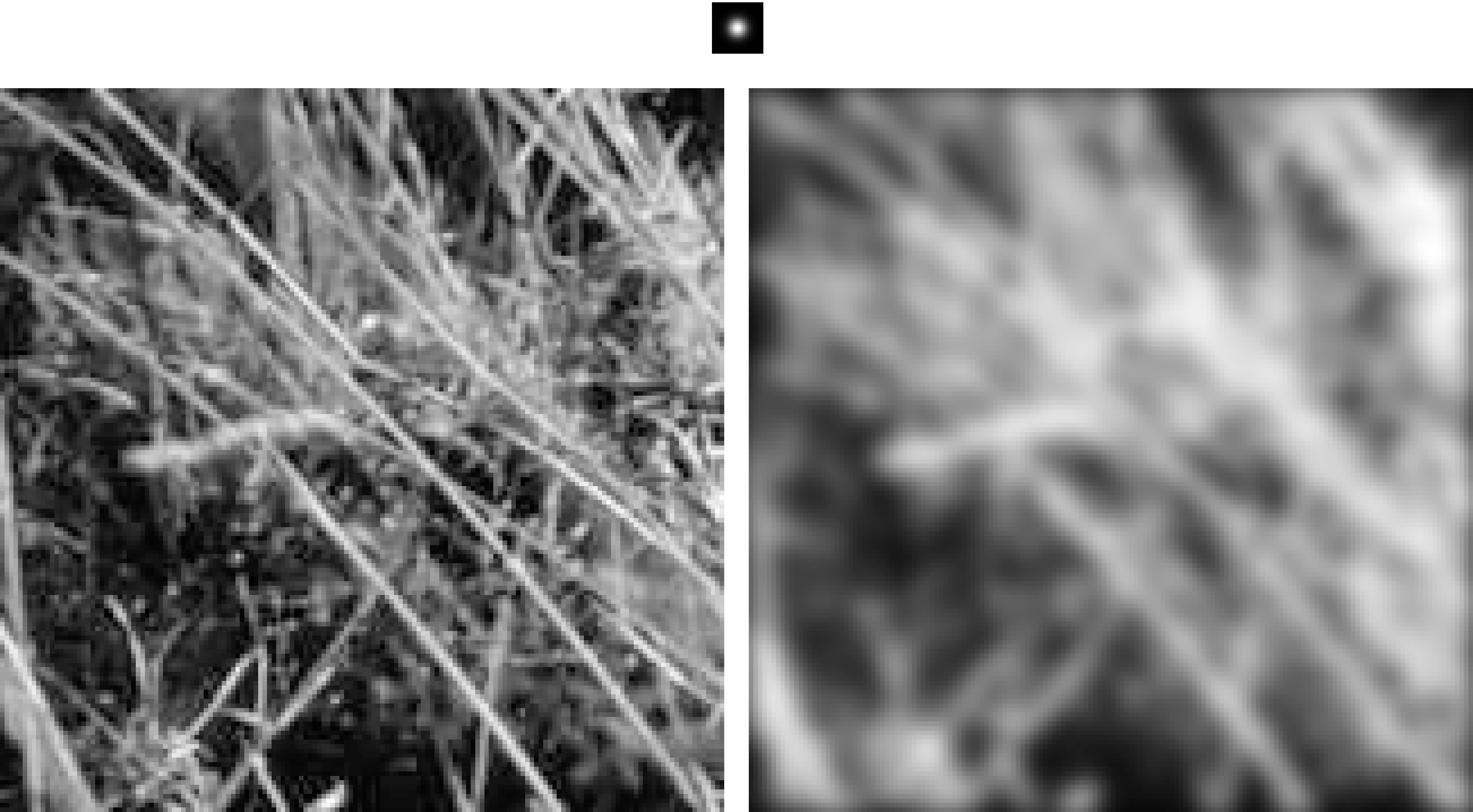
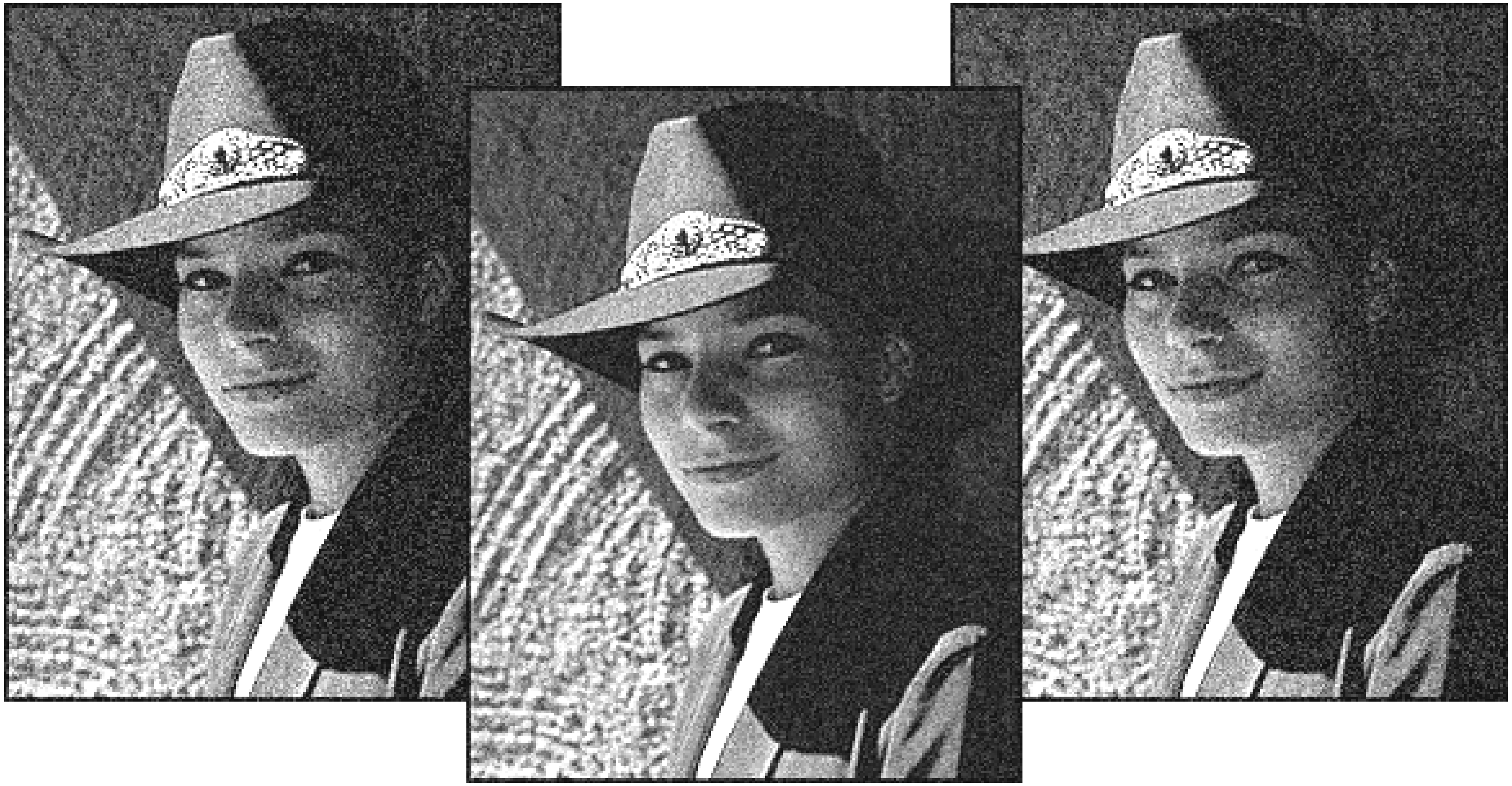
Linear filtering



Motivation: Noise reduction

Given a camera and a still scene, how can you reduce noise?



Take lots of images and average them!

What's the next best thing?

Source: S. Seitz

Moving average

- Let’s replace each pixel with a *weighted* average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for a 3x3 moving average?

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

“box filter”

Source: D. Lowe

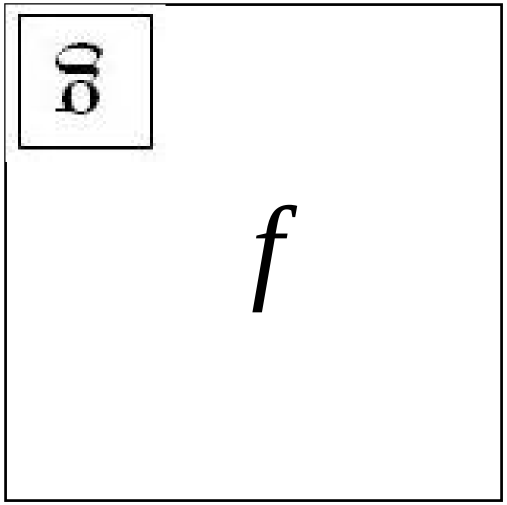
Review: Color

- What are some linear color spaces?
- What are some non-linear color spaces?
- What is a perceptually uniform color space?
- What is color constancy?
- What are some applications of color in computer vision?

