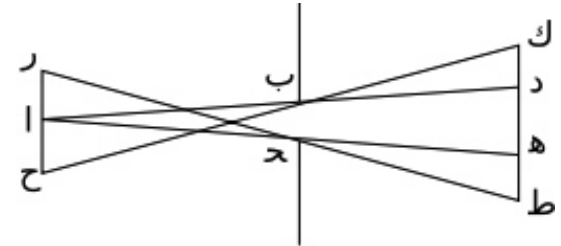


Review

- **Pinhole projection model**
 - What are vanishing points and vanishing lines?
 - What is orthographic projection?
 - How can we approximate orthographic projection?
- **Lenses**
 - Why do we need lenses?
 - What is depth of field?
 - What controls depth of field?
 - What is field of view?
 - What controls field of view?
 - What are some kinds of lens aberrations?
- **Digital cameras**
 - What are the two major types of sensor technologies?
 - How can we capture color with a digital camera?

Historical context

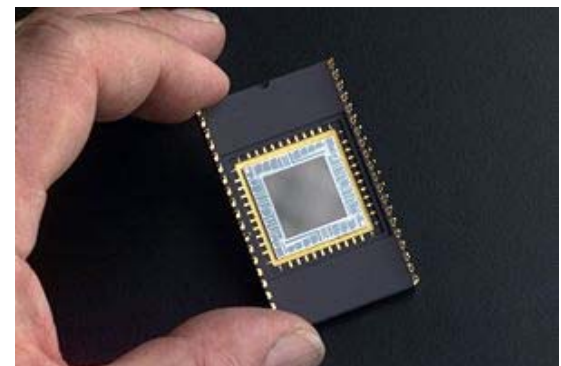
- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film:** Eastman (1889)
- **Cinema:** Lumière Brothers (1895)
- **Color Photography:** Lumière Brothers (1908)
- **Television:** Baird, Farnsworth, Zworykin (1920s)
- **First digitally scanned photograph:** Russell Kirsch, NIST (1957)
- **First consumer camera with CCD:** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes



Niepce, "La Table Servie," 1822



CCD chip

10 Early Firsts In Photography

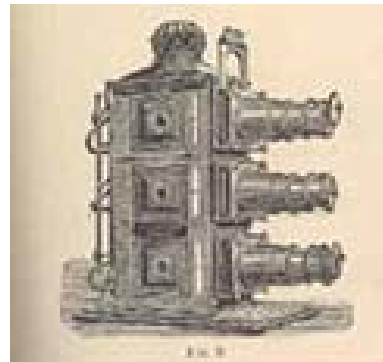
<http://listverse.com/history/top-10-incredible-early-firsts-in-photography/>



Early color photography

Sergey Prokudin-Gorsky (1863-1944)

Photographs of the Russian empire
(1909-1916)



Lantern
projector



http://en.wikipedia.org/wiki/Sergei_Mikhailovich_Prokudin-Gorskii

<http://www.loc.gov/exhibits/empire/>

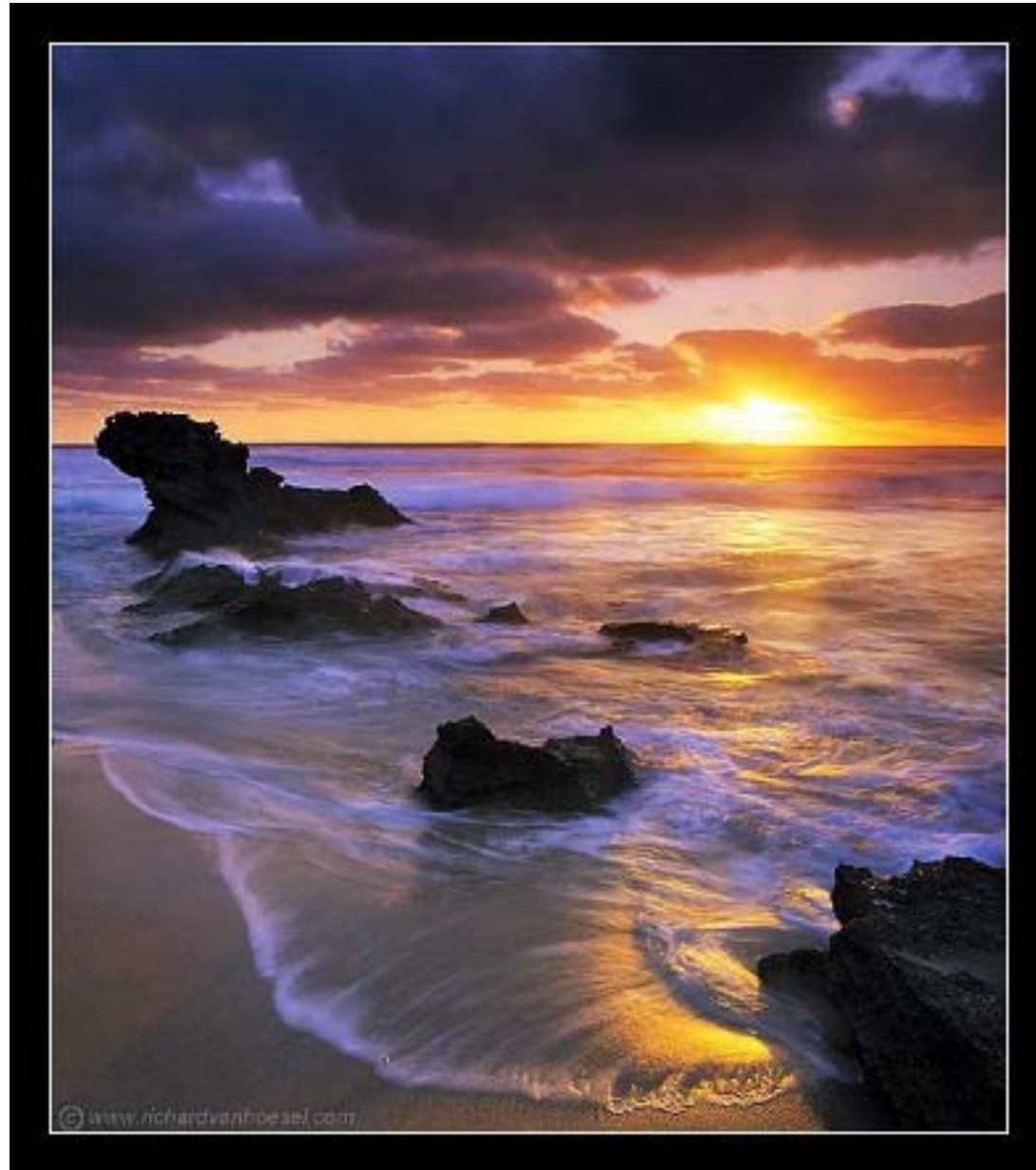
“Fake miniatures”



Create your own fake miniatures: <http://tiltshiftmaker.com/>
<http://tiltshiftmaker.com/tilt-shift-photo-gallery.php>

Idea for class participation: if you find interesting (and relevant) links, send them to me or (better yet) to the class mailing list (comp776@cs.unc.edu).

