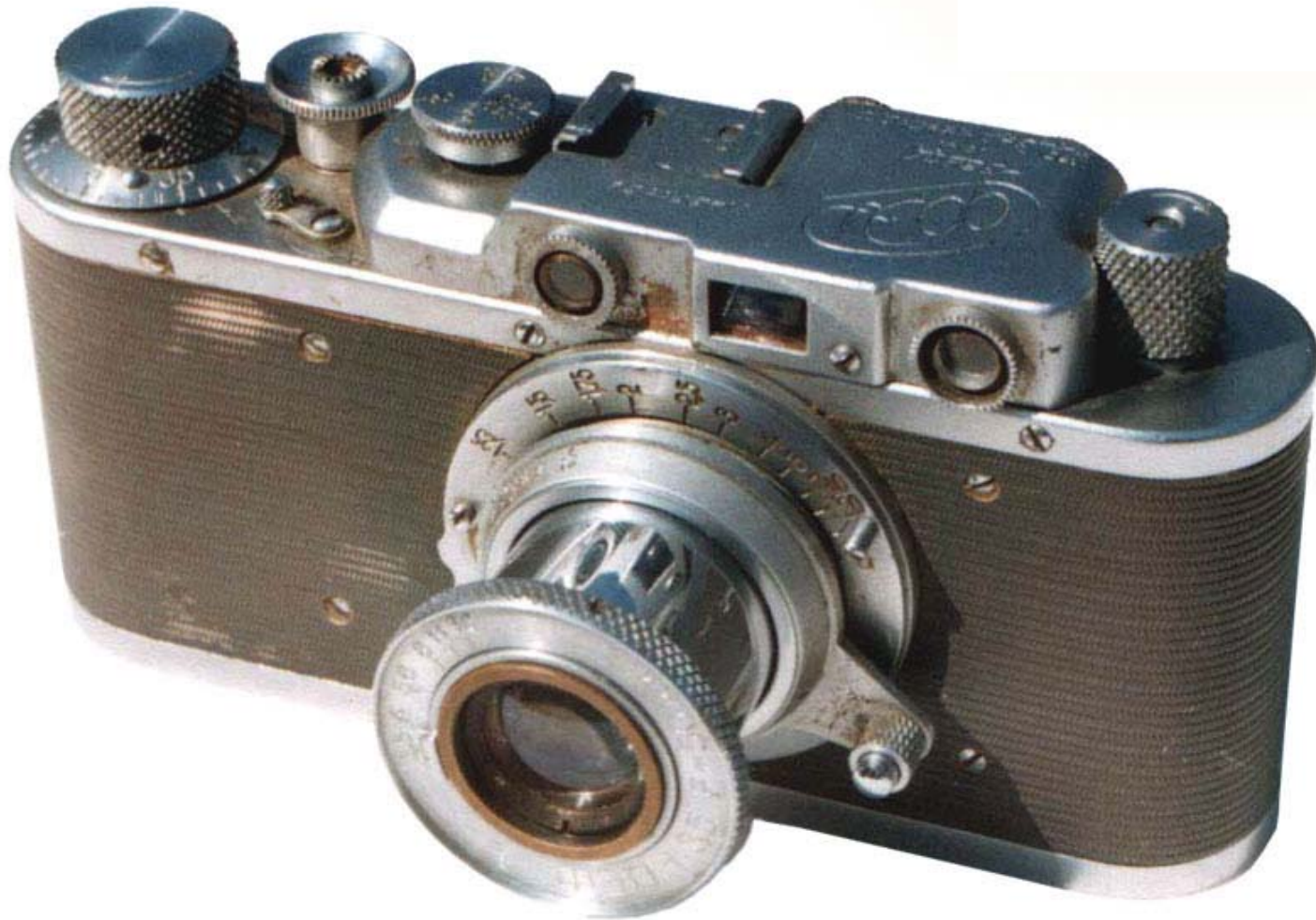


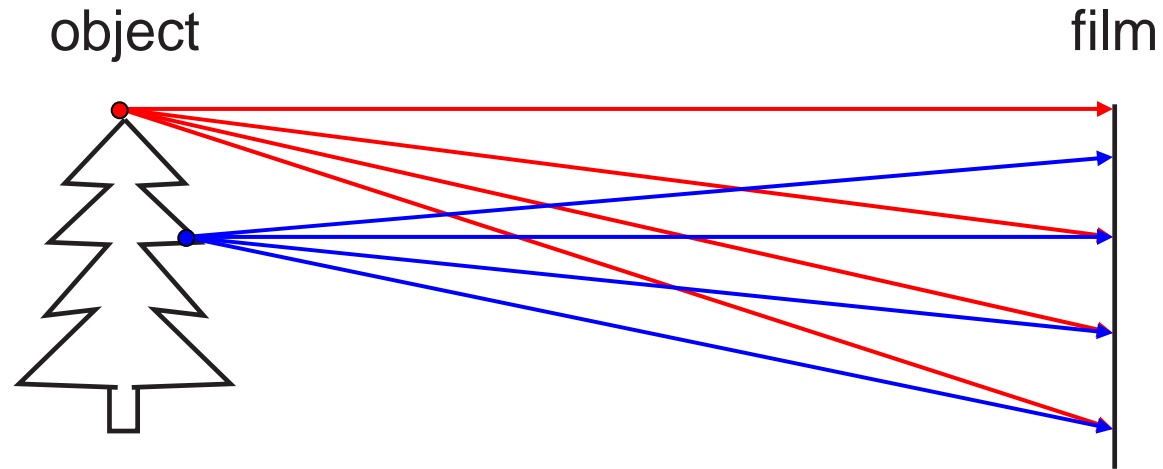
Today: The Camera



Overview

- The pinhole projection model
 - Qualitative properties
 - Perspective projection matrix
- Cameras with lenses
 - Depth of focus
 - Field of view
 - Lens aberrations
- Digital cameras
 - Types of sensors
 - Color

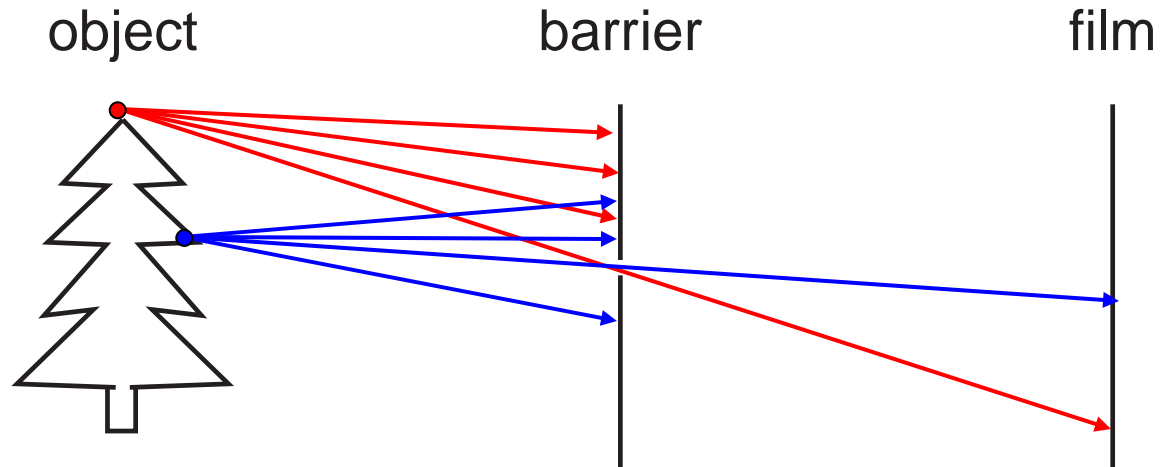
How do we see the world?



Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

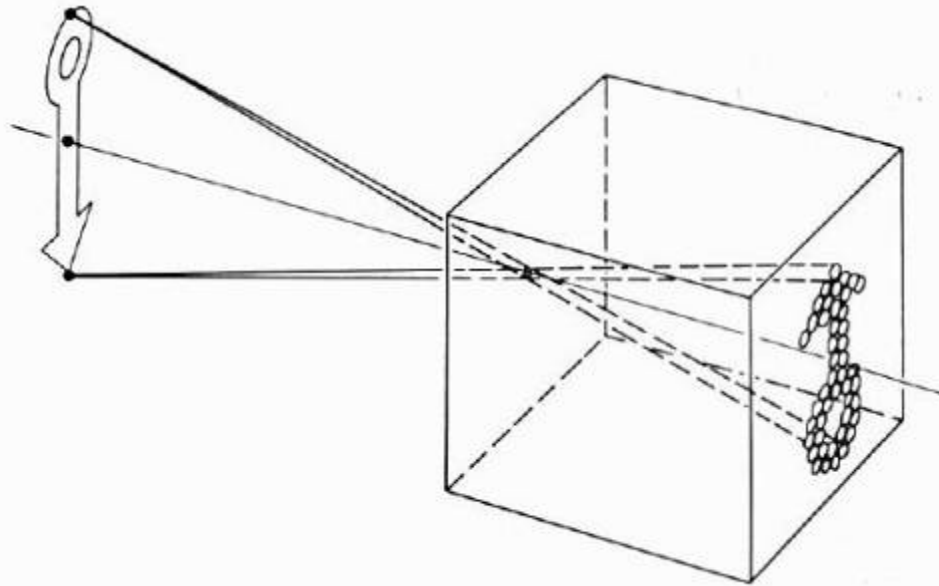
Pinhole camera



Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Pinhole camera model

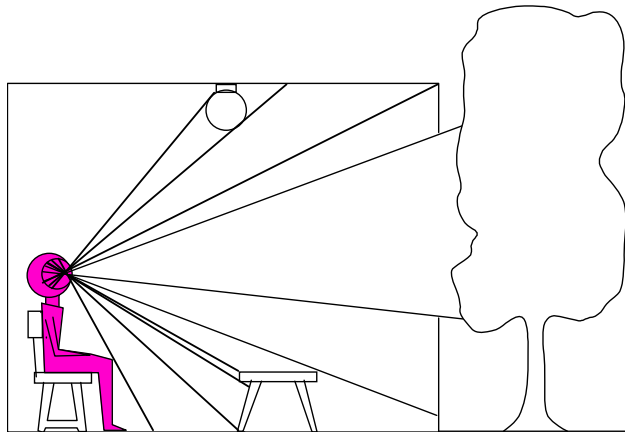


Pinhole model:

- Captures **pencil of rays** – all rays through a single point
- The point is called **Center of Projection (focal point)**
- The image is formed on the **Image Plane**

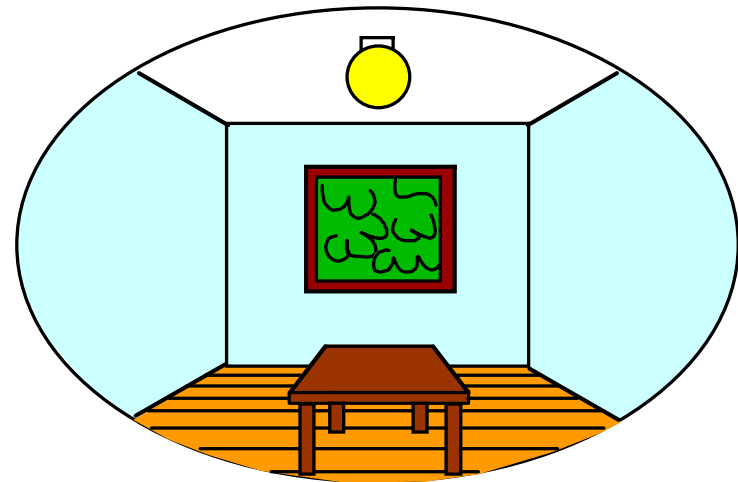
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