Defining convolution

- Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

$$(f * g)[m,n] = \sum_{k,l} f[m-k, n-l]\, g[k,l]$$



- Convention: kernel is “flipped”
- MATLAB: conv2 vs. filter2 (also imfilter)

Source: F. Durand

Key properties

- **Linearity:** filter($f_1 + f_2$) = filter(f_1) + filter(f_2)
- **Shift invariance:** same behavior regardless of pixel location: filter(shift(f)) = shift(filter(f))
- Theoretical result: any linear shift-invariant operator can be represented as a convolution

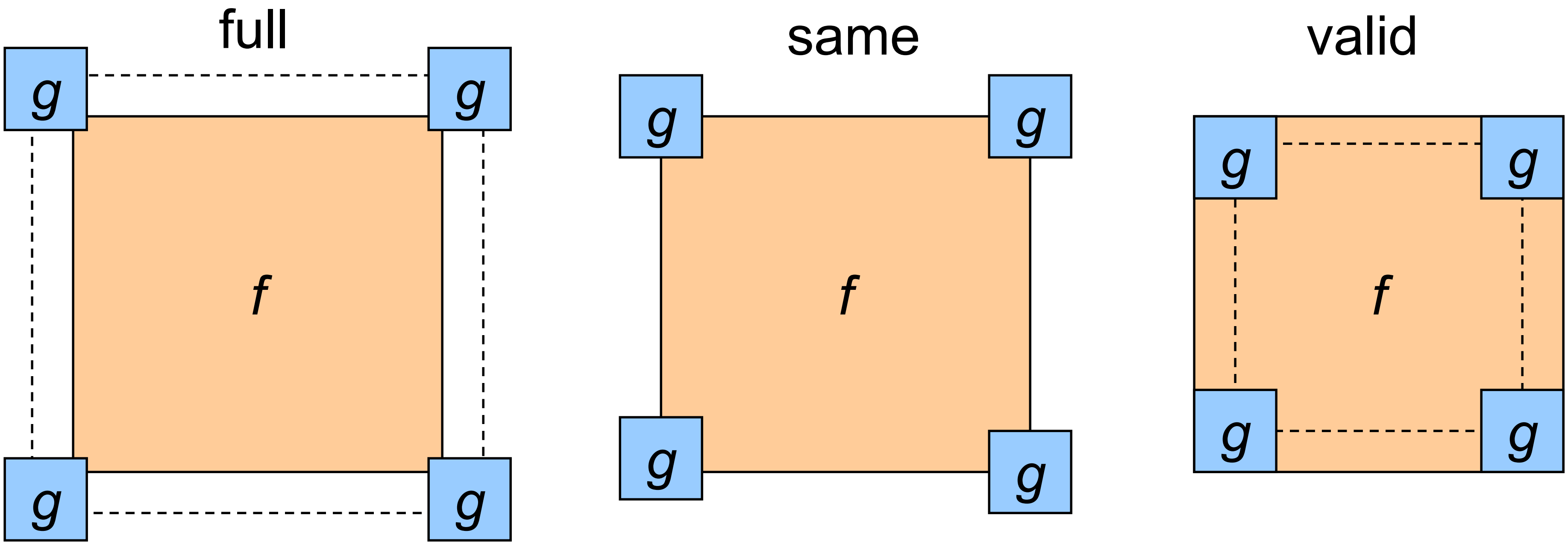
Properties in more detail

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k\,(a * b)$
- Identity: unit impulse e = [..., 0, 0, 1, 0, 0, ...],
  $a * e = a$

Annoying details

What is the size of the output?

- MATLAB: filter2(g, f, *shape*)
  - *shape* = ‘full’: output size is sum of sizes of f and g
  - *shape* = ‘same’: output size is same as f
  - *shape* = ‘valid’: output size is difference of sizes of f and g



Annoying details

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods:
  - clip filter (black)
  - wrap around
  - copy edge
  - reflect across edge

Source: S. Marschner

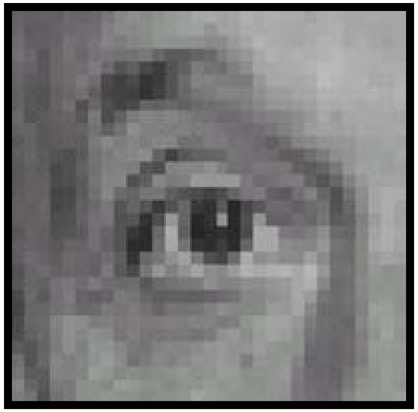
Annoying details

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods (MATLAB):
  - clip filter (black): imfilter(f, g, 0)
  - wrap around: imfilter(f, g, ‘circular’)
  - copy edge: imfilter(f, g, ‘replicate’)
  - reflect across edge: imfilter(f, g, ‘symmetric’)

Source: S. Marschner

Practice with linear filters



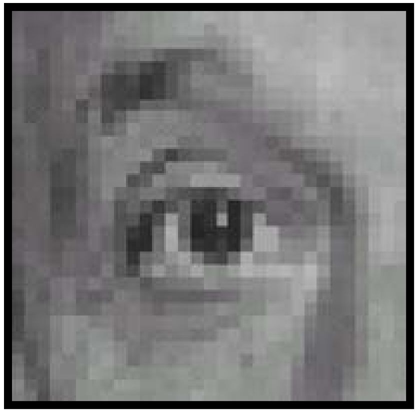
Original

0	0	0
0	1	0
0	0	0

?

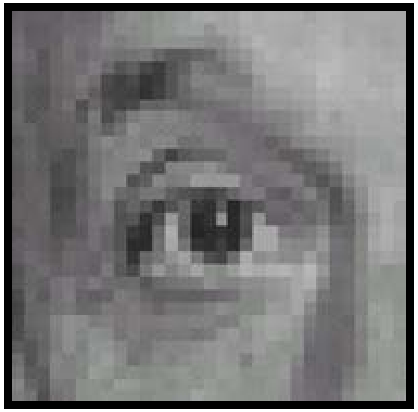
Source: D. Lowe

Practice with linear filters



Original

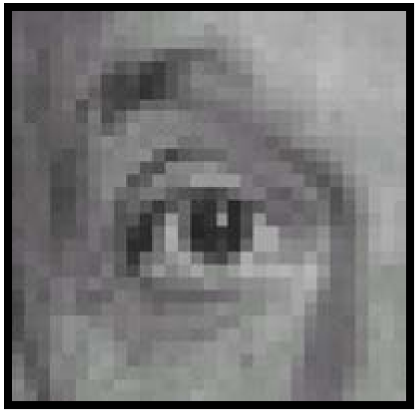
0	0	0
0	1	0
0	0	0



Filtered
(no change)

Source: D. Lowe

Practice with linear filters



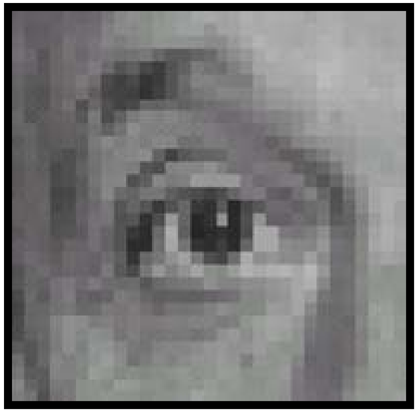
Original

0	0	0
0	0	1
0	0	0

?

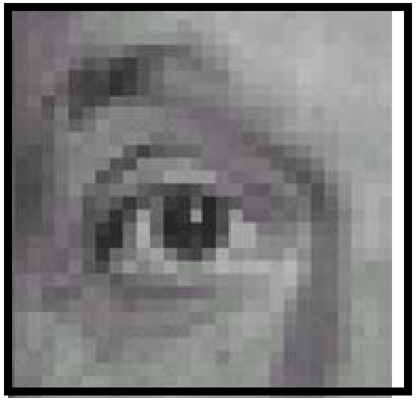
Source: D. Lowe

Practice with linear filters



Original

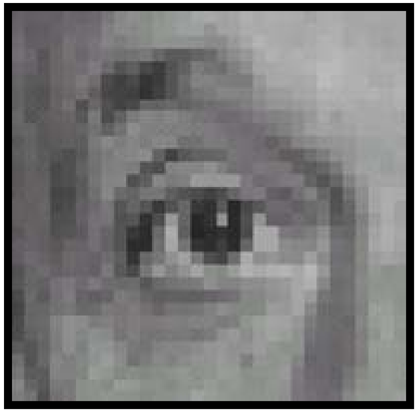
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Source: D. Lowe

Practice with linear filters



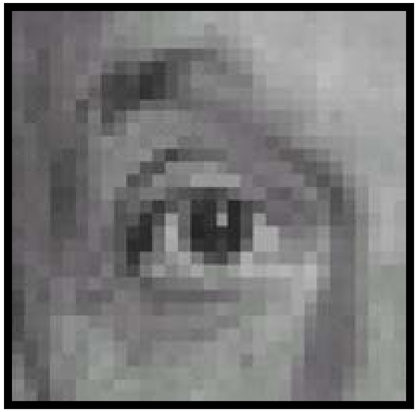
Original

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

?

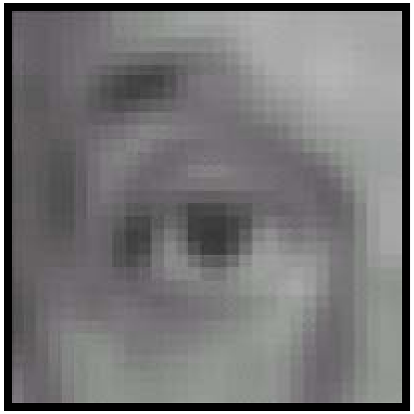
Source: D. Lowe

Practice with linear filters



Original

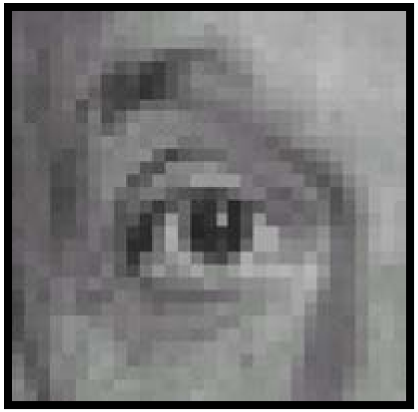
$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters



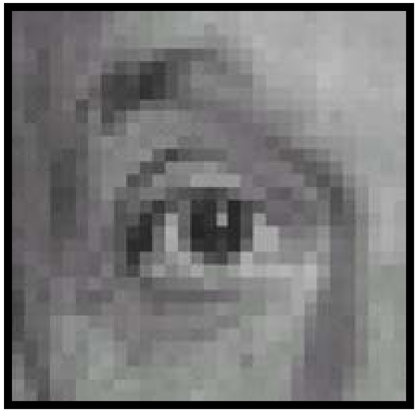
Original

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \qquad ?$$

(Note that filter sums to 1)