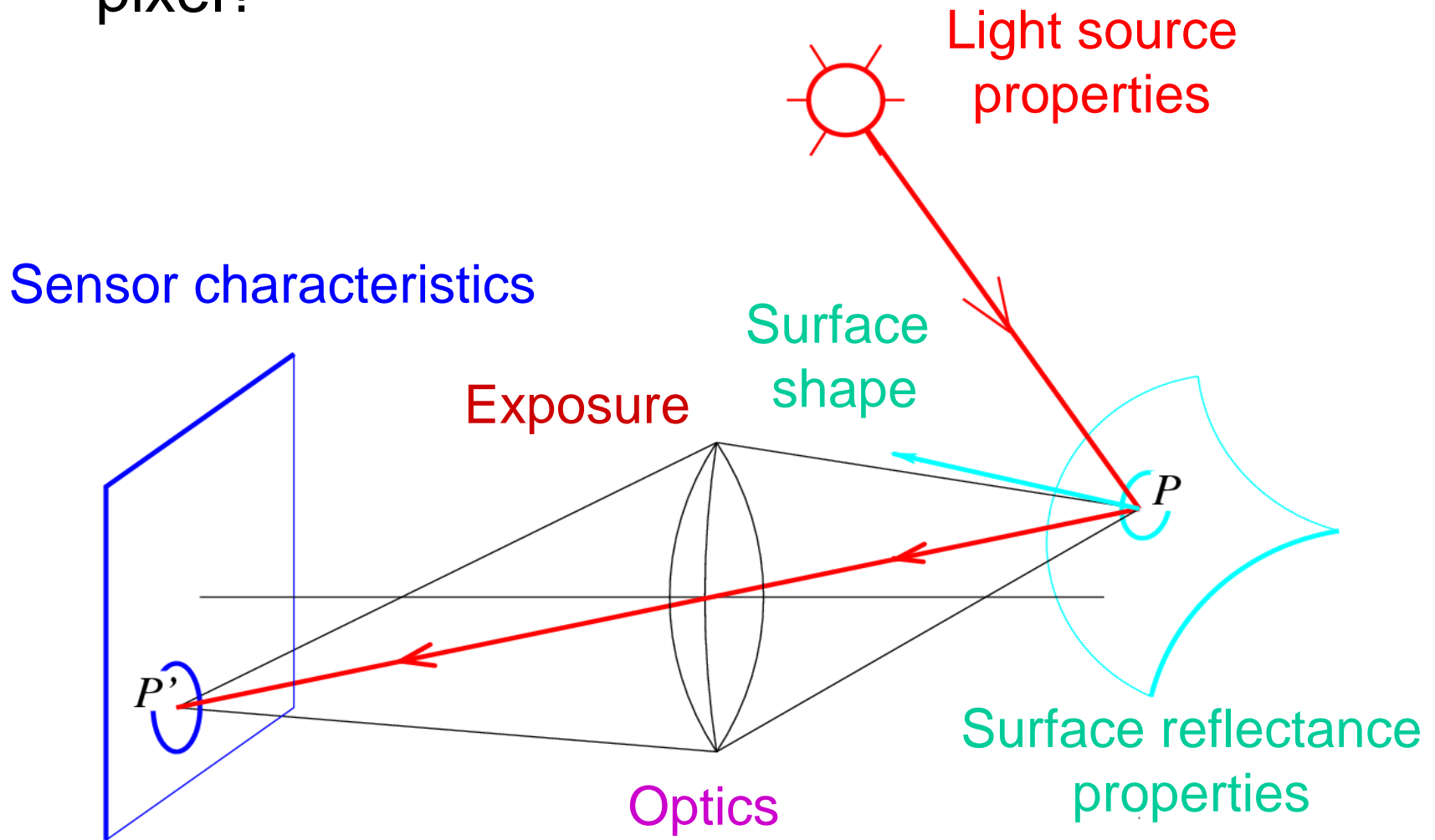
Today: Capturing light



Source: A. Efros

Radiometry

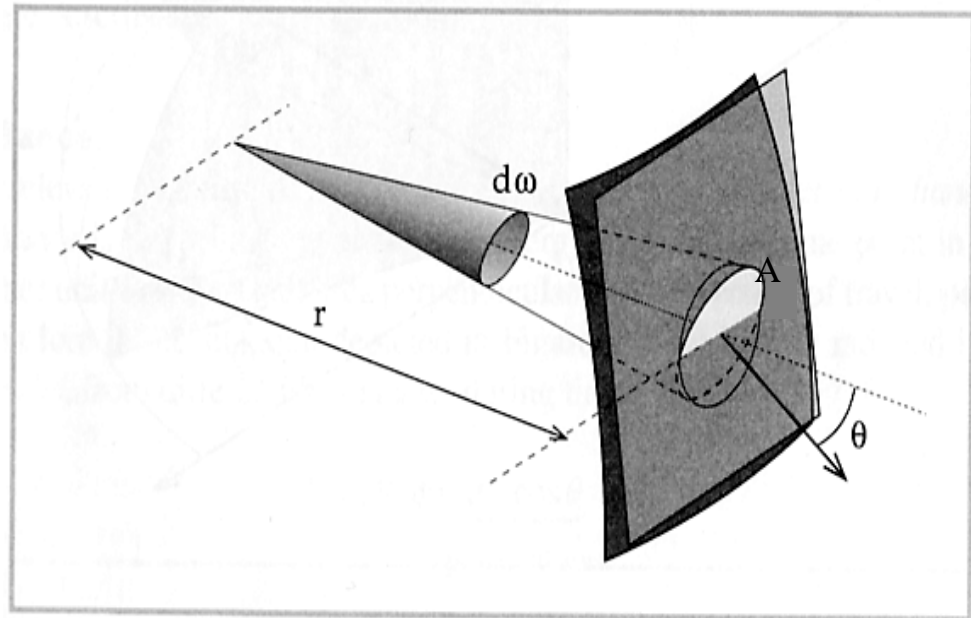
What determines the brightness of an image pixel?



Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle $d\omega$ subtended by a patch of area dA is given by:

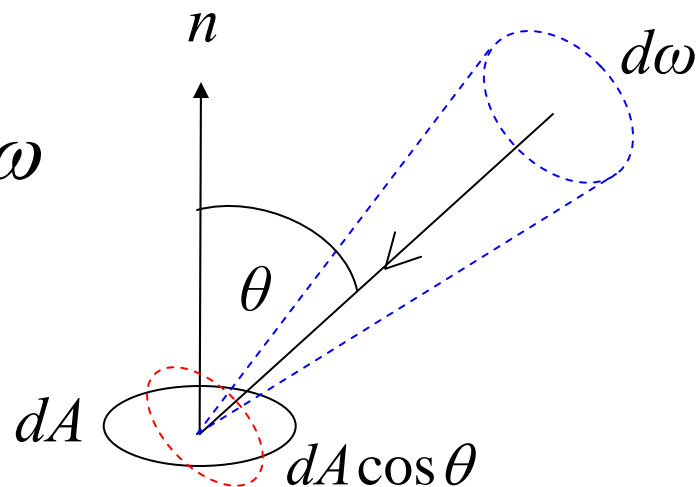
$$d\omega = \frac{dA \cos \theta}{r^2}$$



Radiometry

- Radiance (L): energy carried by a ray
 - Power per unit area perpendicular to the direction of travel, per unit solid angle
 - Units: Watts per square meter per steradian ($\text{W m}^{-2} \text{sr}^{-1}$)
- Irradiance (E): energy arriving at a surface
 - Incident power in a given direction per unit area
 - Units: W m^{-2}
 - For a surface receiving radiance $L(x, \theta, \phi)$ coming in from $d\omega$ the corresponding irradiance is