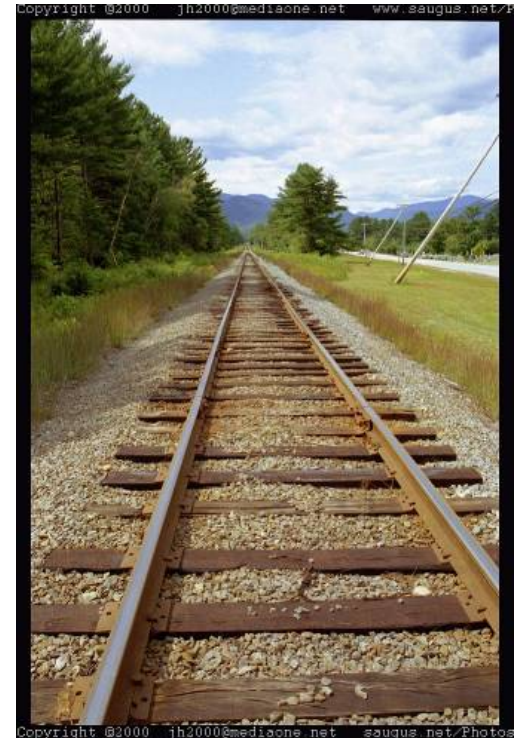
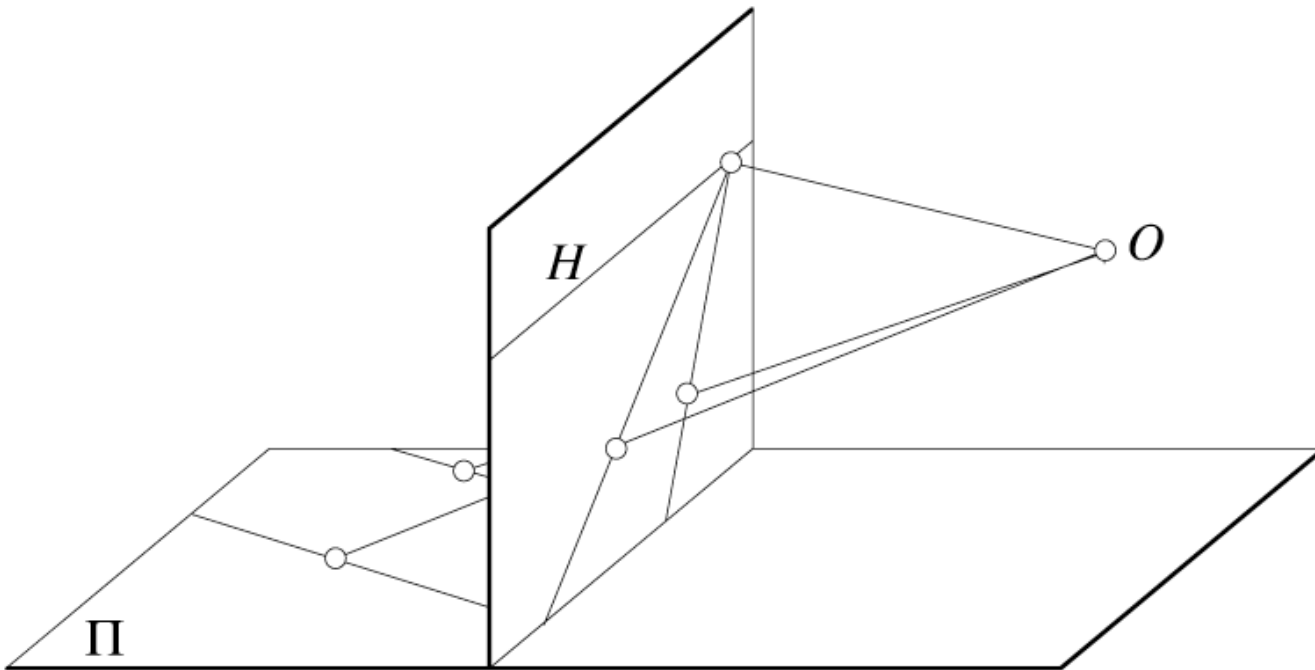
- Angles
- Distances (lengths)

Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points \rightarrow points
 - But projection of points on focal plane is undefined
- Lines \rightarrow lines (collinearity is preserved)
 - But line through focal point projects to a point
- Planes \rightarrow planes (or half-planes)
 - But plane through focal point projects to line

Projection properties

- Parallel lines converge at a vanishing point
 - Each direction in space has its own vanishing point
 - But parallels parallel to the image plane remain parallel
 - All directions in the same plane have vanishing points on the same line

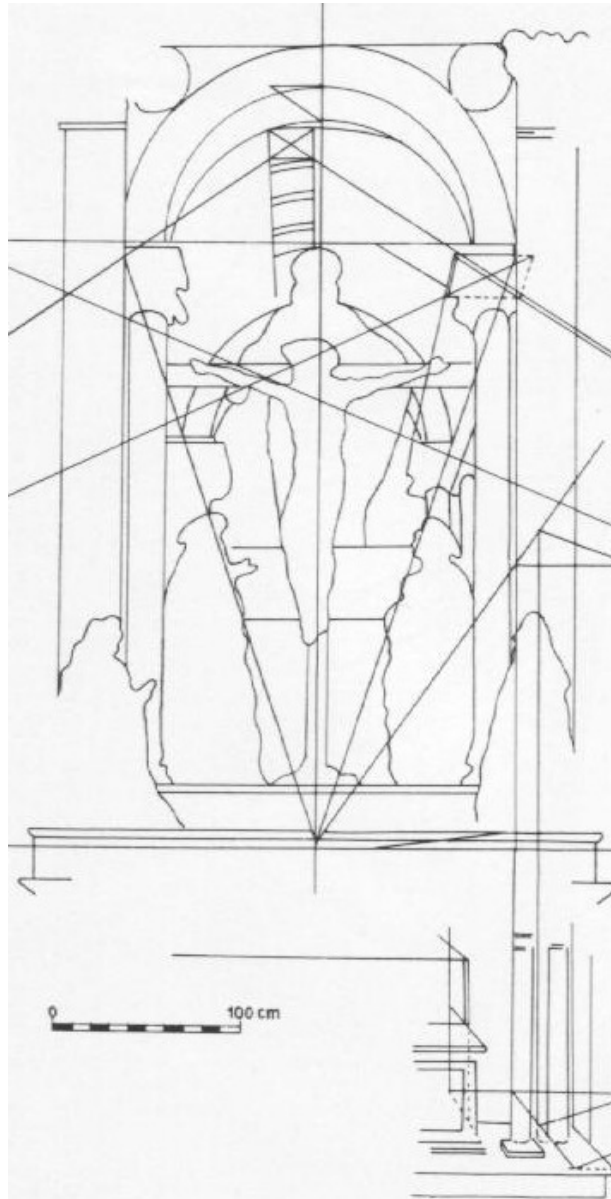
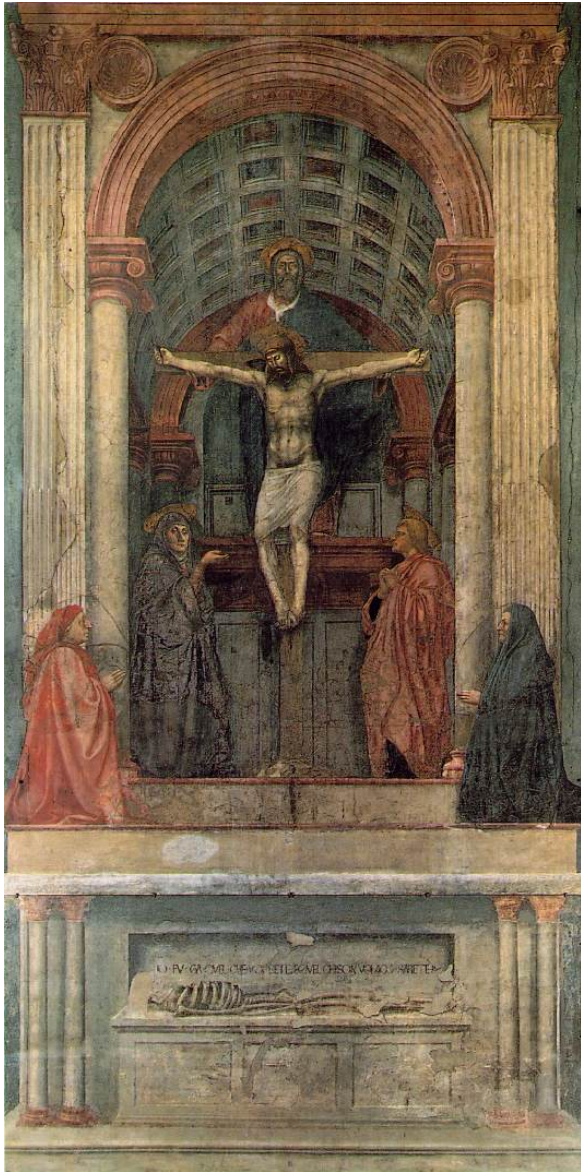


How do we construct the vanishing point/line?

One-point perspective

Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

First consistent use of perspective in Western art?



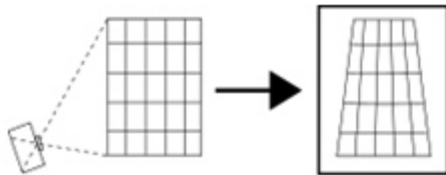
Perspective distortion

- Problem for architectural photography: converging verticals

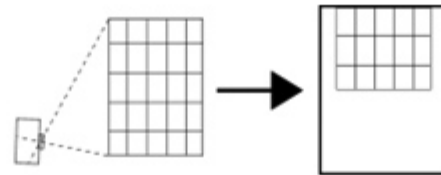


Perspective distortion

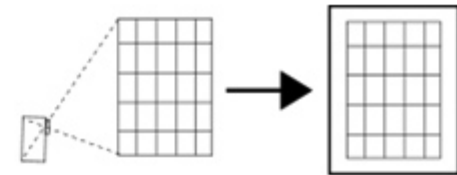
- Problem for architectural photography: converging verticals



Tilting the camera upwards results in converging verticals

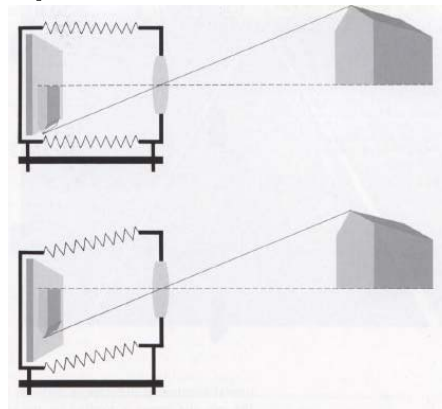
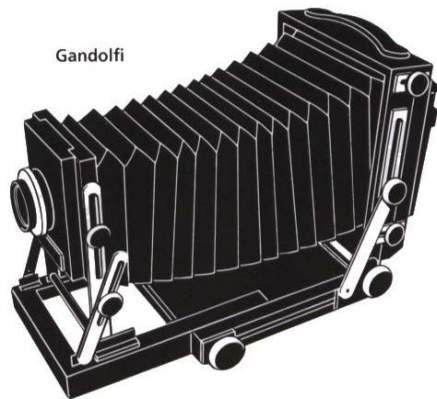


Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building



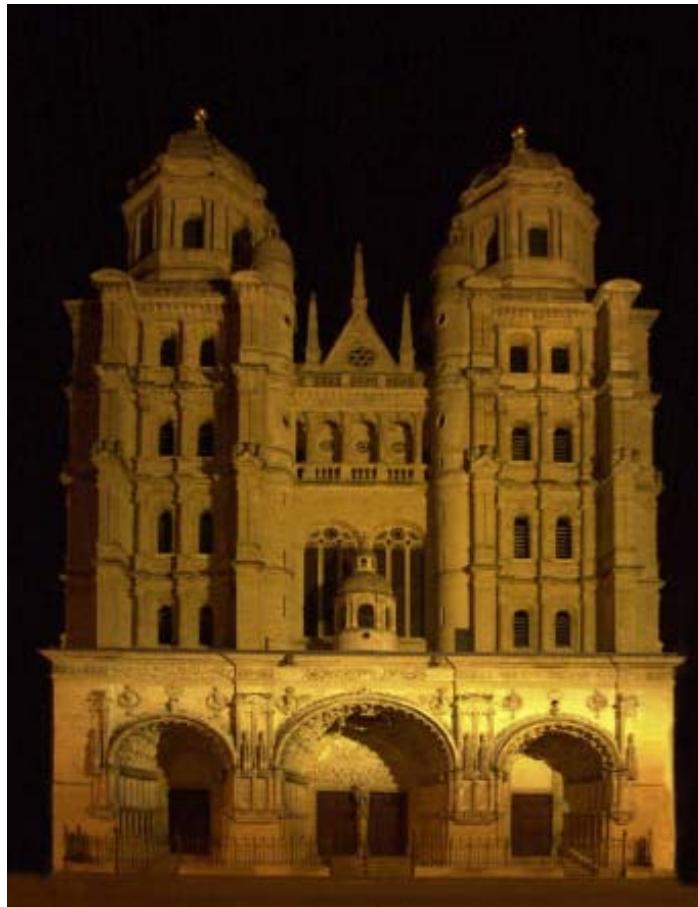
Shifting the lens upwards results in a picture of the entire subject

- Solution: view camera (lens shifted w.r.t. film)



Perspective distortion

- Problem for architectural photography: converging verticals
- Result:



Perspective distortion

- However, converging verticals work quite well for horror movies...



Perspective distortion

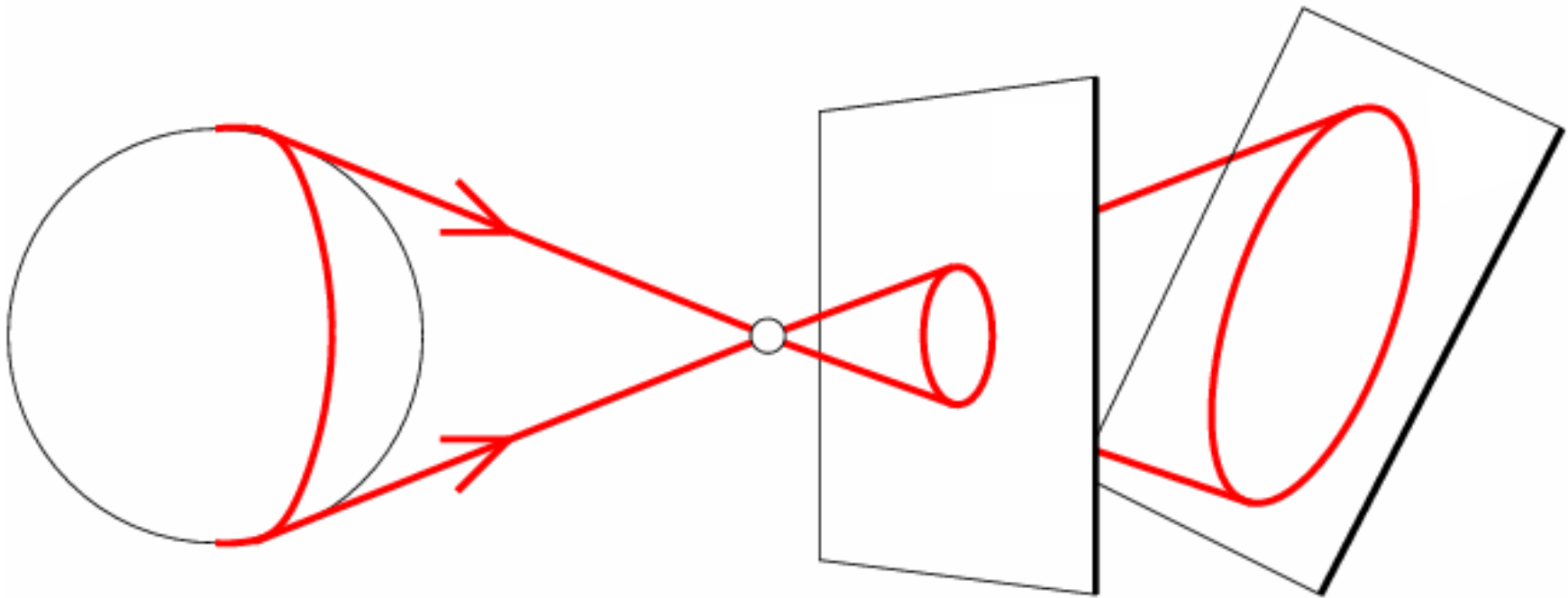
- What does a sphere project to?



Image source: F. Durand

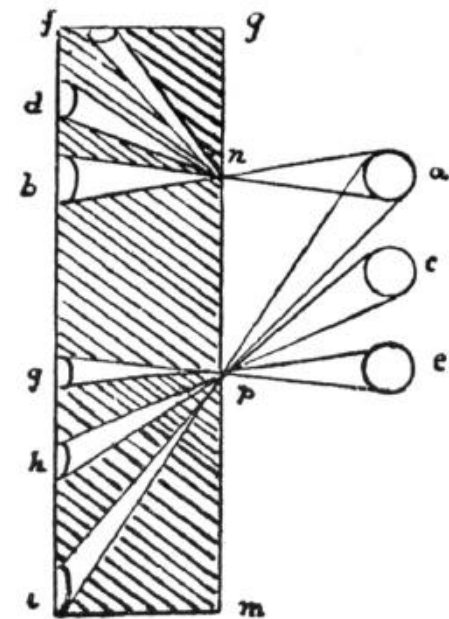
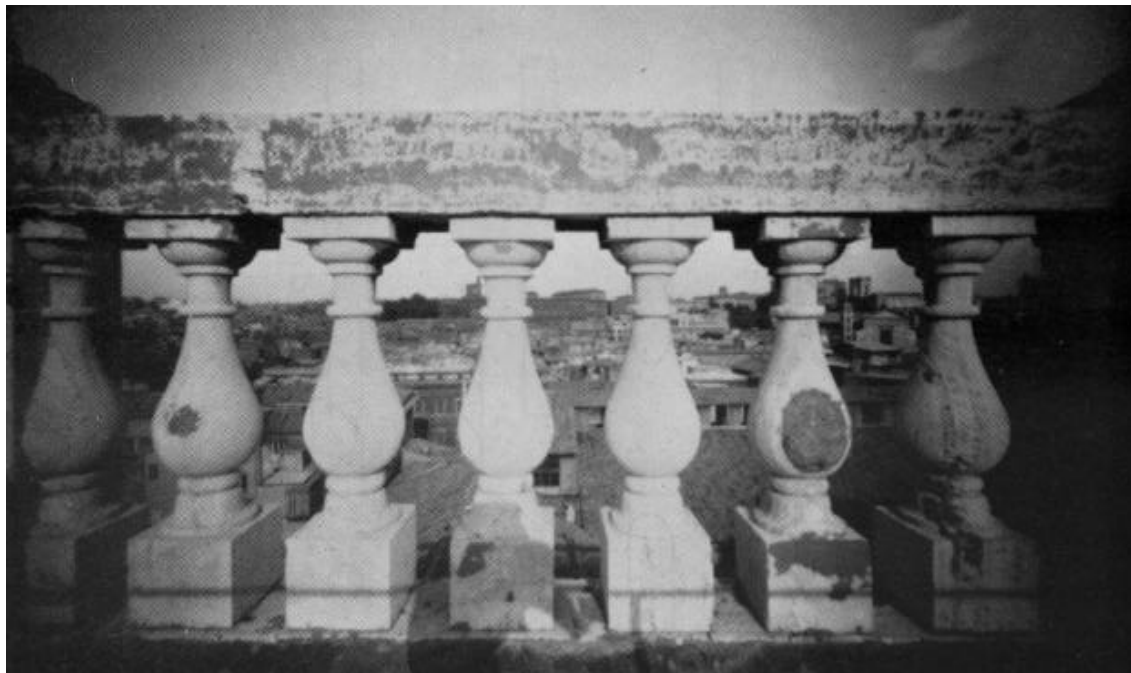
Perspective distortion

- What does a sphere project to?



Perspective distortion

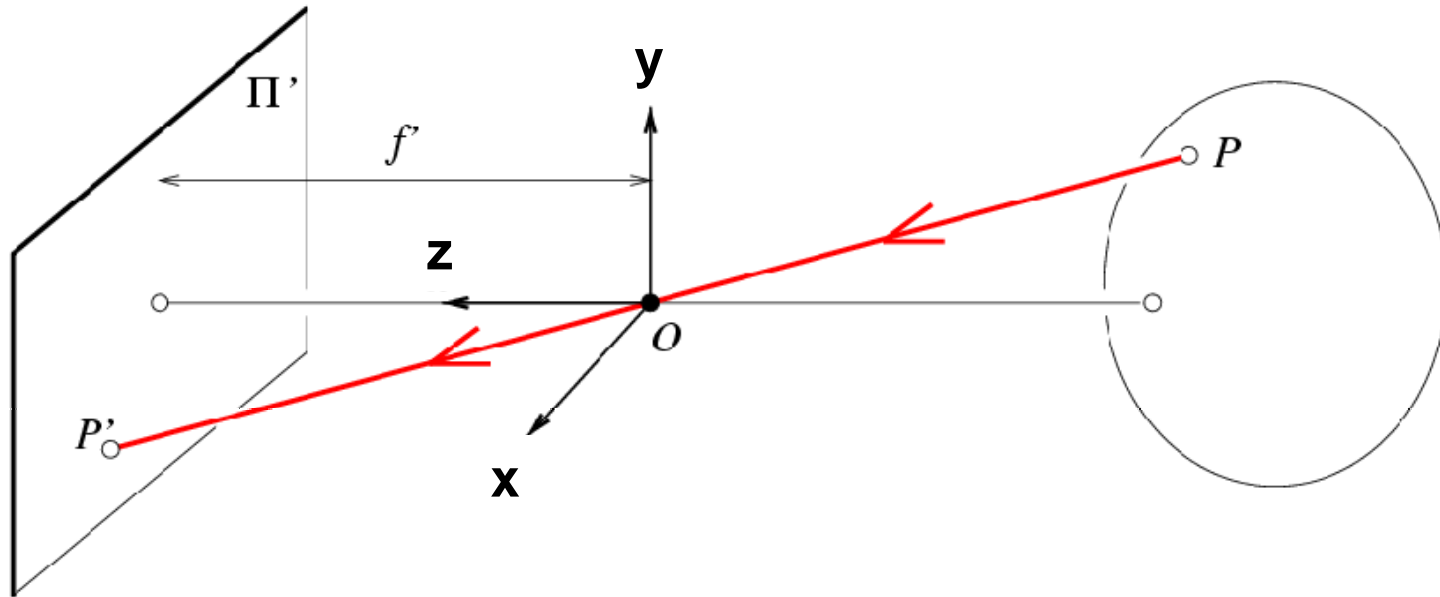
- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci