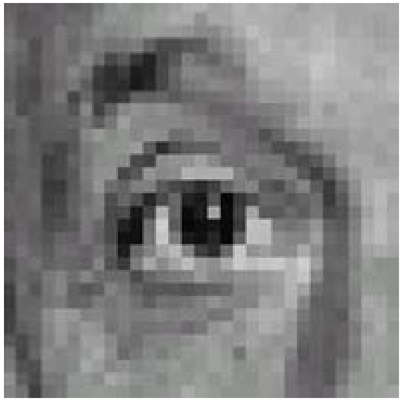
Source: D. Lowe

Practice with linear filters



Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

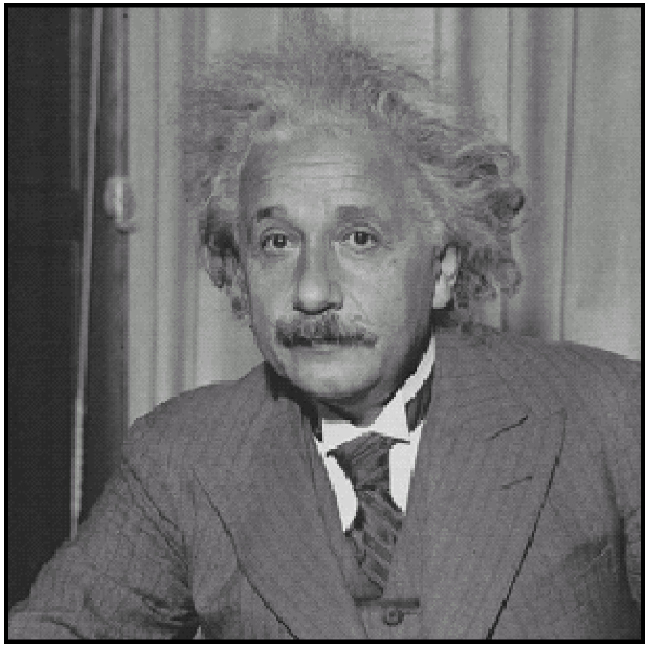


Sharpening filter

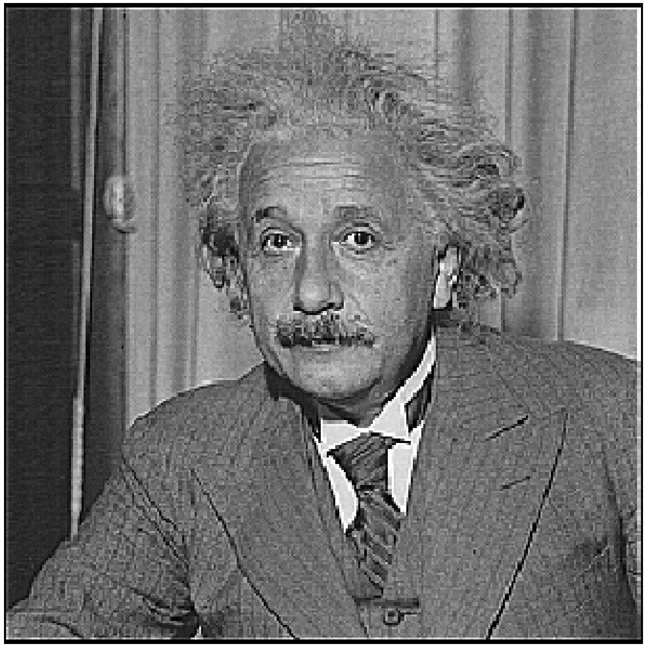
- Accentuates differences with local average

Source: D. Lowe

Sharpening



before

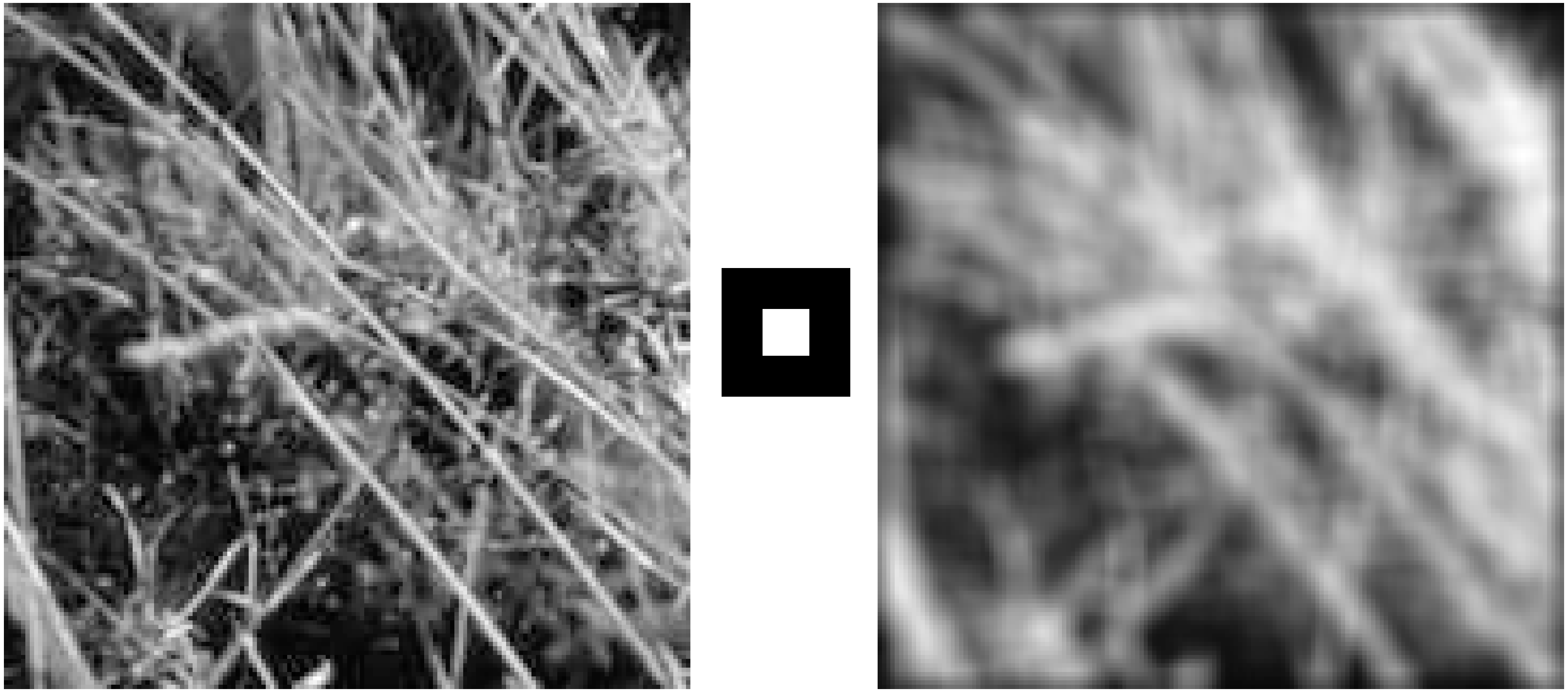


after

Source: D. Lowe

Smoothing with box filter revisited

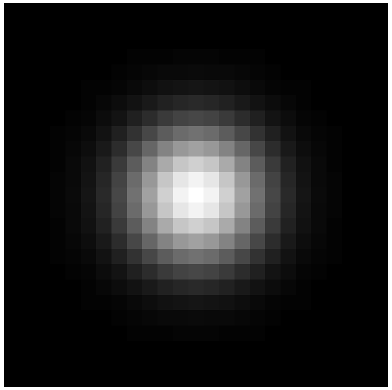
- Smoothing with an average actually doesn't compare at all well with a defocused lens
- Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob; but the averaging process would give a little square



Source: D. Forsyth

Smoothing with box filter revisited

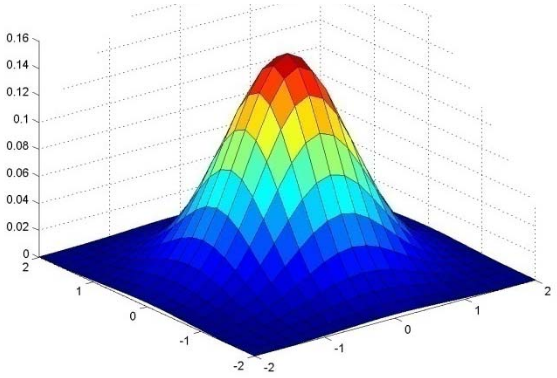
- Smoothing with an average actually doesn’t compare at all well with a defocused lens
- Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob; but the averaging process would give a little square
- Better idea: to eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center, like so:

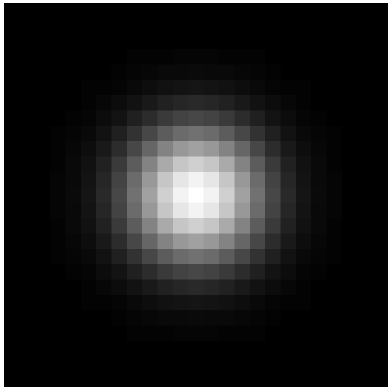


“fuzzy blob”

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

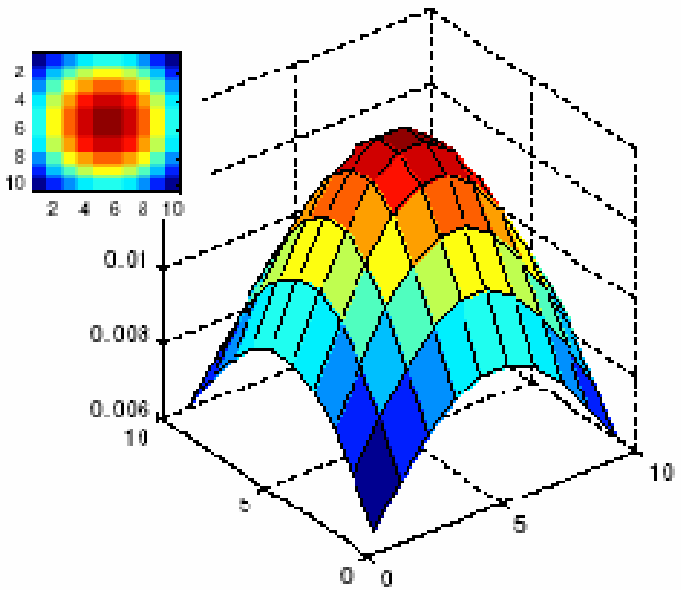
5 x 5, σ = 1

- Constant factor at front makes volume sum to 1 (can be ignored, as we should re-normalize weights to sum to 1 in any case)