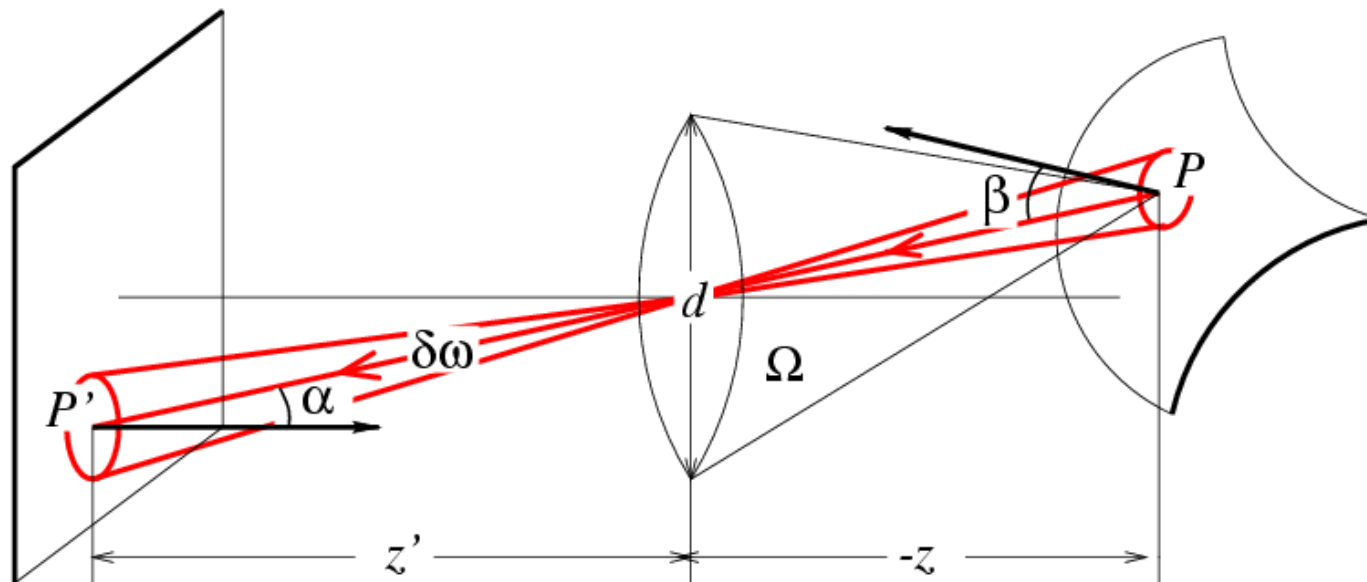
$$E(\theta, \phi) = L(\theta, \phi) \cos \theta d\omega$$



Radiometry of thin lenses

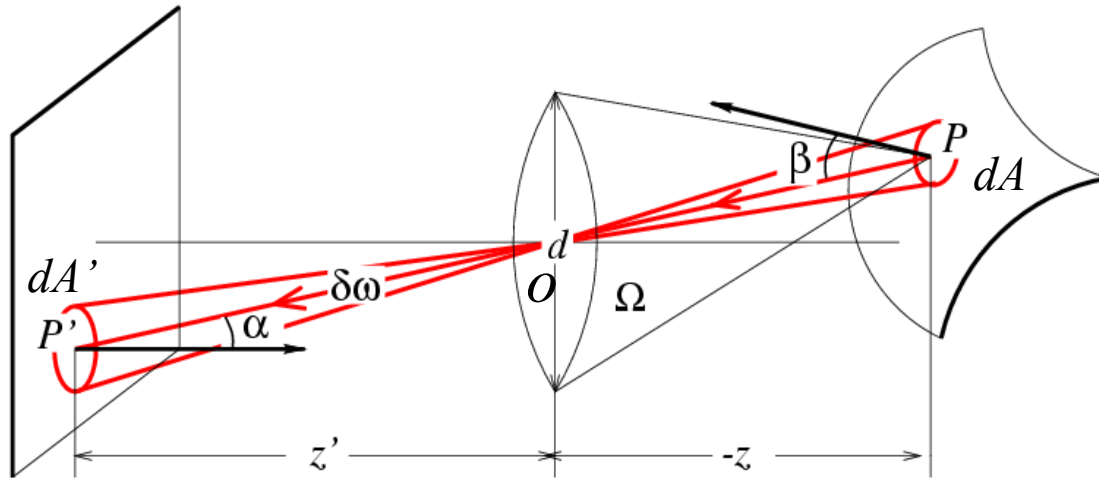
L : Radiance emitted from P toward P'

E : Irradiance falling on P' from the lens



What is the relationship between E and L ?

Example: Radiometry of thin lenses



$$|OP| = \frac{z}{\cos \alpha}$$

$$|OP'| = \frac{z'}{\cos \alpha}$$

$$\text{Area of the lens: } \frac{\pi d^2}{4}$$

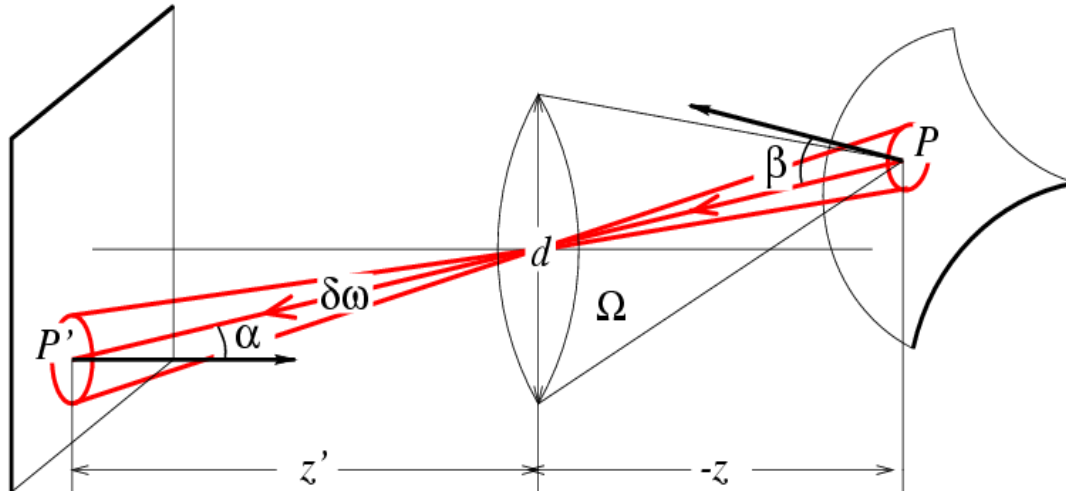
The power δP received by the lens from P is
$$\delta P = L \left(\frac{\pi d^2}{4} \right) \cos \alpha \delta \omega$$

The radiance emitted from the lens towards dA' is
$$\frac{\delta P}{\left(\frac{\pi d^2}{4} \right) \cos \alpha \delta \omega} = L$$

The irradiance received at P' is

$$E = L \cos \alpha \left(\frac{\pi d^2 \cos \alpha}{4 (z' / \cos \alpha)^2} \right) = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

Radiometry of thin lenses



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

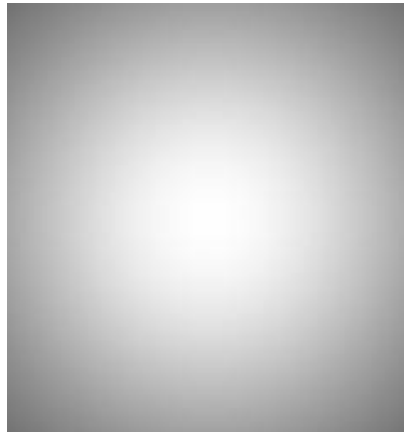
- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Radiometry of thin lenses