Perspective distortion: People



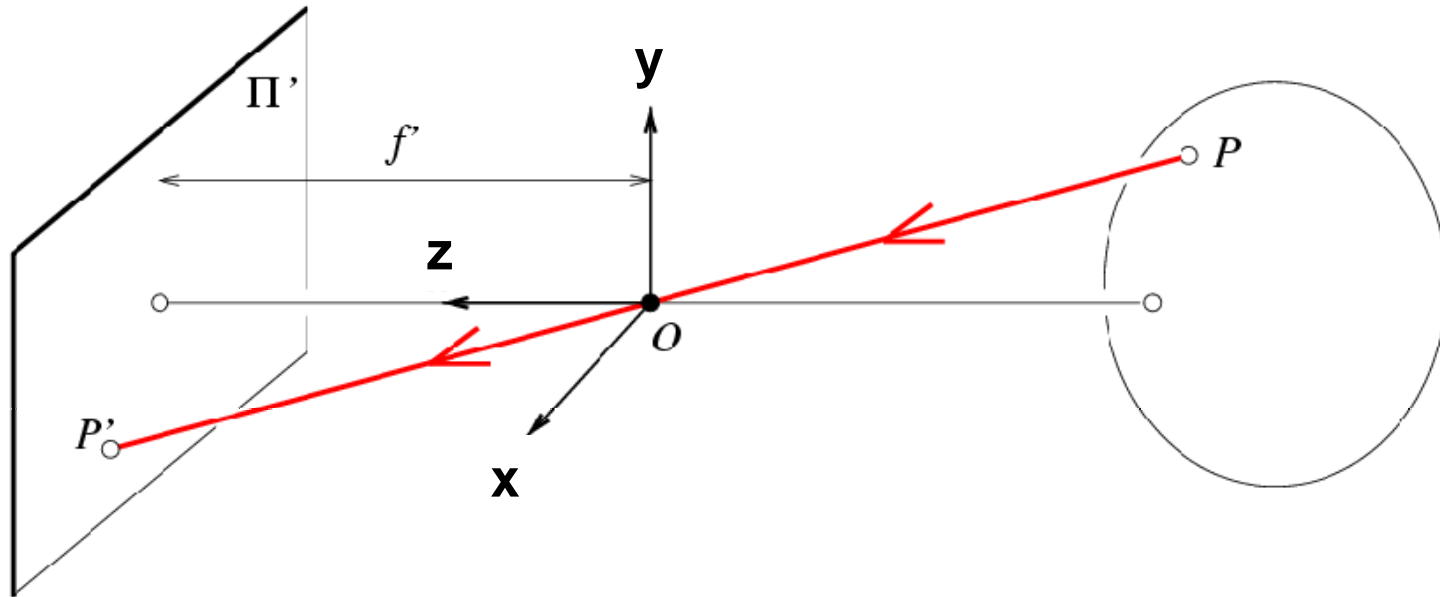
Modeling projection



The coordinate system

- We will use the pinhole model as an approximation
- Put the optical center (O) at the origin
- Put the image plane (Π') *in front* of O

Modeling projection



Projection equations

- Compute intersection with Π' of ray from $P = (x,y,z)$ to O
- Derived using similar triangles

Homogeneous coordinates

$$(x, y, z) \rightarrow \left(f' \frac{x}{z}, f' \frac{y}{z} \right)$$

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \end{bmatrix} \Rightarrow \left(f' \frac{x}{z}, f' \frac{y}{z} \right)$$

divide by the third coordinate

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \end{bmatrix} \Rightarrow \left(f' \frac{x}{z}, f' \frac{y}{z} \right)$$

divide by the third coordinate

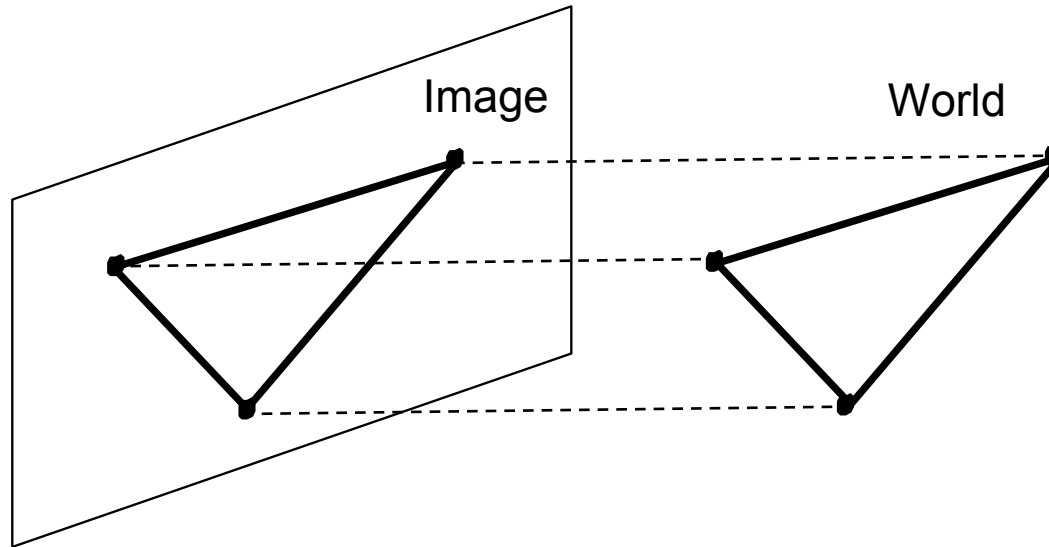
In practice: lots of coordinate transformations...

$$\begin{pmatrix} \text{2D} \\ \text{point} \\ (3 \times 1) \end{pmatrix} = \begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ (3 \times 3) \end{pmatrix} \begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ (3 \times 4) \end{pmatrix} \begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ (4 \times 4) \end{pmatrix} \begin{pmatrix} \text{3D} \\ \text{point} \\ (4 \times 1) \end{pmatrix}$$

Orthographic Projection

Special case of perspective projection

- Distance from center of projection to image plane is infinite



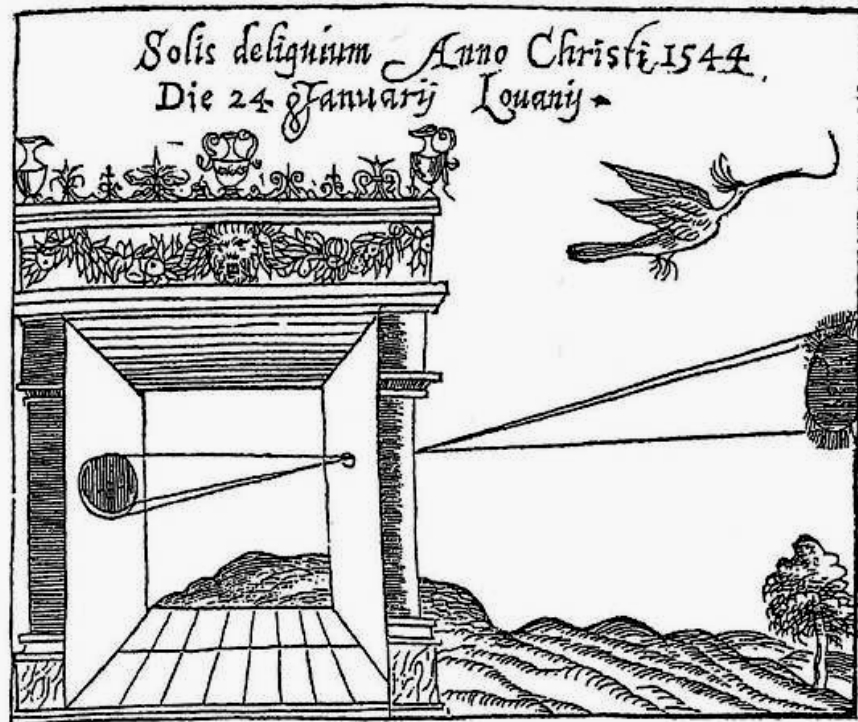
- Also called “parallel projection”
- What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Building a real camera



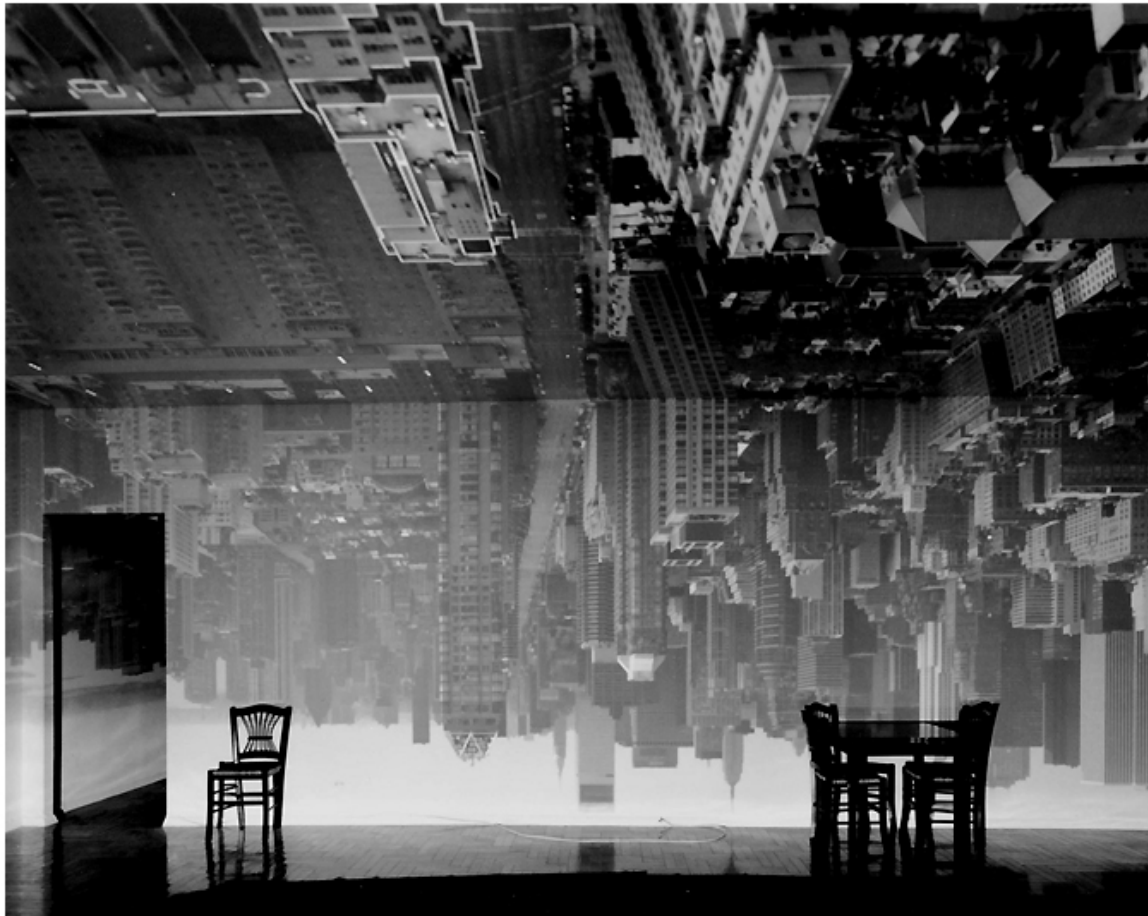
Camera Obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Abelardo Morell



Camera Obscura Image of Manhattan View
Looking South in Large Room, 1996

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

From *Grand Images Through a Tiny Opening*, **Photo District News**, February 2005