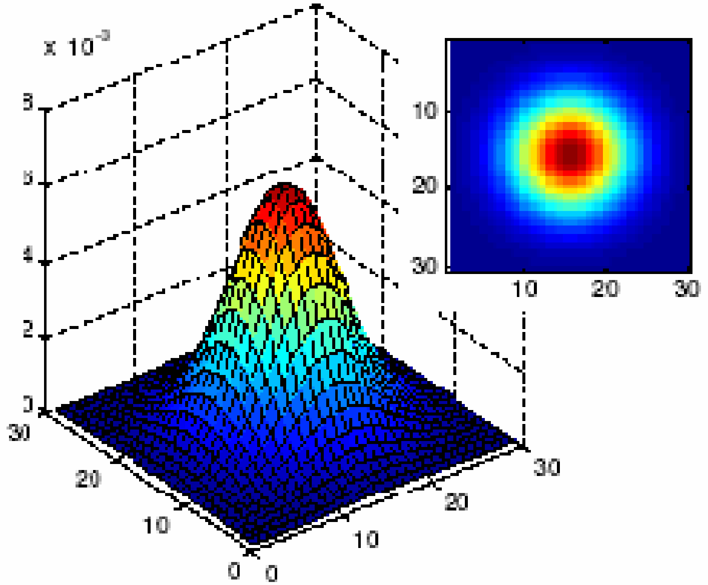
Source: C. Rasmussen

Choosing kernel width

- Gaussian filters have infinite support, but discrete filters use finite kernels



σ = 5 with 10x10 kernel

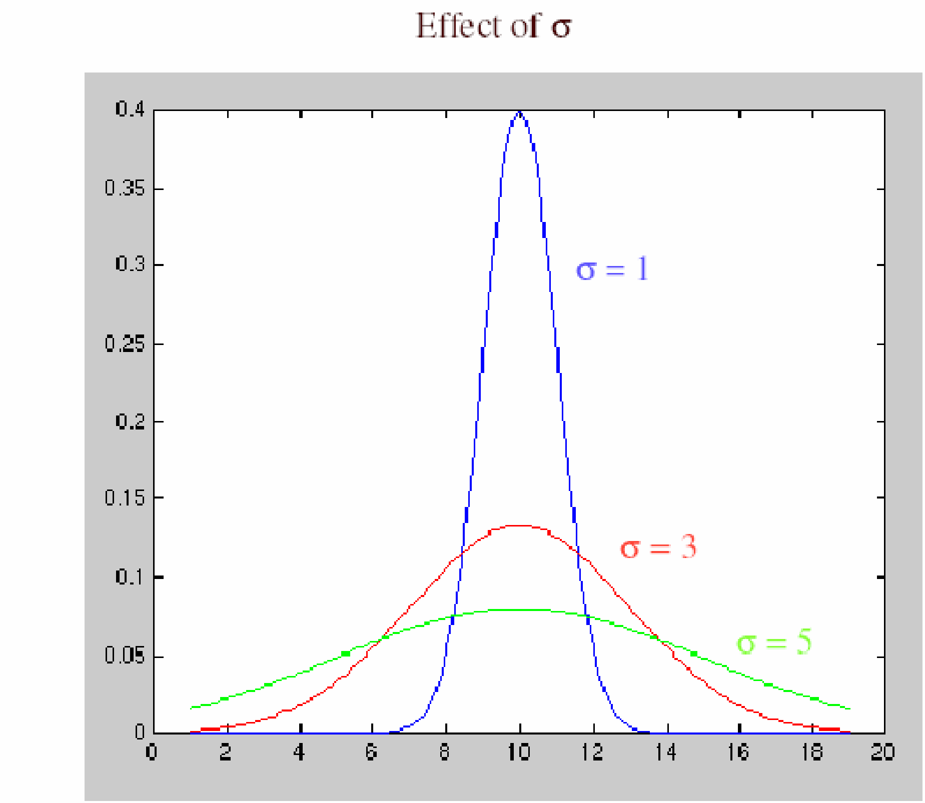


σ = 5 with 30x30 kernel

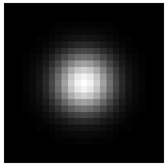
Source: K. Grauman

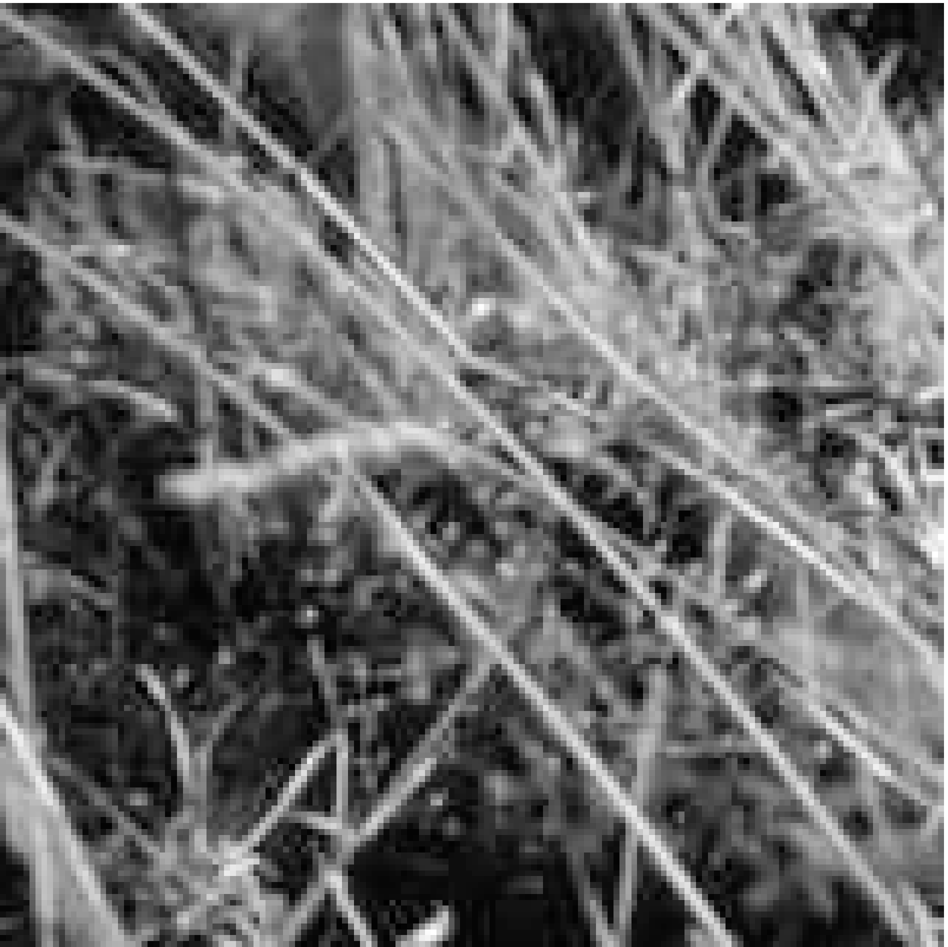
Choosing kernel width

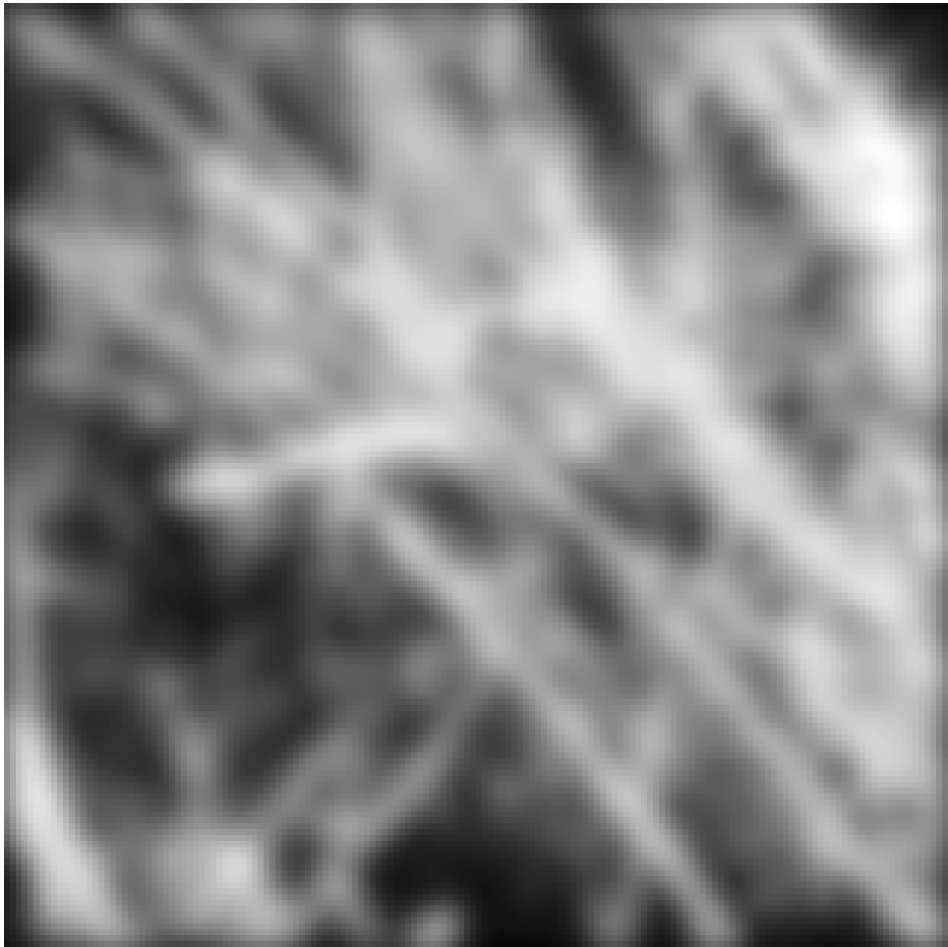
- Rule of thumb: set filter half-width to about 3 σ