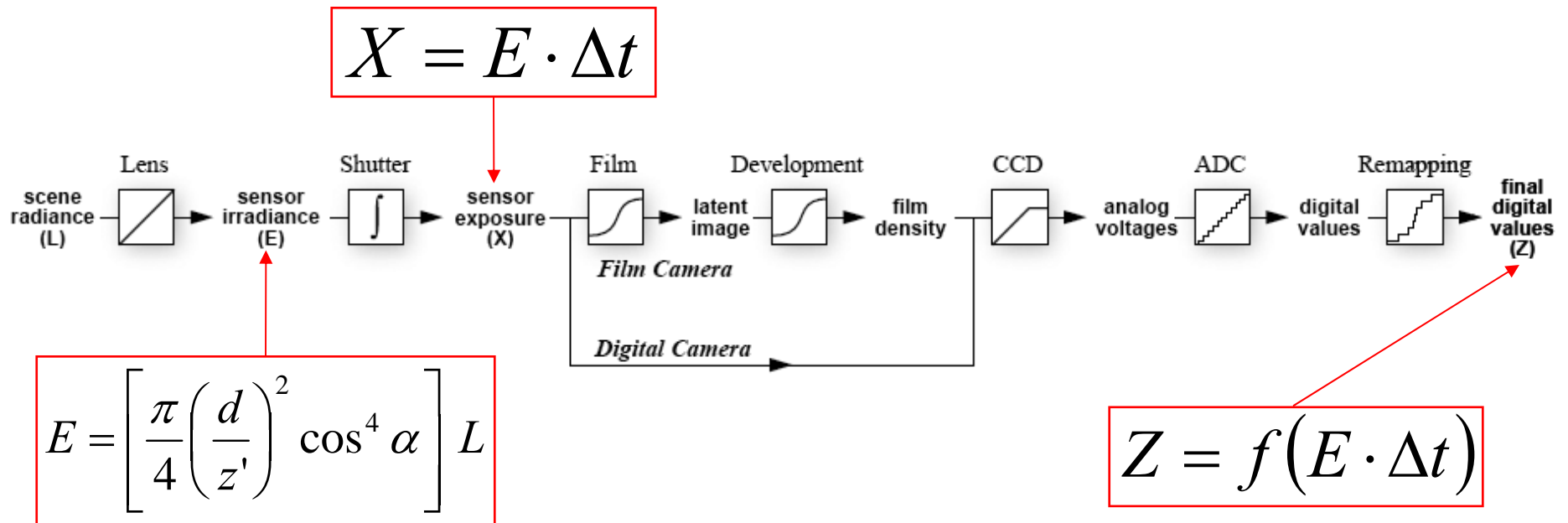
$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

- Application:

- S. B. Kang and R. Weiss, [Can we calibrate a camera using an image of a flat, textureless Lambertian surface?](#) ECCV 2000.

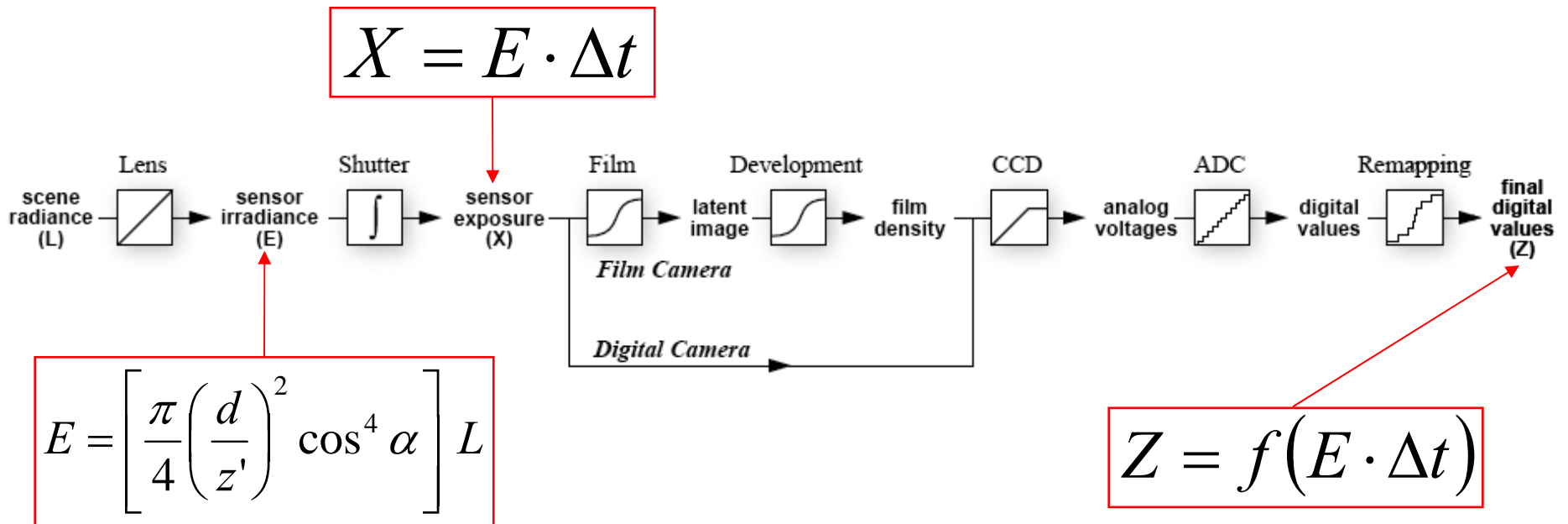


The journey of the light ray



- Camera response function: the mapping f from irradiance to pixel values
 - Useful if we want to estimate material properties
 - Enables us to create high dynamic range images

The journey of the light ray



- Camera response function: the mapping f from irradiance to pixel values

For more info

- P. E. Debevec and J. Malik. [Recovering High Dynamic Range Radiance Maps from Photographs](#). In [SIGGRAPH 97](#), August 1997

The interaction of light and surfaces

What happens when a light ray hits a point on an object?

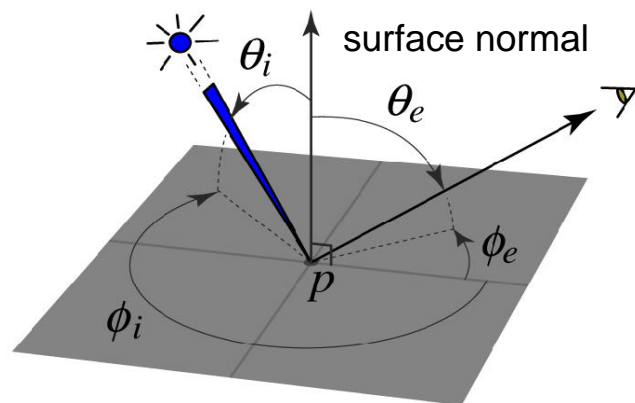
- Some of the light gets absorbed
 - converted to other forms of energy (e.g., heat)
- Some gets transmitted through the object
 - possibly bent, through “refraction”
- Some gets reflected
 - possibly in multiple directions at once
- Really complicated things can happen
 - fluorescence

Let's consider the case of reflection in detail

- In the most general case, a single incoming ray could be reflected in all directions. How can we describe the amount of light reflected in each direction?

Bidirectional reflectance distribution function (BRDF)

- Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the outgoing direction to irradiance in the incident direction

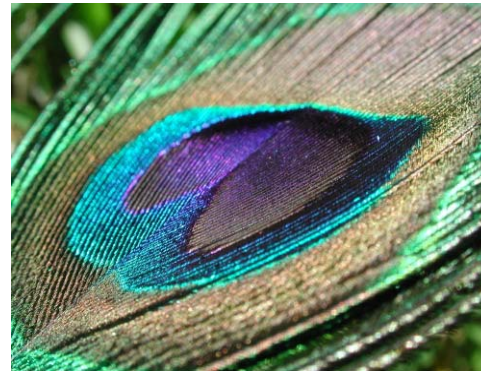


$$\rho(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L_e(\theta_e, \phi_e)}{E_i(\theta_i, \phi_i)} = \frac{L_e(\theta_e, \phi_e)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega}$$

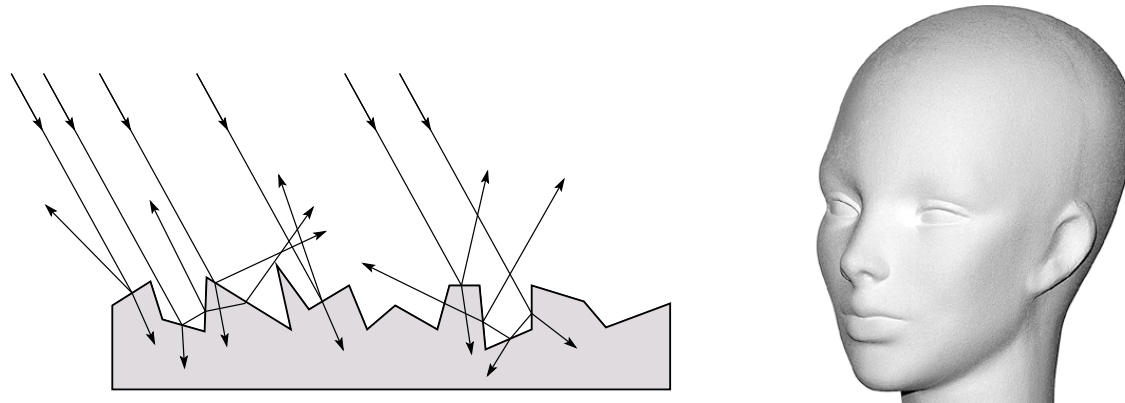
- Radiance leaving a surface in a particular direction: add contributions from every incoming direction

$$\int_{\Omega} \rho(\theta_i, \phi_i, \theta_e, \phi_e) L_i(\theta_i, \phi_i) \cos \theta_i d\omega_i$$

BRDF's can be incredibly complicated...