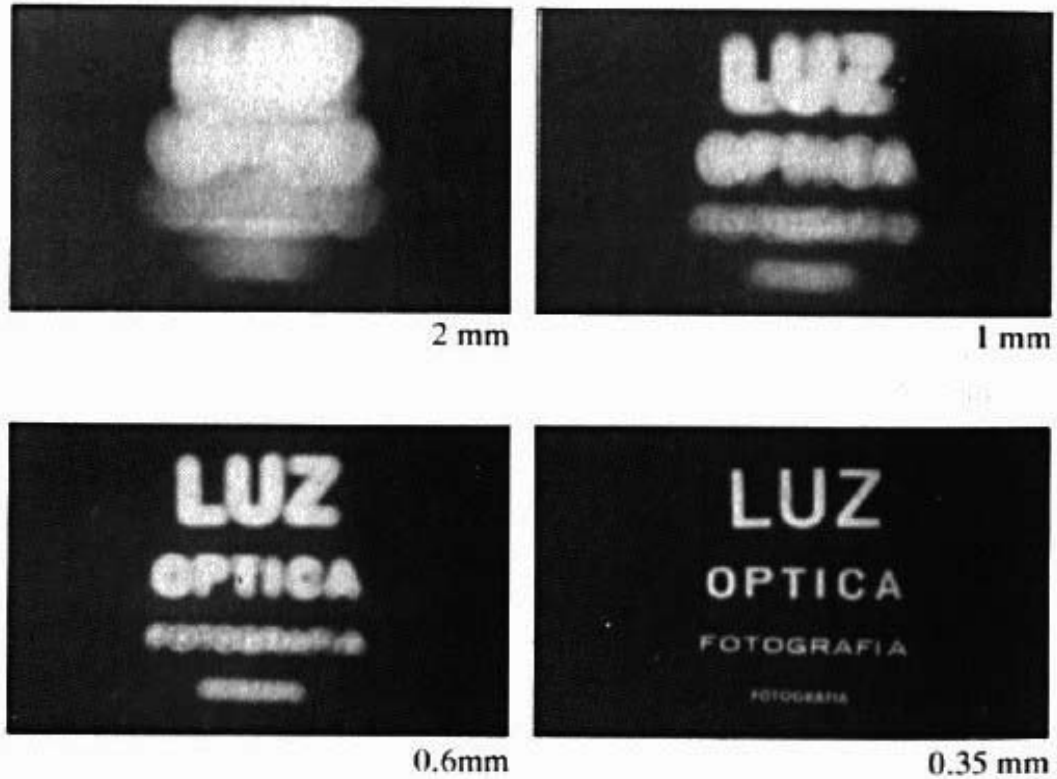
http://www.abelardomorell.net/camera_obscura1.html

Home-made pinhole camera



Why so blurry?

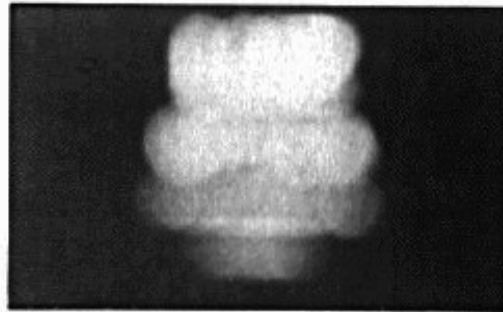
Shrinking the aperture



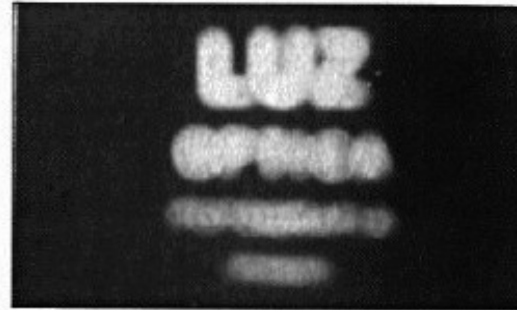
Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects...

Shrinking the aperture



2 mm



1 mm



0.6mm



0.35 mm

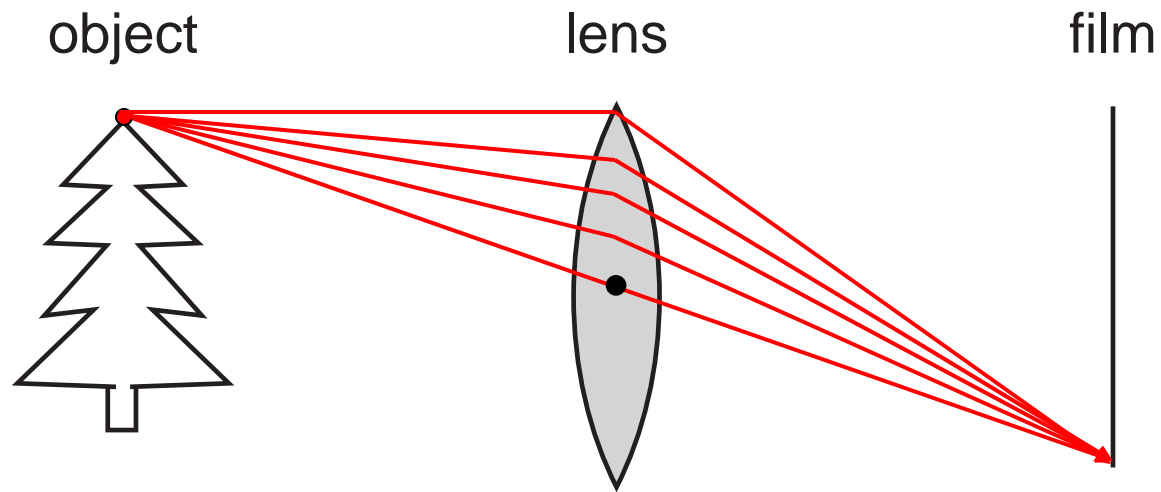


0.15 mm



0.07 mm

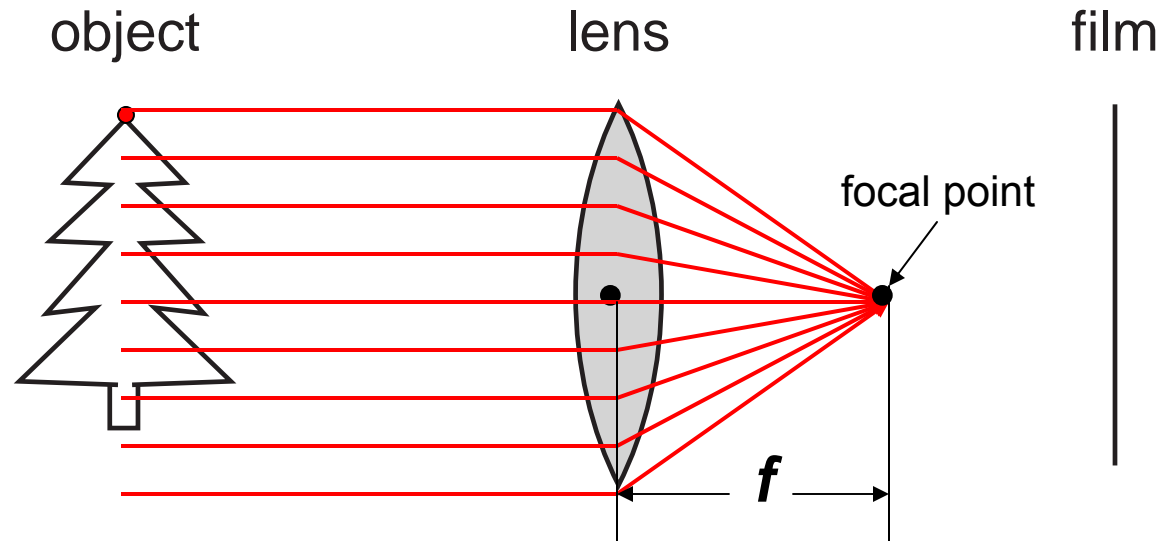
Adding a lens



A lens focuses light onto the film

- Rays passing through the center are not deviated

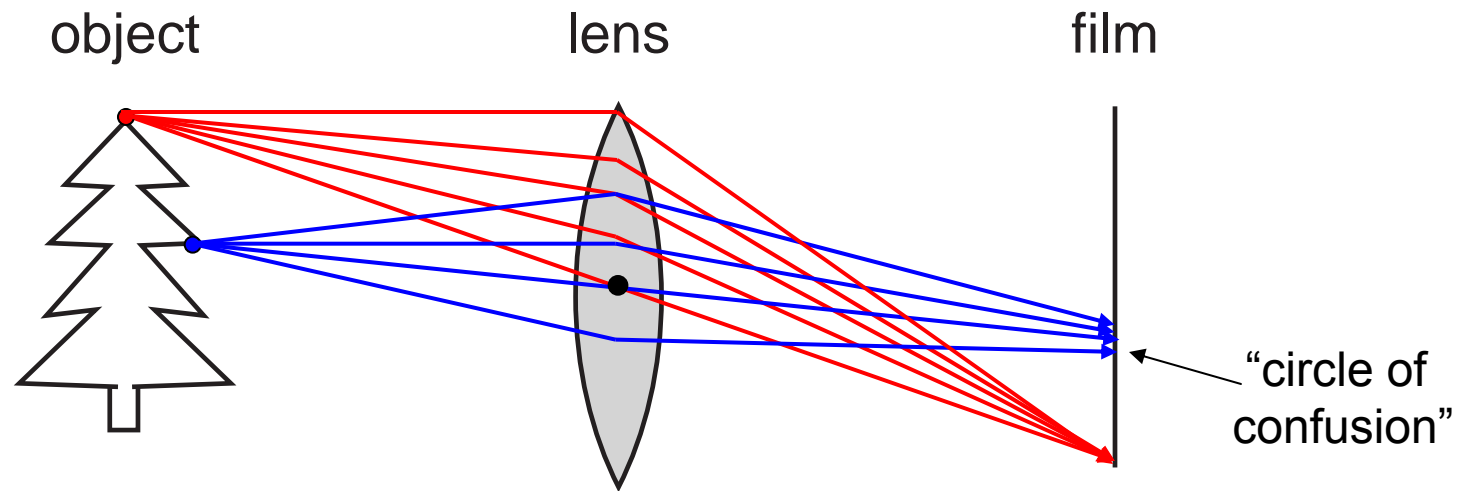
Adding a lens



A lens focuses light onto the film

- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the *focal length* f

Adding a lens

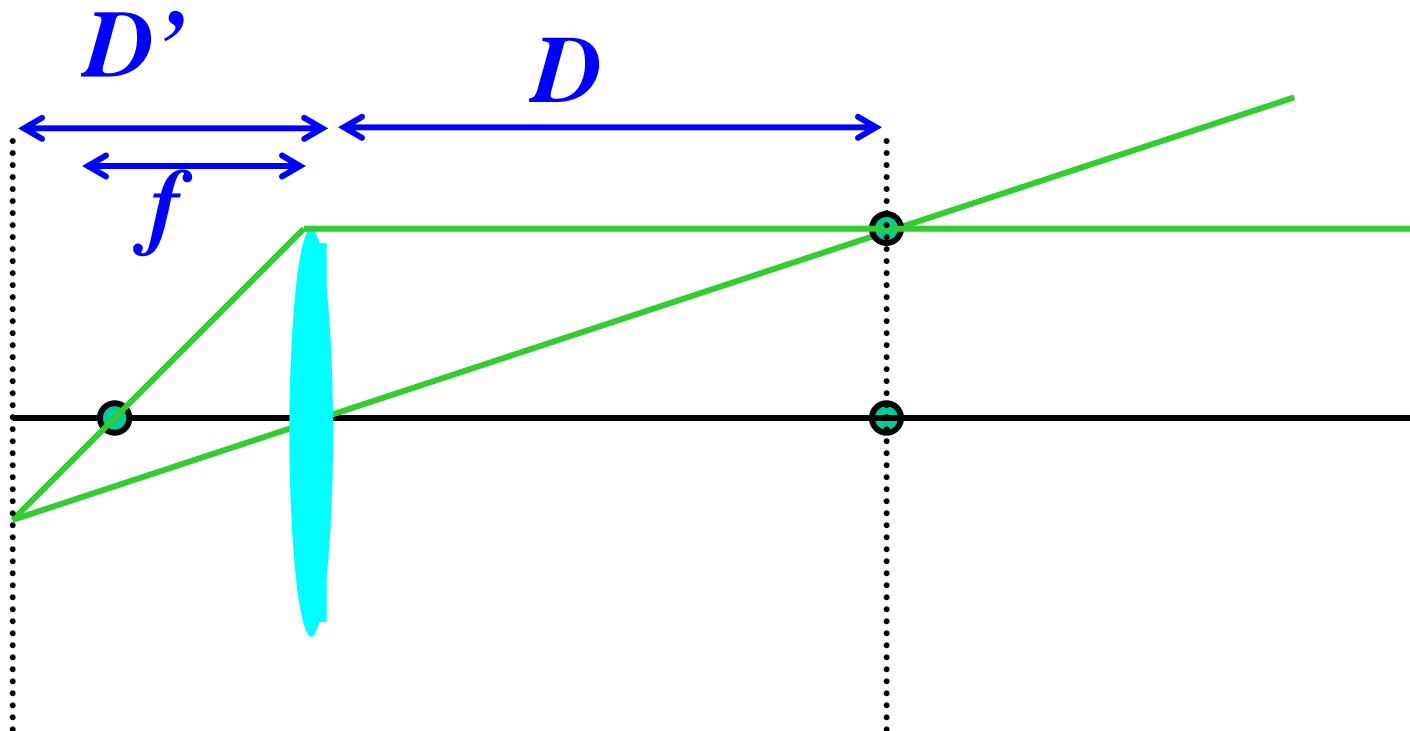


A lens focuses light onto the film

- There is a specific distance at which objects are "in focus"
 - other points project to a "circle of confusion" in the image