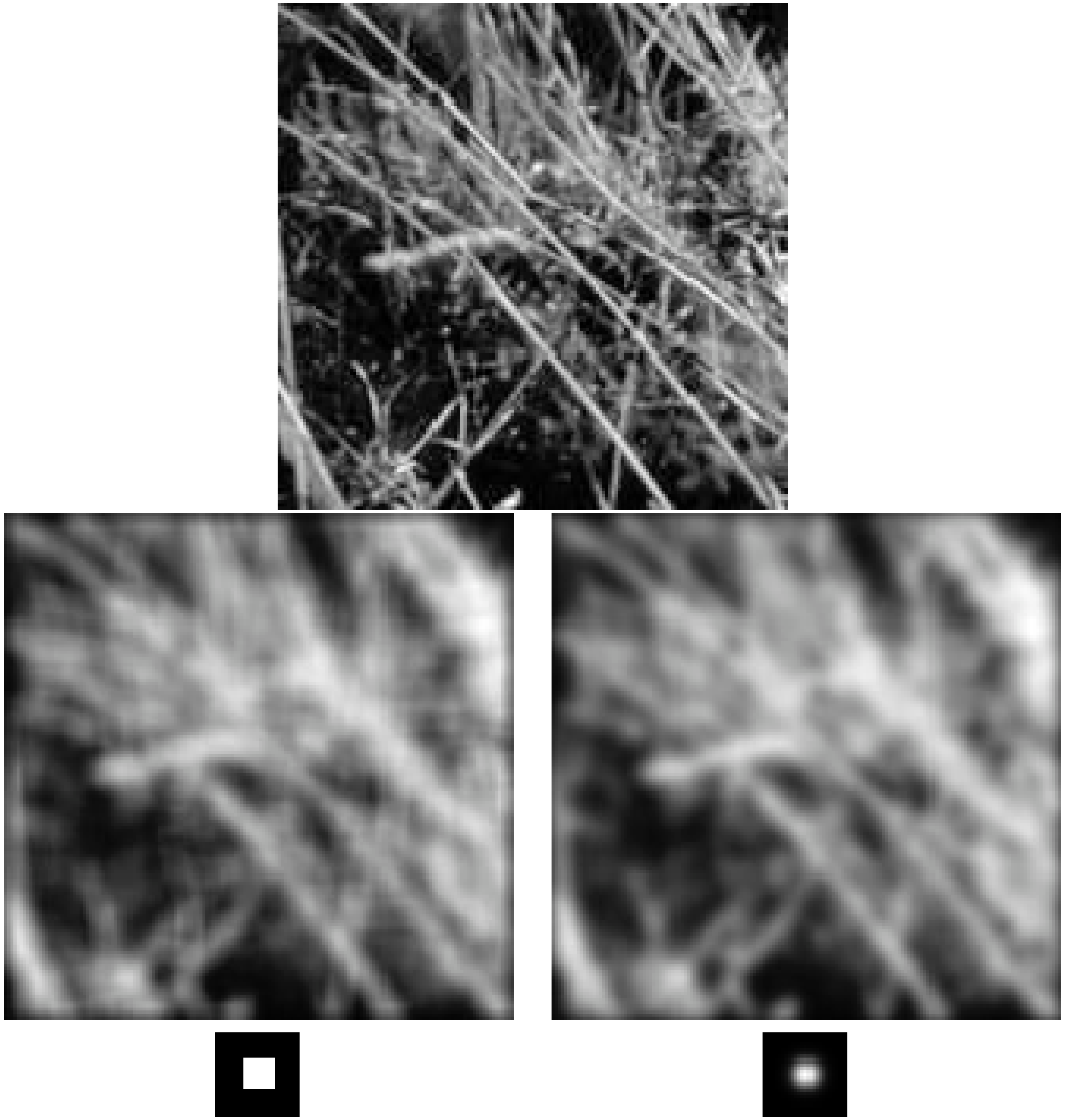
Example: Smoothing with a Gaussian







Mean vs. Gaussian filtering



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of *x* and the other a function of *y*

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array}$$

The filter factors
into a product of 1D
filters:

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} \text{ x } \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array}$$

Perform convolution
along rows:

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline & 11 & \\ \hline & 18 & \\ \hline & 18 & \\ \hline \end{array}$$

Followed by convolution
along the remaining column:

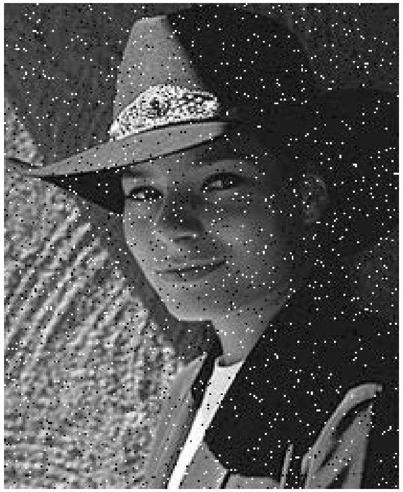
Source: K. Grauman

Separability

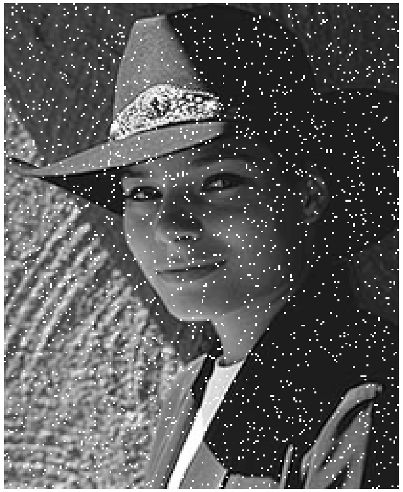
- Why is separability useful in practice?

Noise

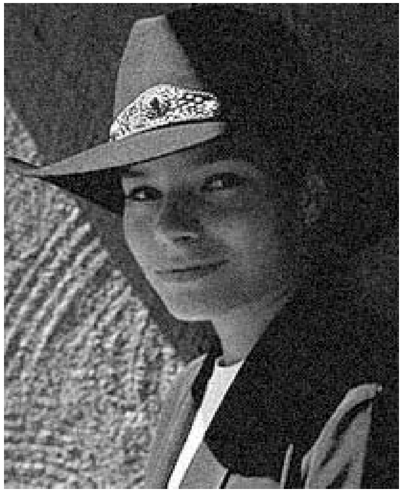
Original



Salt and pepper noise



Impulse noise



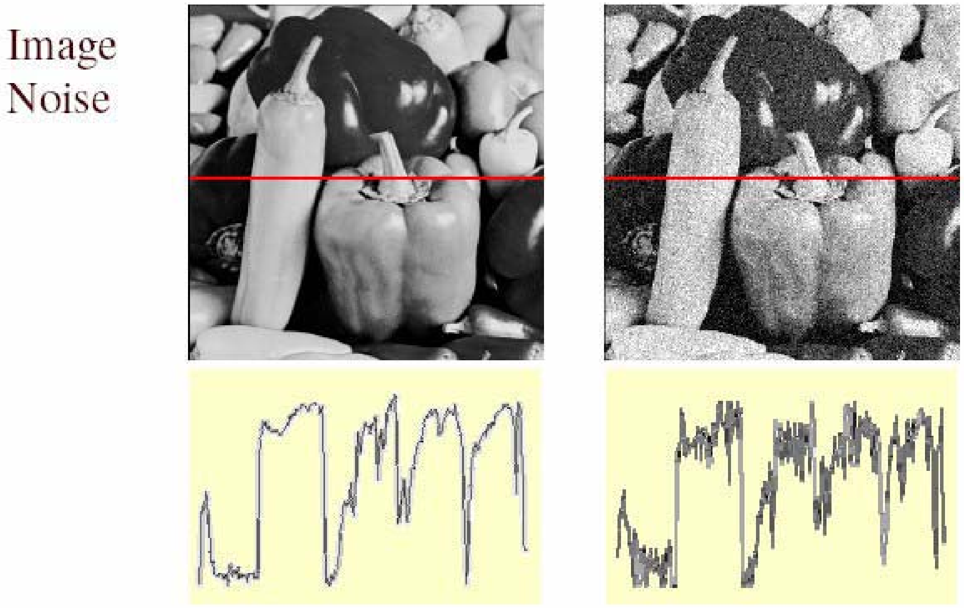
Gaussian noise

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Source: S. Seitz

Gaussian noise

- Mathematical model: sum of many independent factors
- Good for small standard deviations
- Assumption: independent, zero-mean noise