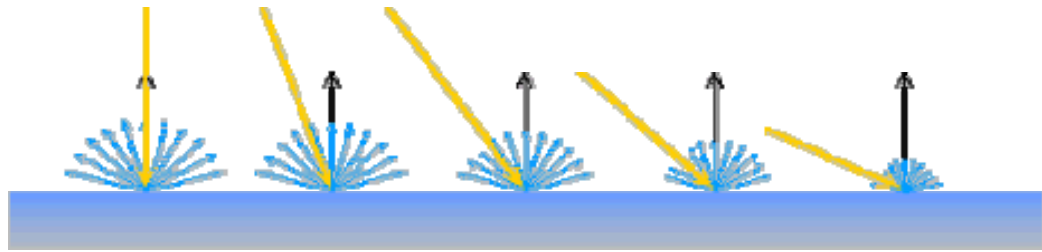
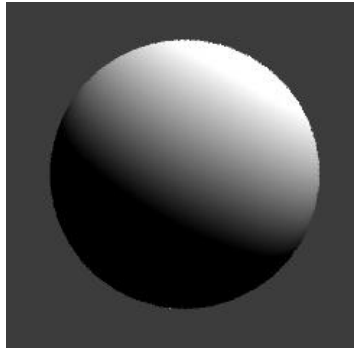
Diffuse reflection



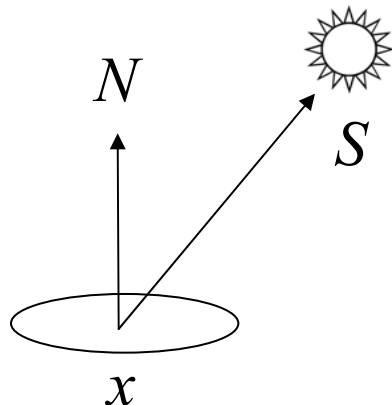
- Light is reflected equally in all directions: **BRDF is constant**
- Dull, matte surfaces like chalk or latex paint
- Microfacets scatter incoming light randomly
- *Albedo*: fraction of incident irradiance reflected by the surface
- *Radiosity*: total power leaving the surface per unit area (regardless of direction)

Diffuse reflection: Lambert's law

- Viewed brightness does not depend on viewing direction, but it *does* depend on direction of illumination



$$B(x) = \rho_d(x)(N(x) \cdot S_d(x))$$



B : radiosity

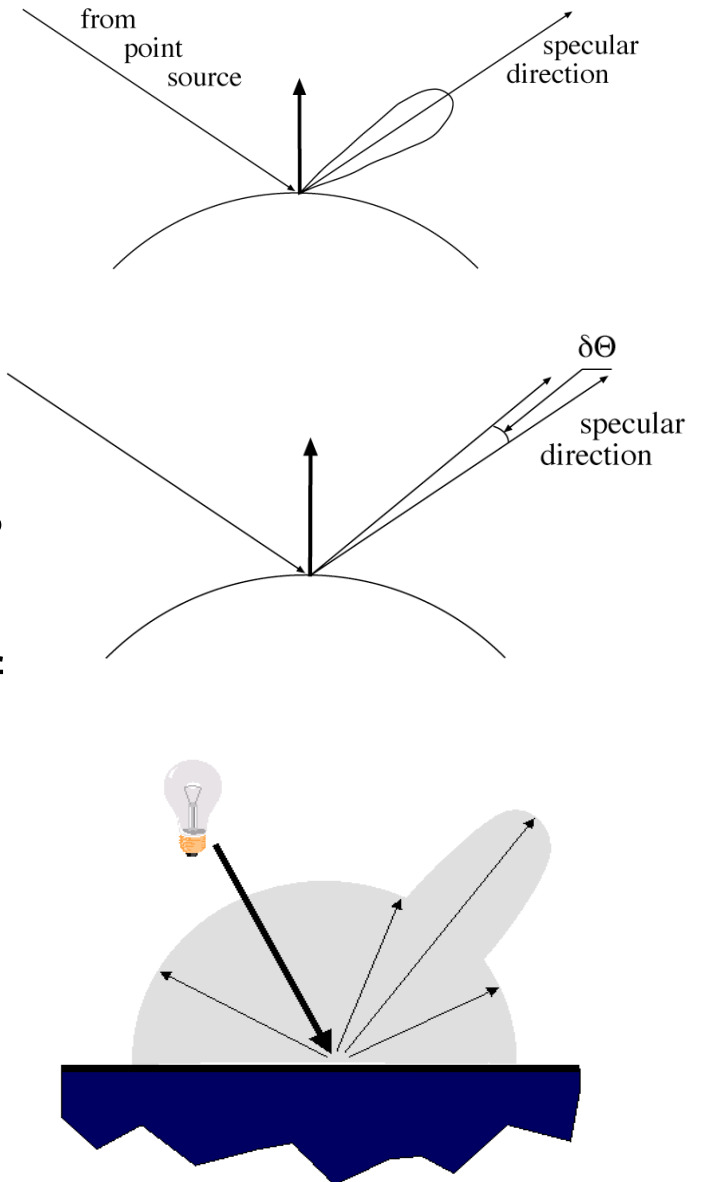
ρ : albedo

N : unit normal

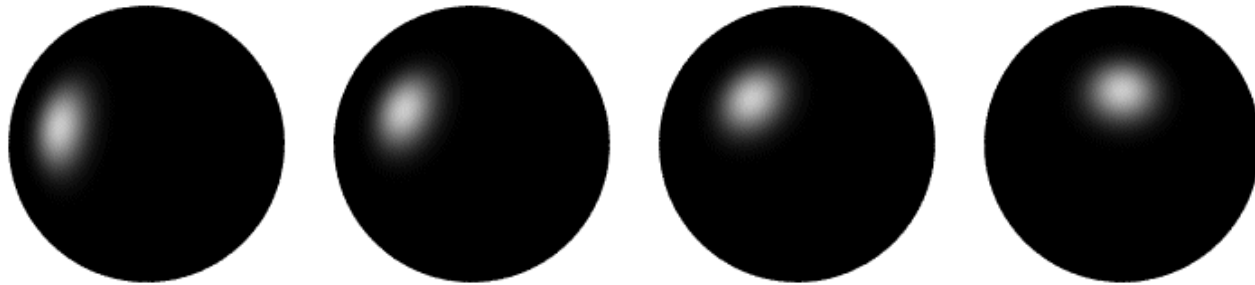
S : source vector (magnitude proportional to intensity of the source)

Specular reflection

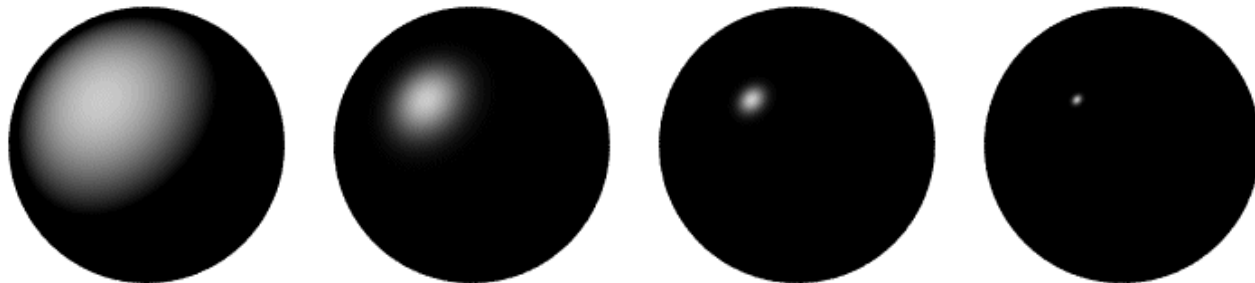
- Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
- Some fraction is absorbed, some reflected
- On real surfaces, energy usually goes into a lobe of directions
- Phong model: reflected energy falls off with $\cos^n(\delta\theta)$
- Lambertian + specular model: sum of diffuse and specular term