Thin lens formula

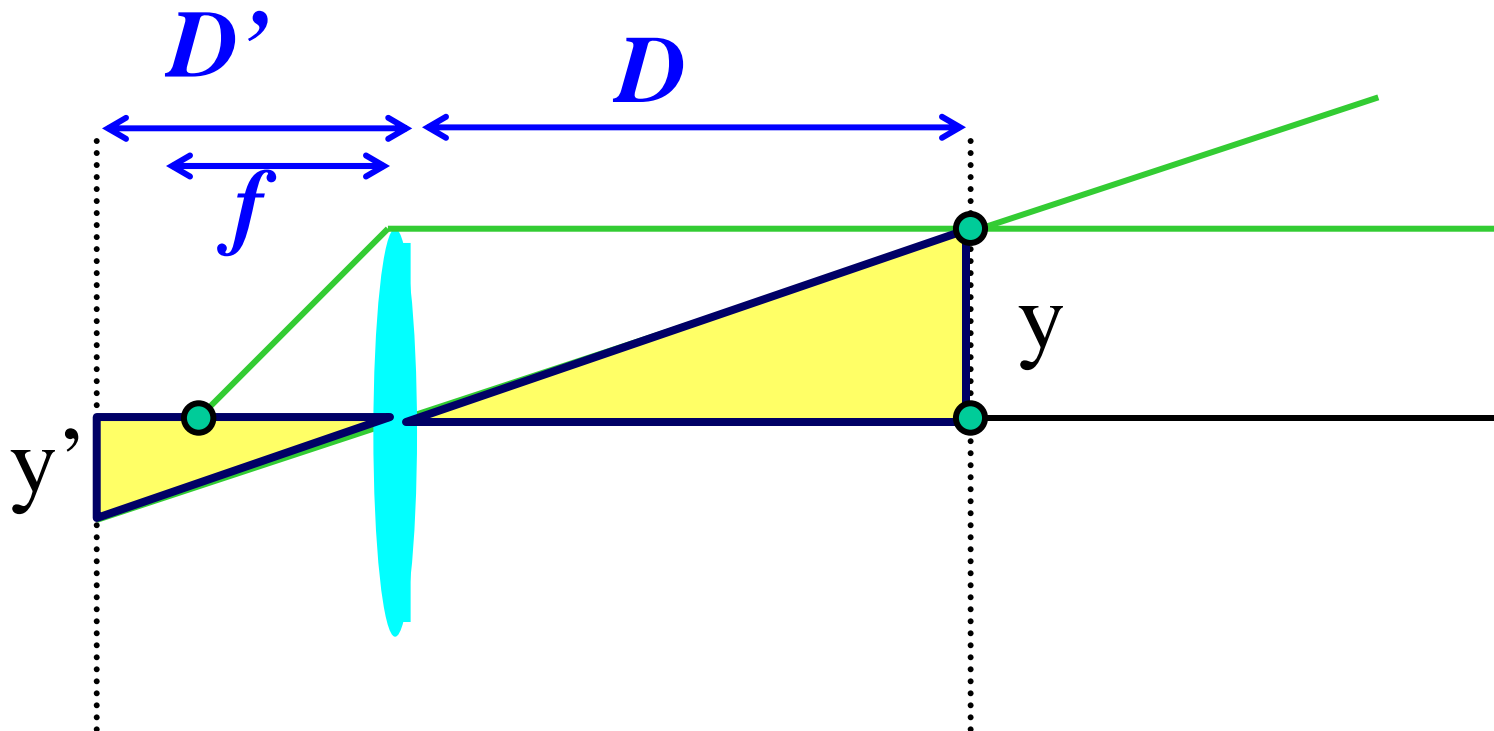
Similar triangles everywhere!



Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$

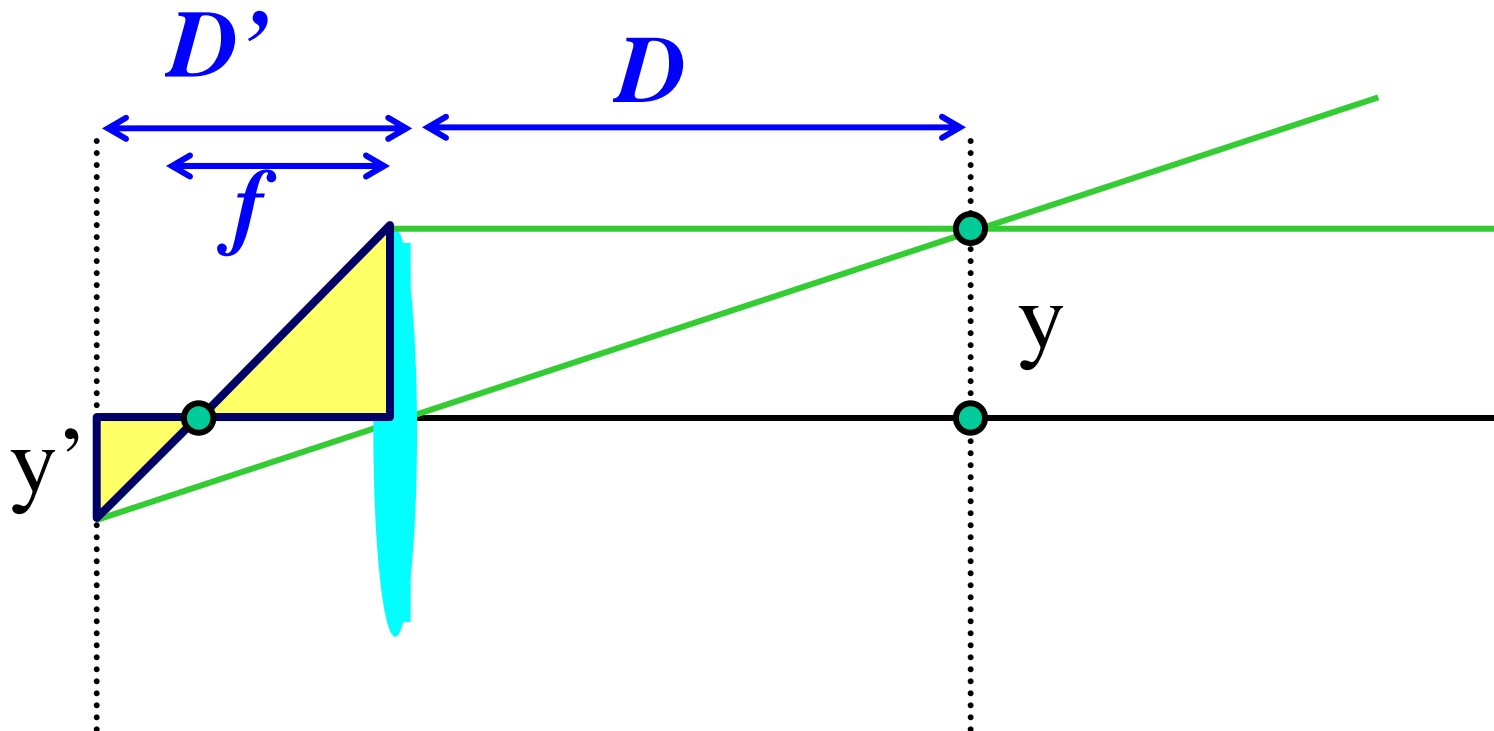


Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$

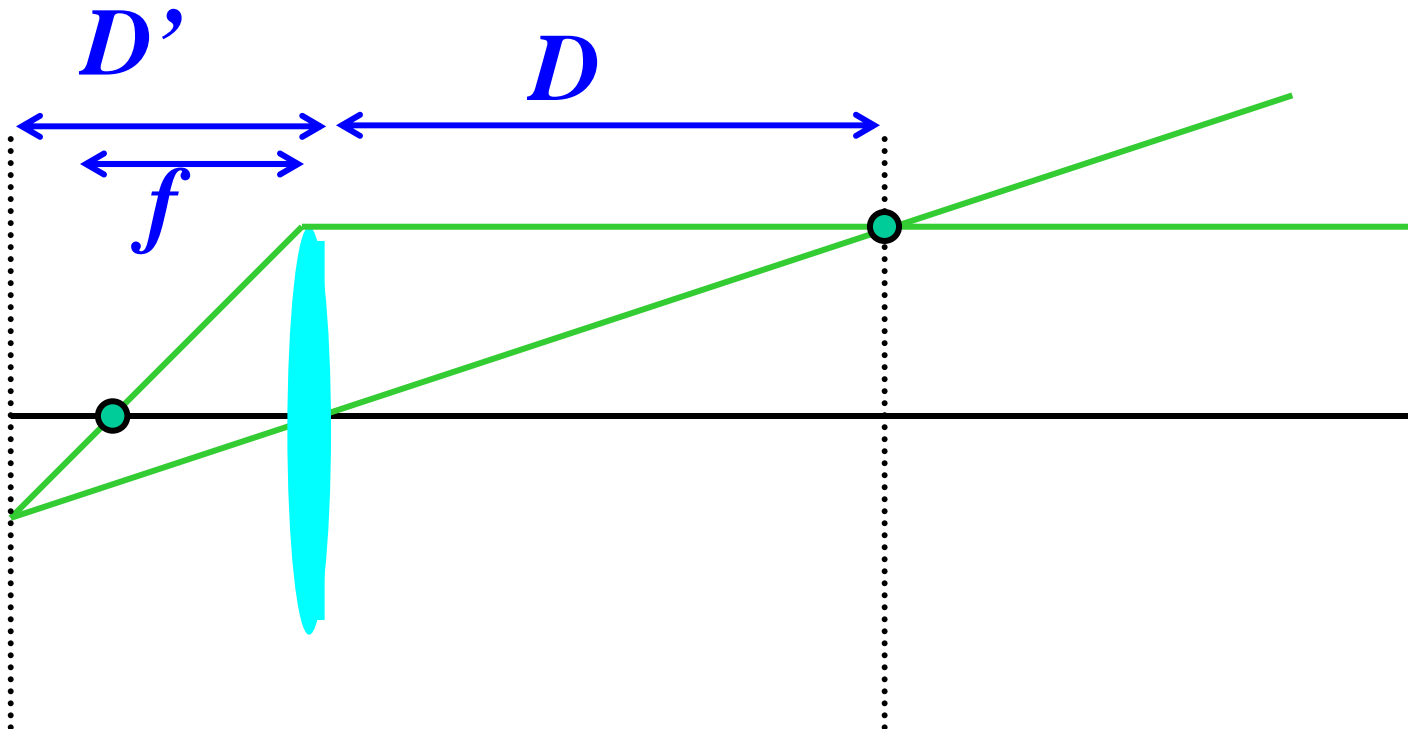
$$y'/y = (D' - f)/D$$



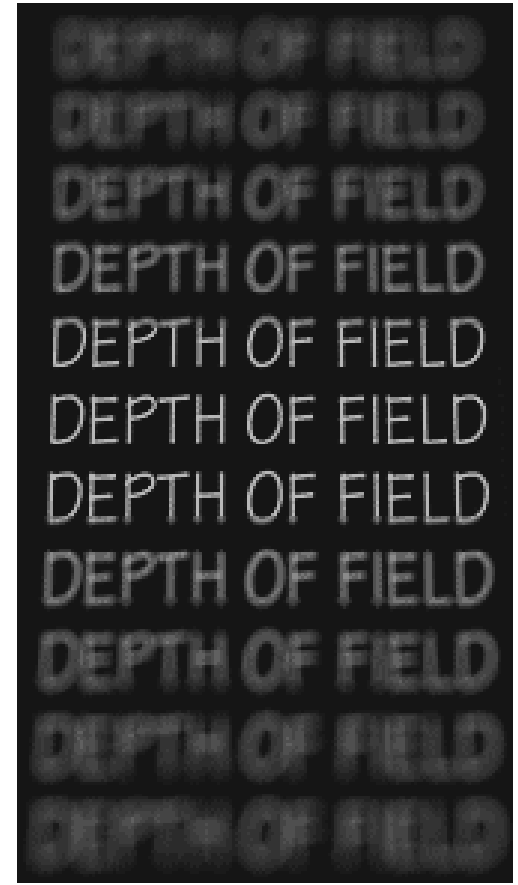
Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

Any point satisfying the thin lens equation is in focus.

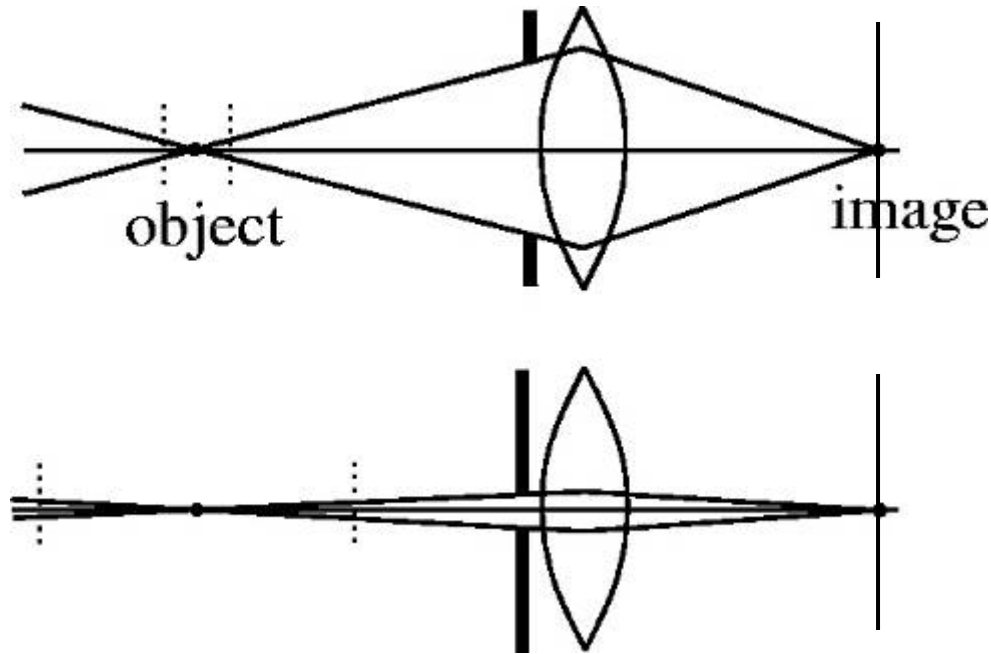


Depth of Field



<http://www.cambridgeincolour.com/tutorials/depth-of-field.htm>

How can we control the depth of field?



Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure

Varying the aperture



Large aperture = small DOF



Small aperture = large DOF

Manipulating the plane of focus

In this image, the plane of focus is almost at a right angle to the image plane

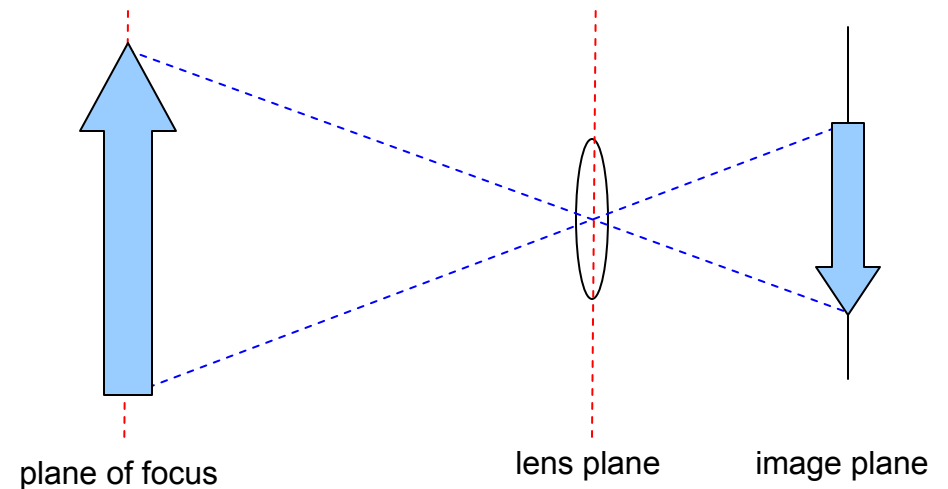
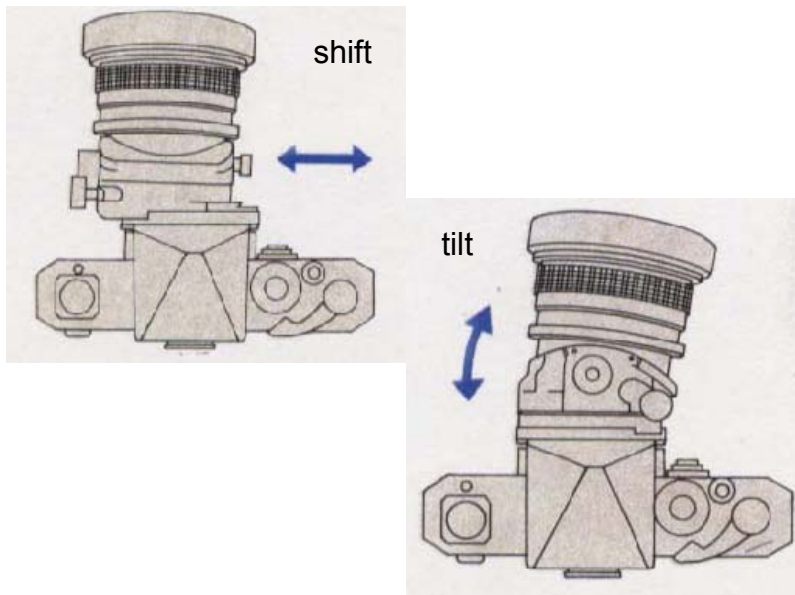


IAN GROOVER Untitled, 1985

Source: F. Durand

Tilt-shift lenses

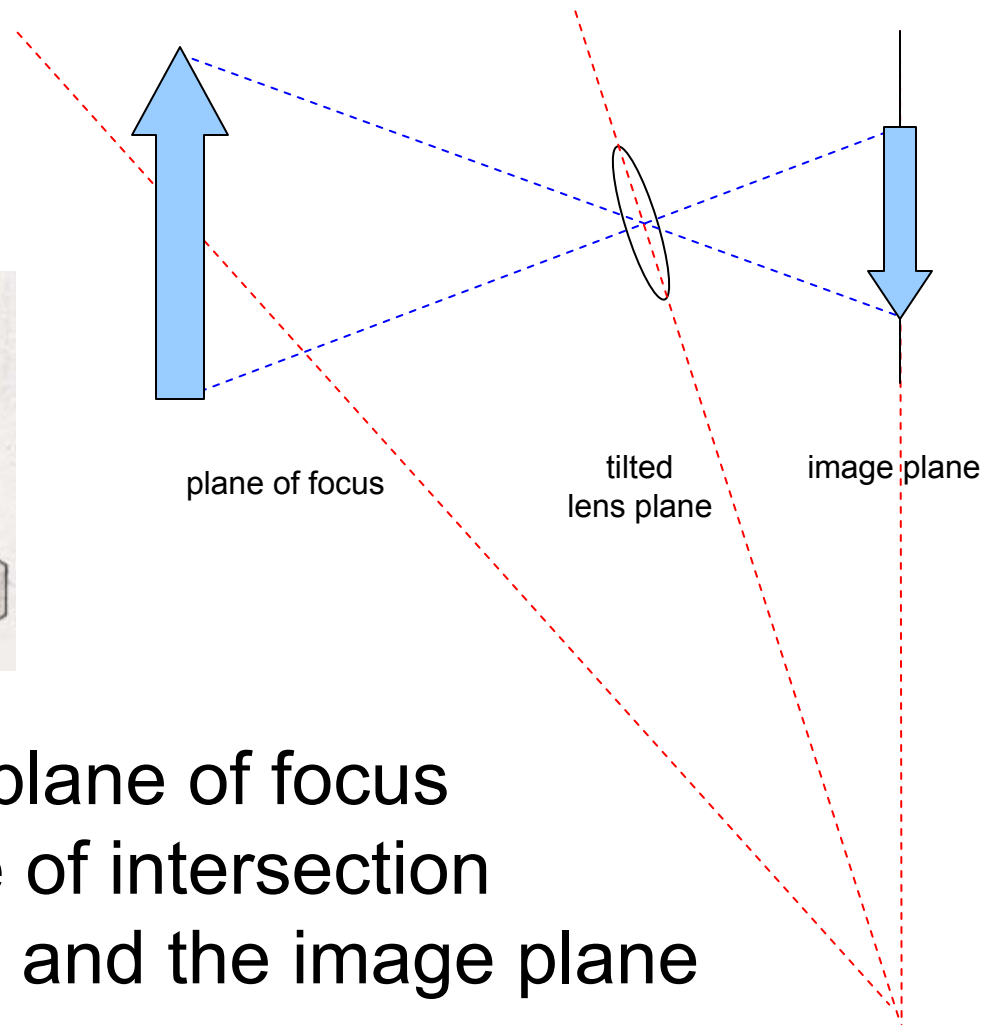
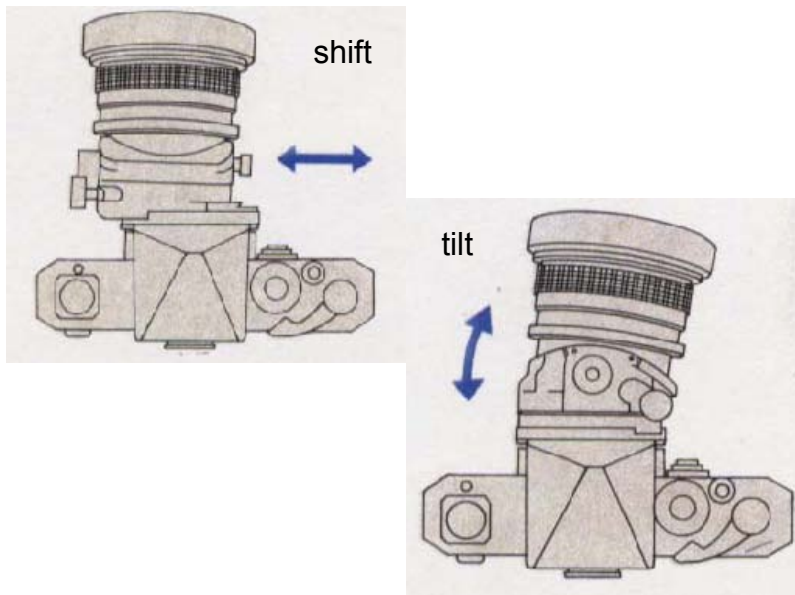
- Tilting the lens with respect to the image plane allows to choose an arbitrary plane of focus