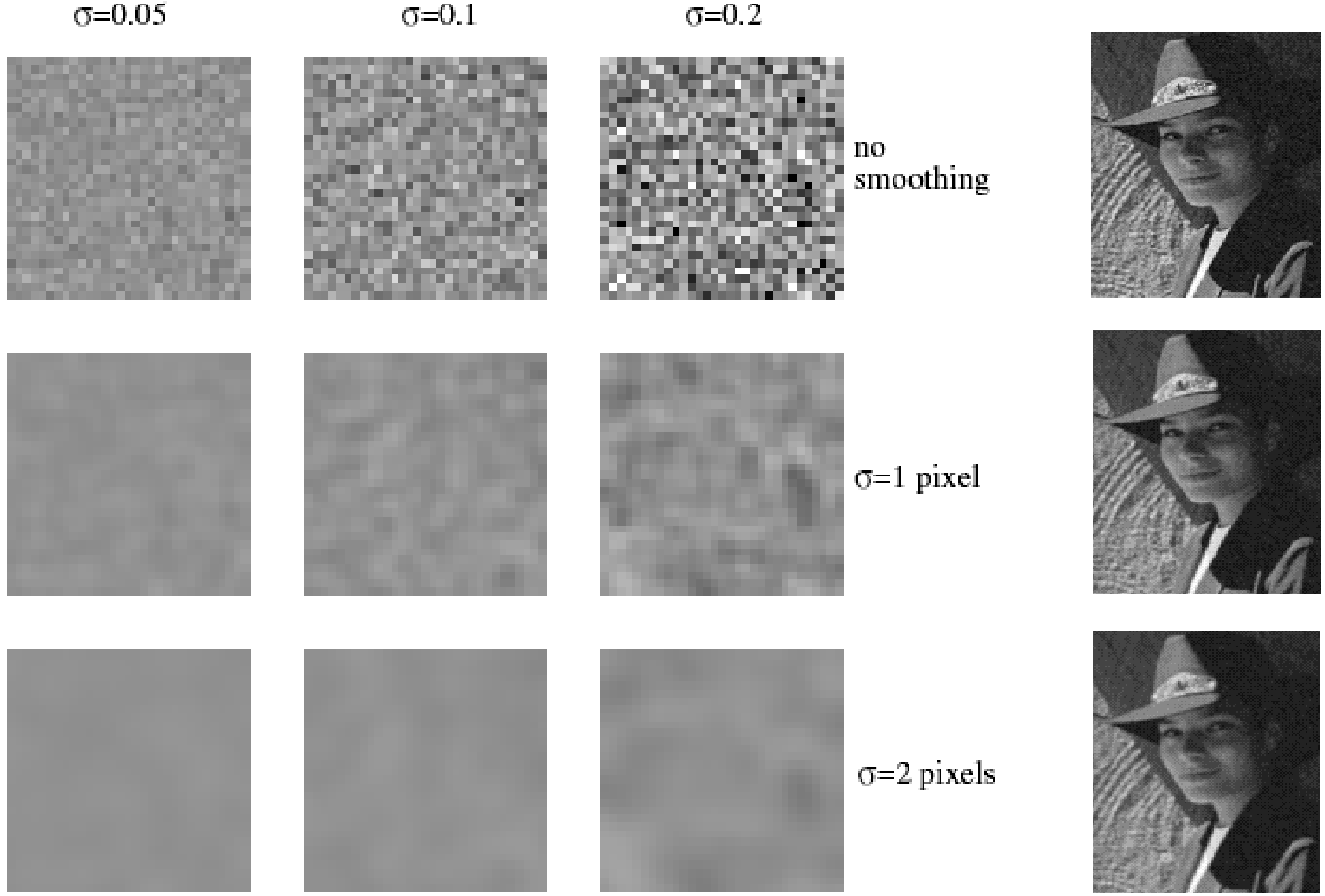


$$f(x,y) = \overbrace{\hat{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

Source: M. Hebert

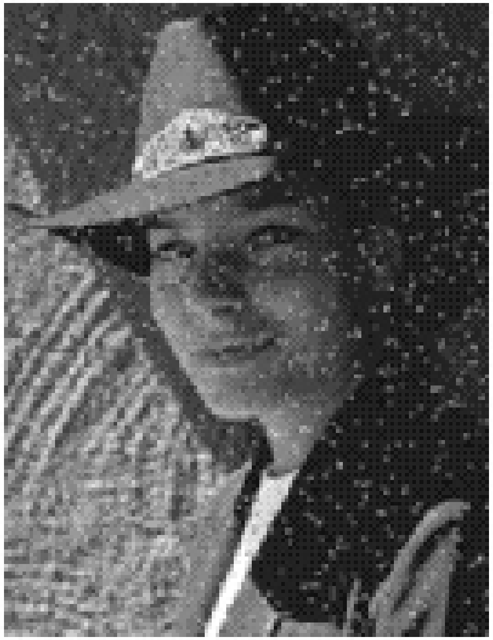
Reducing Gaussian noise



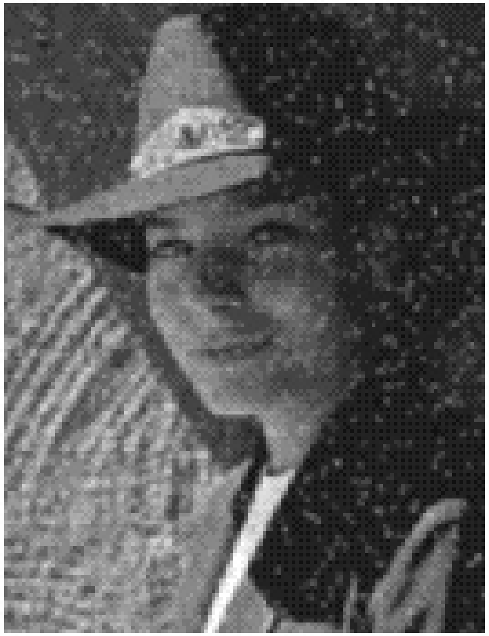
Smoothing with larger standard deviations suppresses noise, but also blurs the image

Reducing salt-and-pepper noise

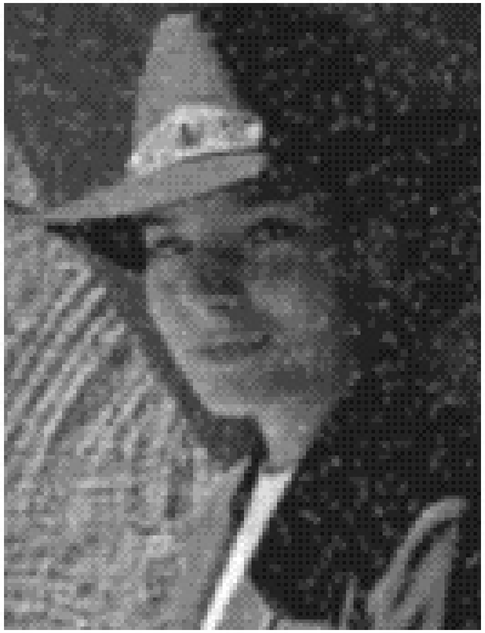
3x3



5x5



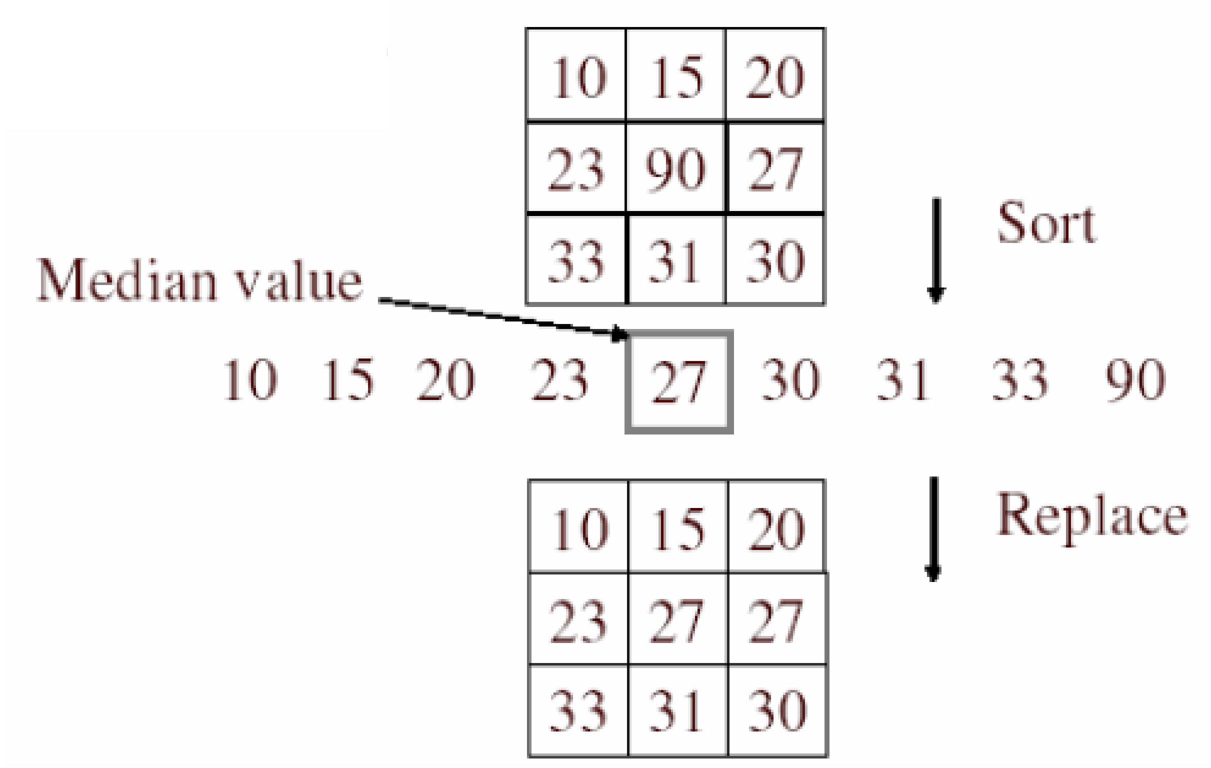
7x7



What’s wrong with the results?

Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



10	15	20
23	90	27
33	31	30

Sort

Median value

10 15 20 23 27 30 31 33 90

Replace

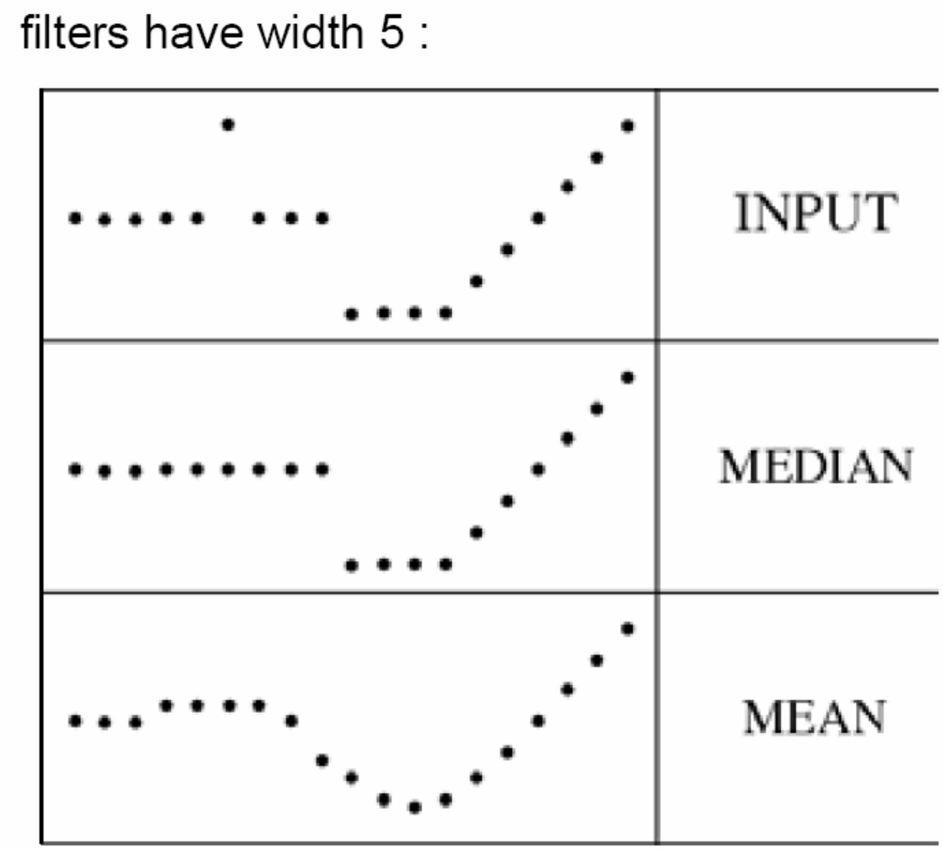
10	15	20
23	27	27
33	31	30

- Is median filtering linear?