Specular reflection



Moving the light source



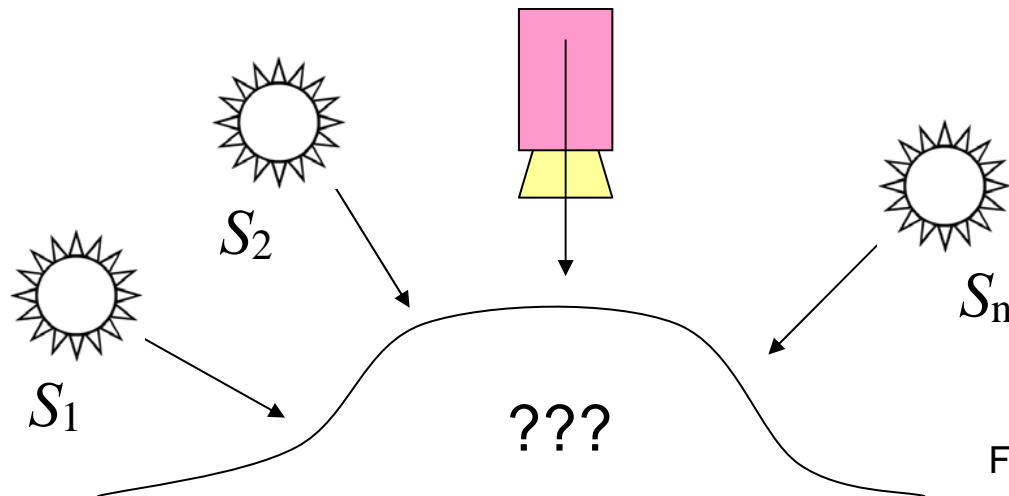
Changing the exponent

Photometric stereo

Assume:

- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo



Surface model: Monge patch

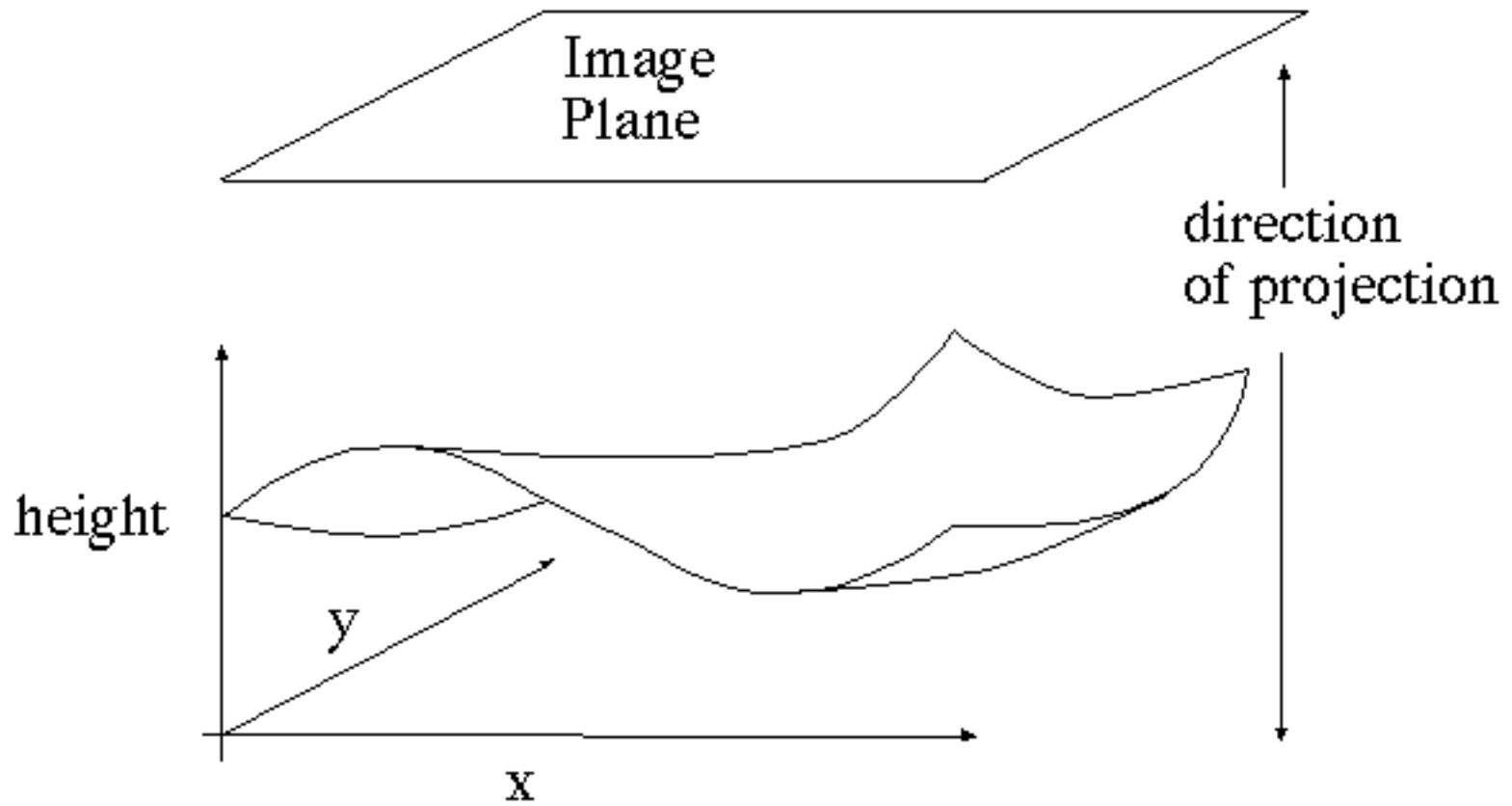


Image model

- Known: source vectors S_j and pixel values $I_j(x,y)$
- We also assume that the response function of the camera is a linear scaling by a factor of k
- Combine the unknown normal $N(x,y)$ and albedo $\rho(x,y)$ into one vector g , and the scaling constant k and source vectors S_j into another vector V_j :

$$\begin{aligned} I_j(x, y) &= k B(x, y) \\ &= k \rho(x, y) (N(x, y) \cdot S_j) \\ &= (\rho(x, y) N(x, y)) \cdot (k S_j) \\ &= g(x, y) \cdot V_j \end{aligned}$$

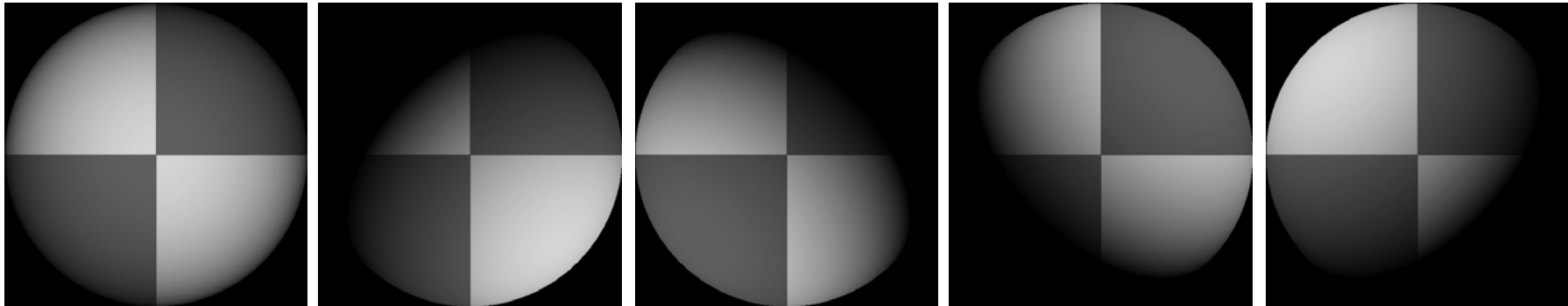
Least squares problem

- For each pixel, we obtain a linear system:

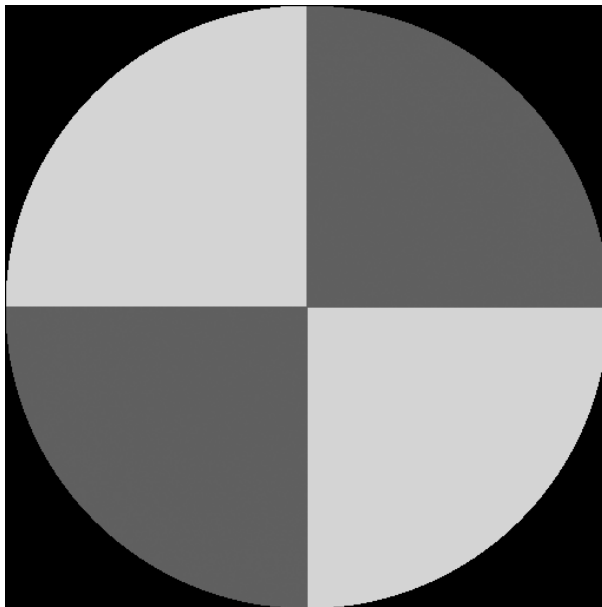
$$\begin{array}{ccc}
 \begin{bmatrix} I_1(x, y) \\ I_2(x, y) \\ \vdots \\ I_n(x, y) \end{bmatrix} & = & \begin{bmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{bmatrix} g(x, y) \\
 \begin{array}{c} | \\ (n \times 1) \\ \text{known} \end{array} & & \begin{array}{c} | \\ (n \times 3) \\ \text{known} \end{array} \quad \begin{array}{c} | \\ (3 \times 1) \\ \text{unknown} \end{array}
 \end{array}$$

- Obtain least-squares solution for $g(x, y)$
- Since $N(x, y)$ is the unit normal, $\rho(x, y)$ is given by the magnitude of $g(x, y)$ (and it should be less than 1)
- Finally, $N(x, y) = g(x, y) / \rho(x, y)$

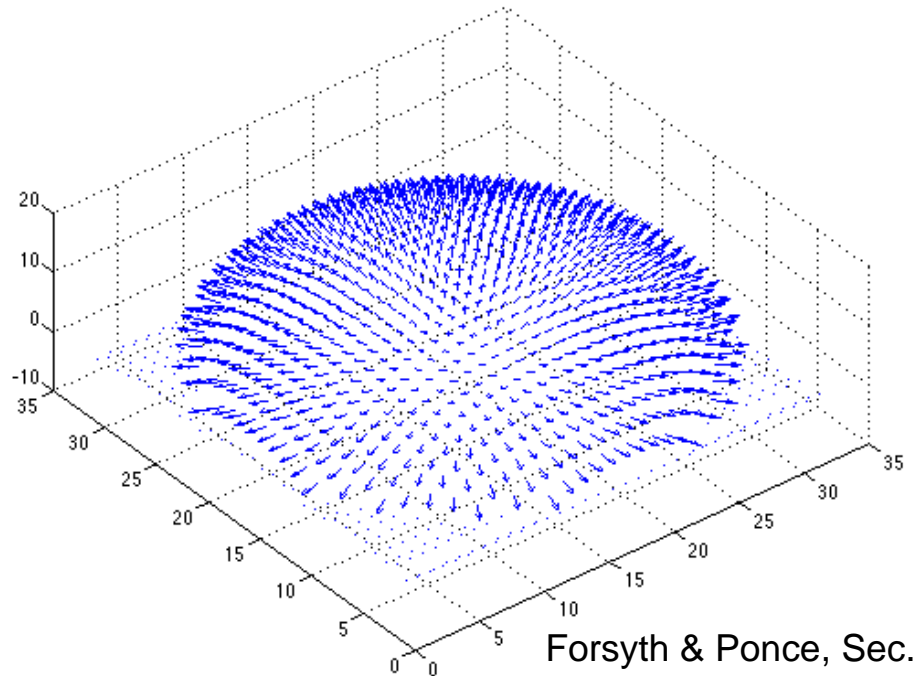
Example



Recovered albedo



Recovered normal field



Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$N(x, y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \right) \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector g as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = (g_1(x, y) / g_3(x, y))$$

$$f_y(x, y) = (g_2(x, y) / g_3(x, y))$$

Recovering a surface from normals

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial(g_1(x, y)/g_3(x, y))}{\partial y} = \frac{\partial(g_2(x, y)/g_3(x, y))}{\partial x}$$

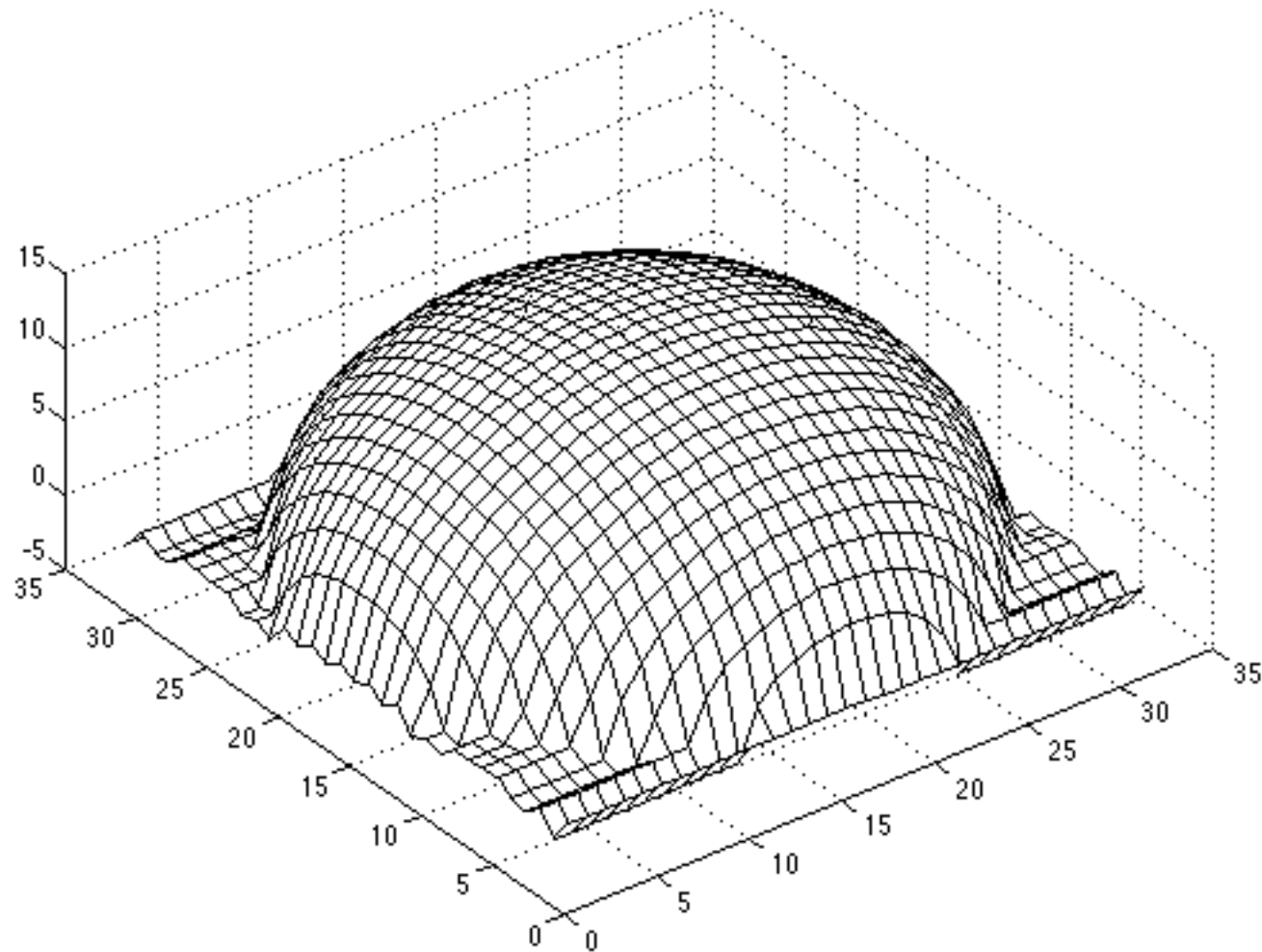
(in practice, they should at least be similar)