Source: K. Grauman

Median filter

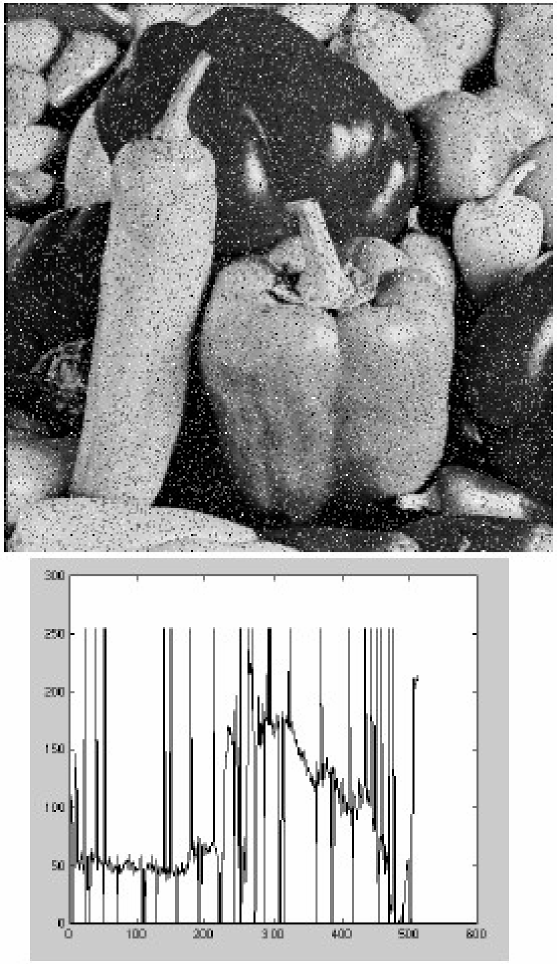
- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers



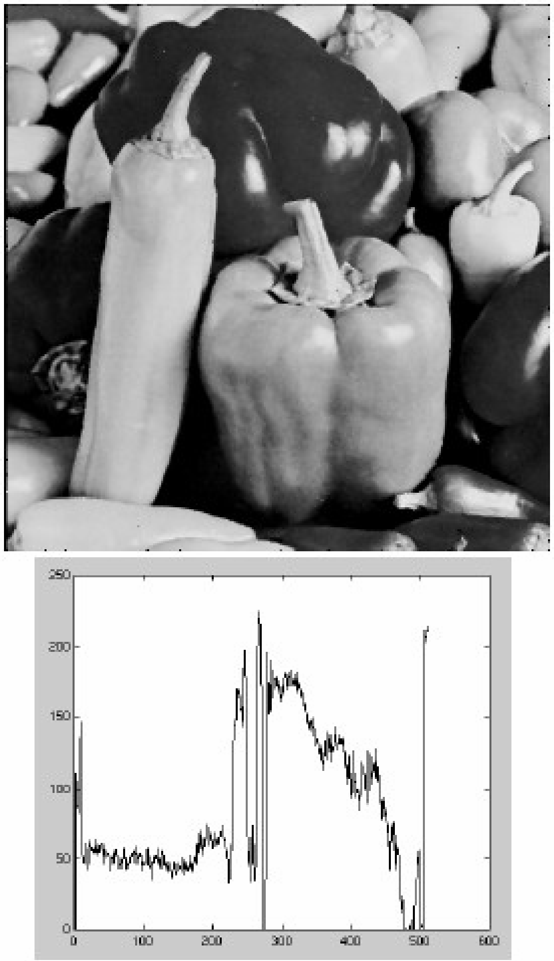
Source: K. Grauman

Median filter

Salt-and-pepper noise



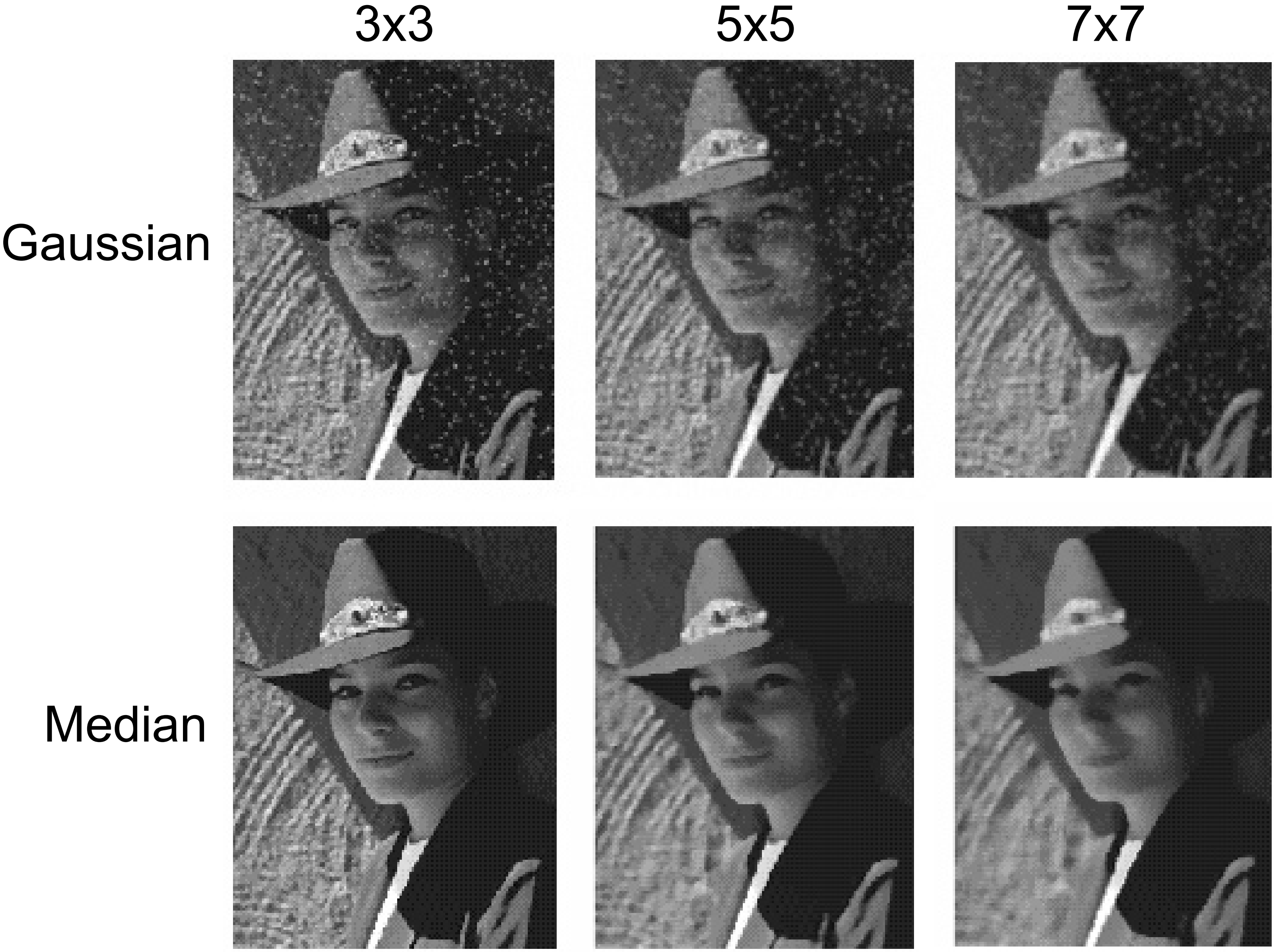
Median filtered



MATLAB: medfilt2(image, [h w])

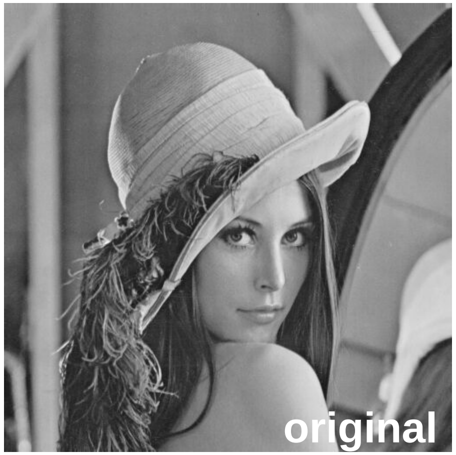
Source: M. Hebert

Median vs. Gaussian filtering



Sharpening revisited

What does blurring take away?