We can now recover the surface height at any point by integration along some path, e.g.

$$f(x, y) = \int_0^x f_x(s, y) ds + \int_0^y f_y(x, t) dt + c$$

(for robustness, can take integrals over many different paths and average the results)

Surface recovered by integration



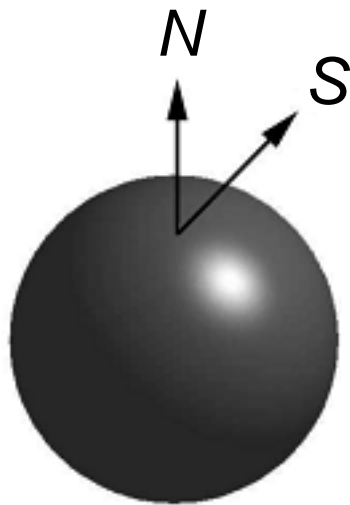
Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

Finding the direction of the light source

$$I(x,y) = N(x,y) \cdot S(x,y) + A$$

Full 3D case:



$$\begin{pmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) & 1 \\ N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \\ A \end{pmatrix} = \begin{pmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{pmatrix}$$

For points on the *occluding contour*:

$$\begin{pmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) & 1 \\ N_x(x_2, y_2) & N_y(x_2, y_2) & 1 \\ \vdots & \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ A \end{pmatrix} = \begin{pmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{pmatrix}$$

Finding the direction of the light source



P. Nillius and J.-O. Eklundh, "Automatic estimation of the projected light source direction," CVPR 2001

Application: Detecting composite photos

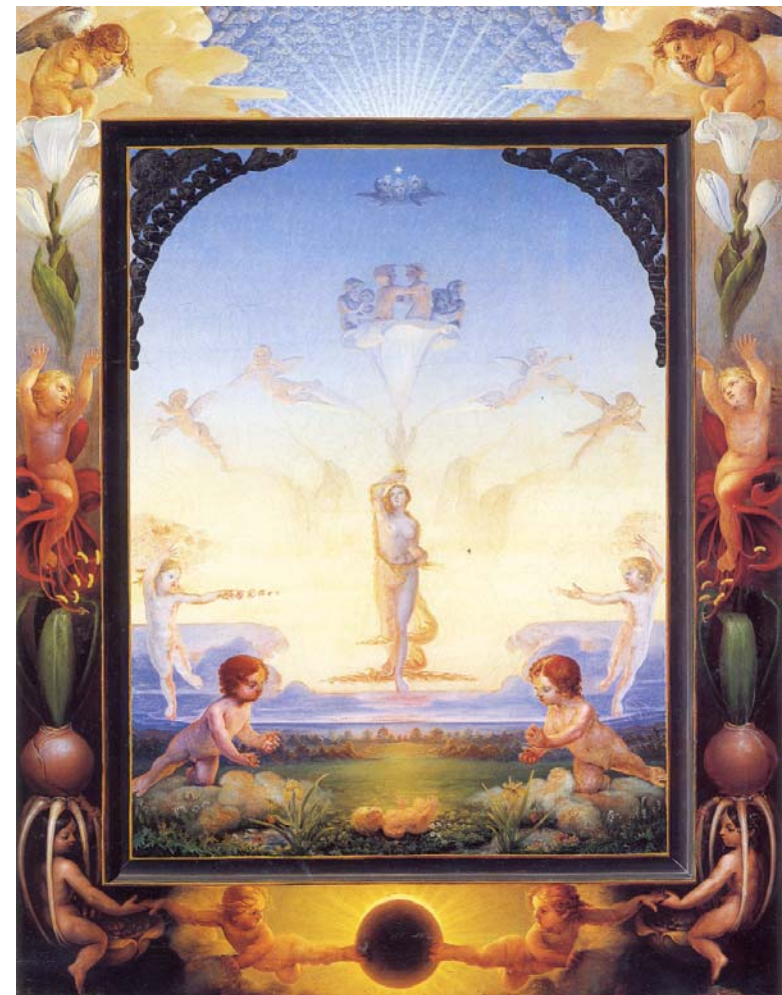
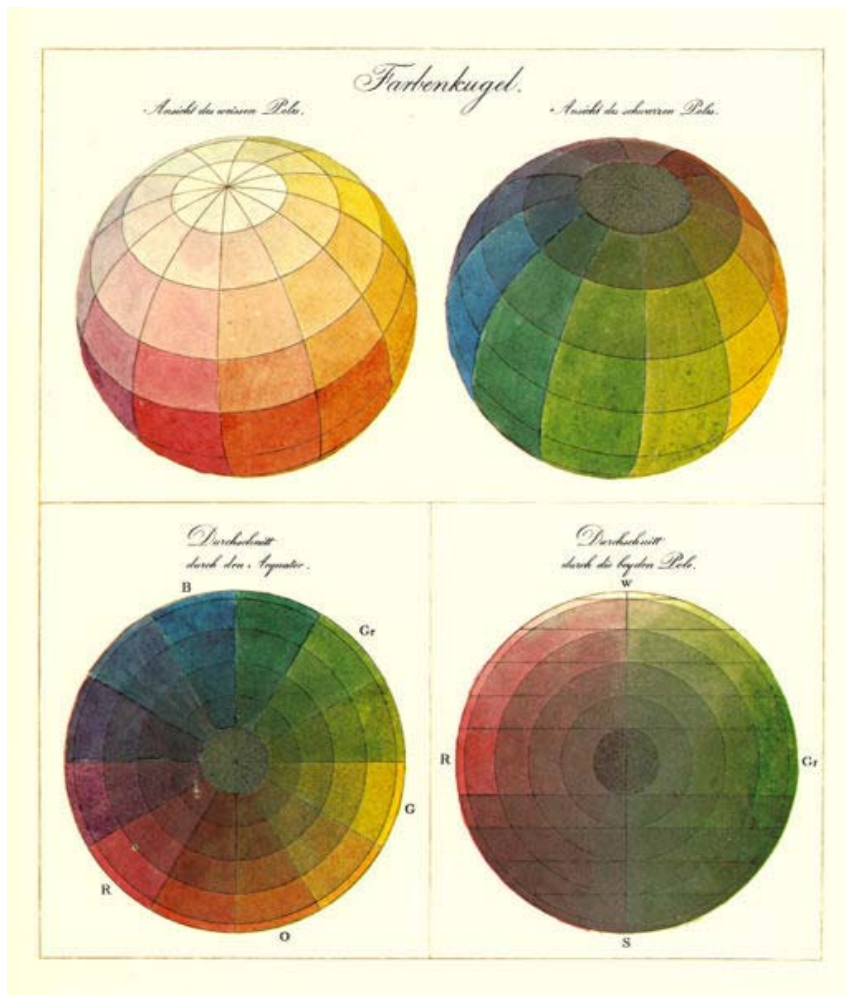
Fake photo



Real photo



Next time: Color



Phillip Otto Runge (1777-1810)