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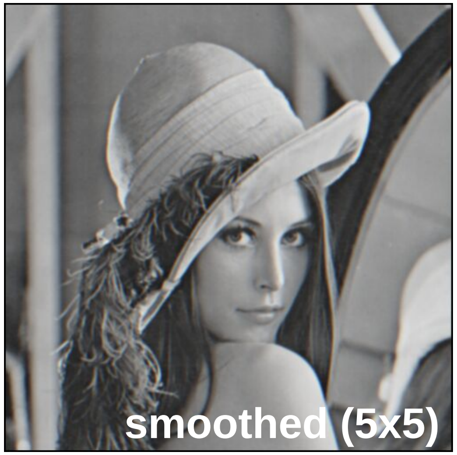

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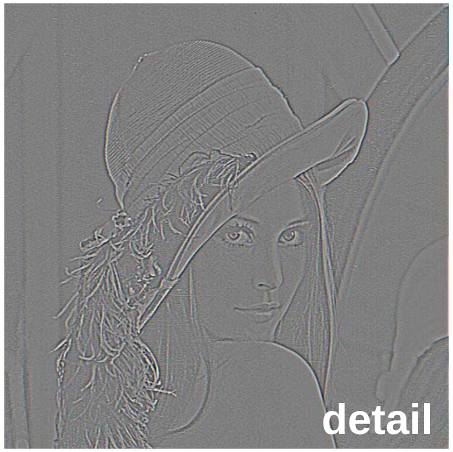
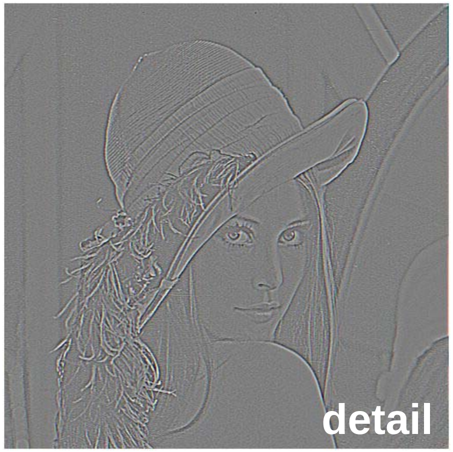


Let’s add it back:

+ α



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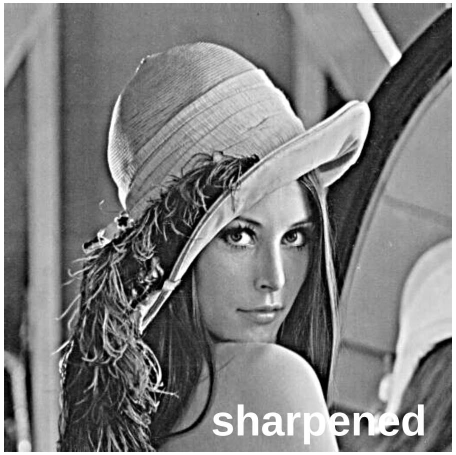


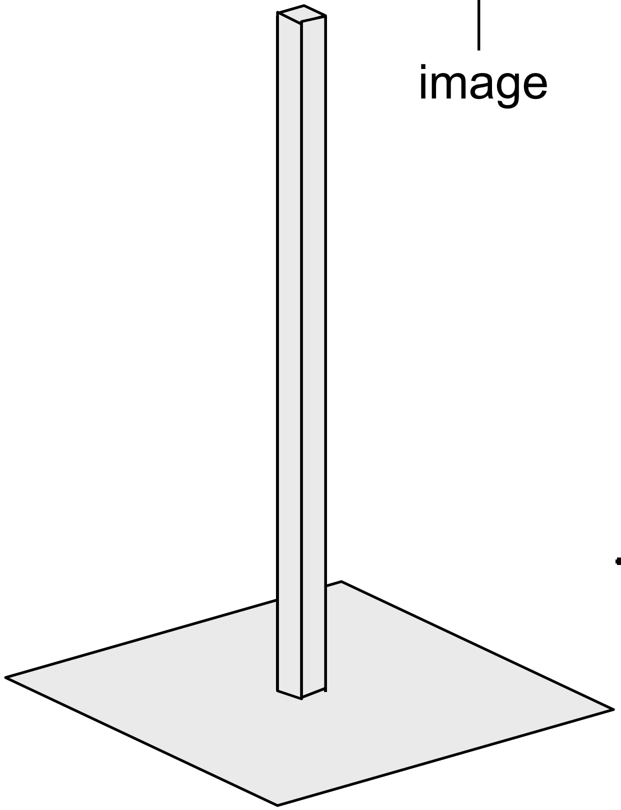
Unsharp mask filter

$$f + \alpha(f - f * g) = (1+\alpha) f - \alpha\, f * g = f * ((1+\alpha)e - g)$$

image

blurred
image

unit impulse
(identity)



unit impulse

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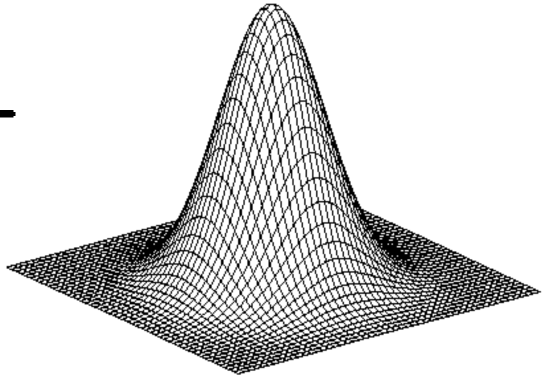
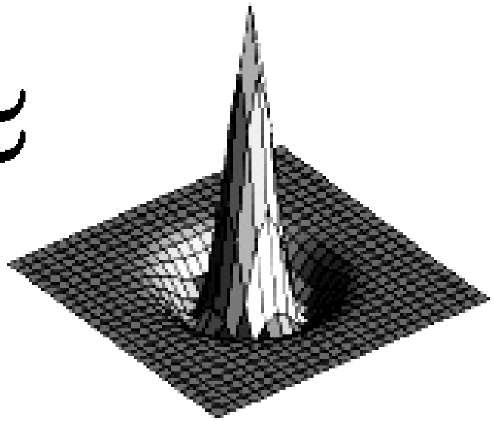


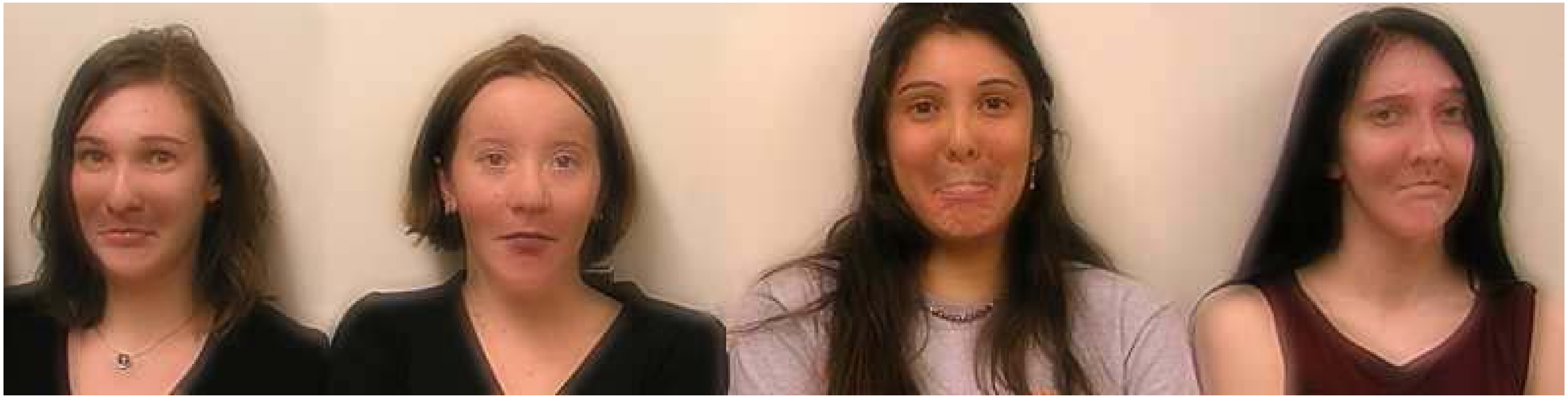
Gaussian

≈



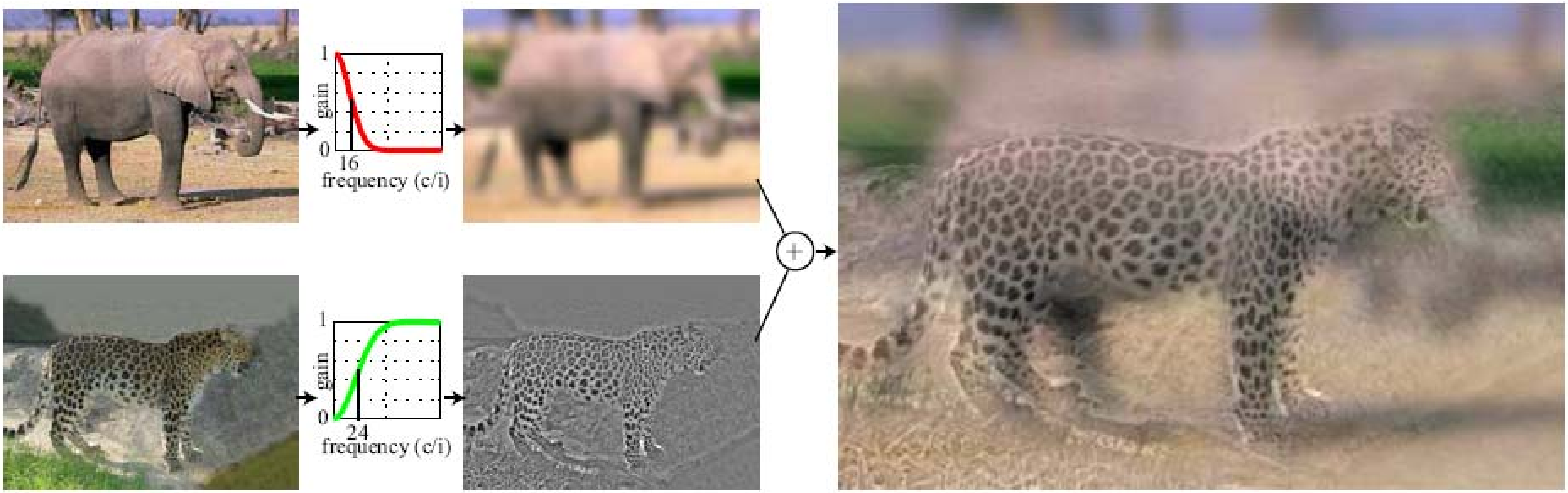
Laplacian of Gaussian

Application: Hybrid Images



A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Application: Hybrid Images



A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006