- Standard setup: plane of focus is parallel to image plane and lens plane

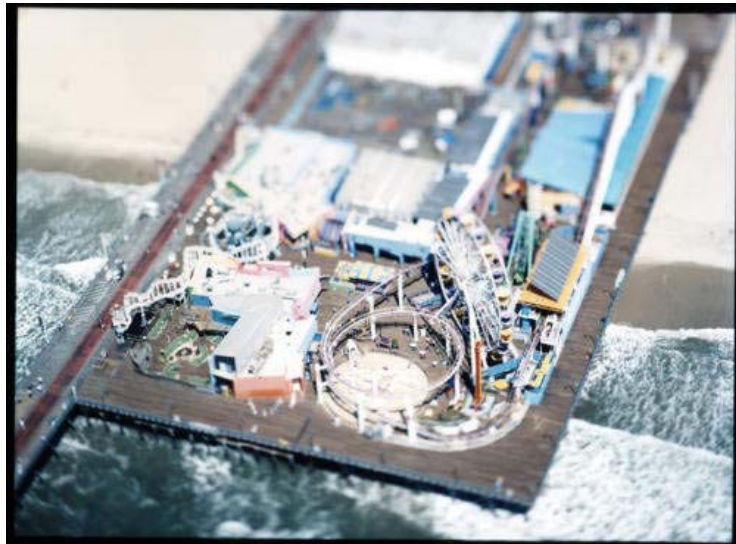
Tilt-shift lenses

- Tilting the lens with respect to the image plane allows to choose an arbitrary plane of focus



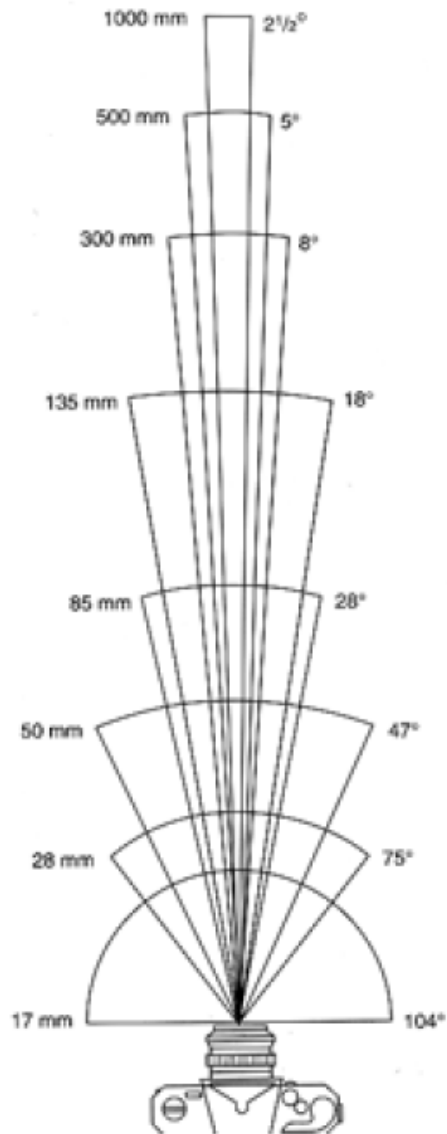
- [Scheimpflug principle](#): plane of focus passes through the line of intersection between the lens plane and the image plane

“Fake miniatures”



Olivo Barbieri: <http://www.metropolismag.com/cda/story.php?artid=1760>

Field of View (Zoom)



17mm



28mm



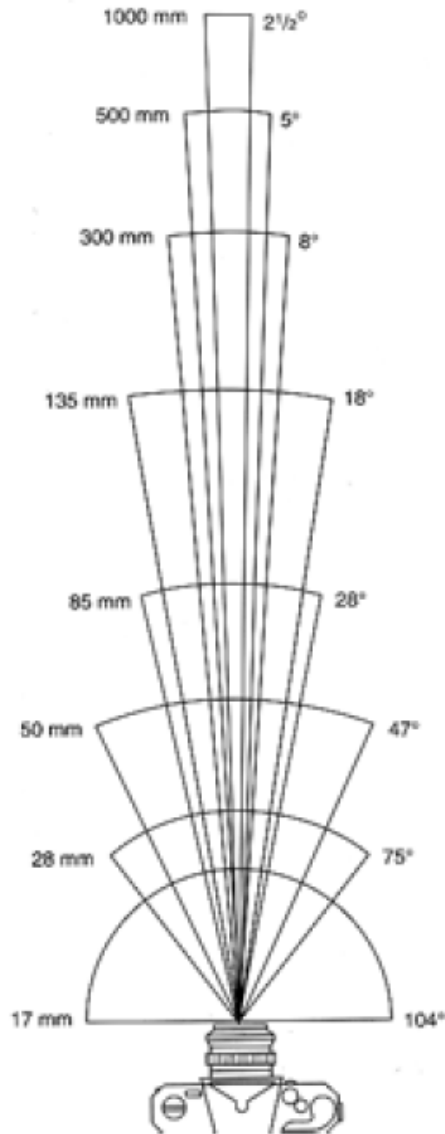
50mm



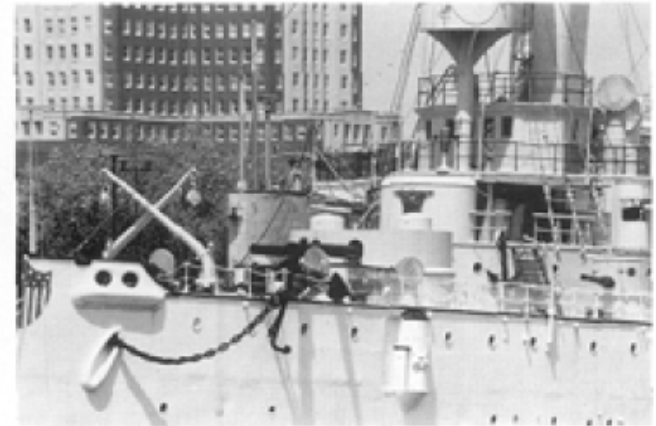
85mm

From London and Upton

Field of View (Zoom)



135mm



300mm



500mm

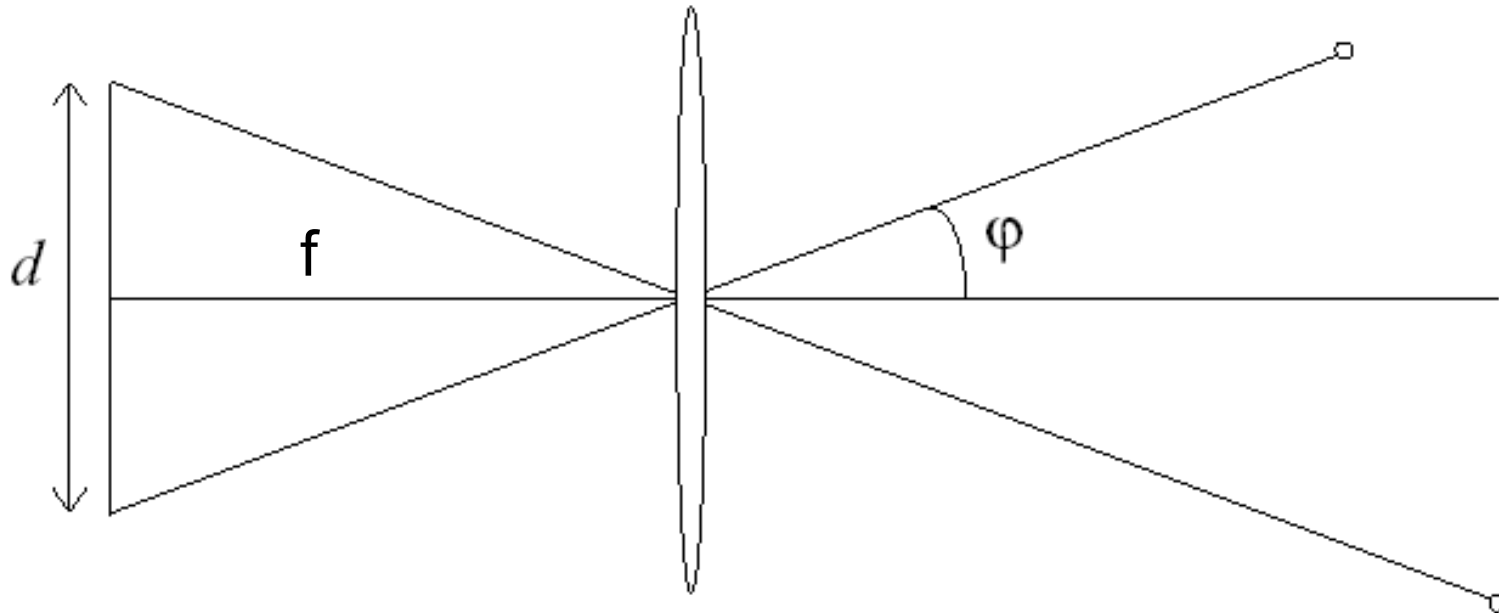


1000mm

From London and Upton

Slide by A. Eφος

Field of View

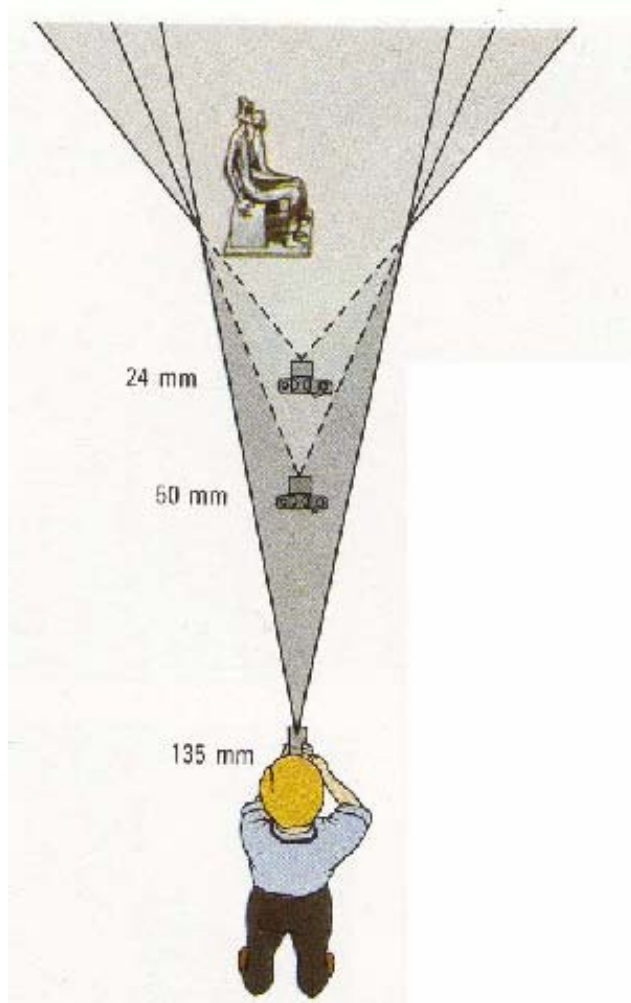


FOV depends on focal length and size of the camera retina

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

Field of View / Focal Length

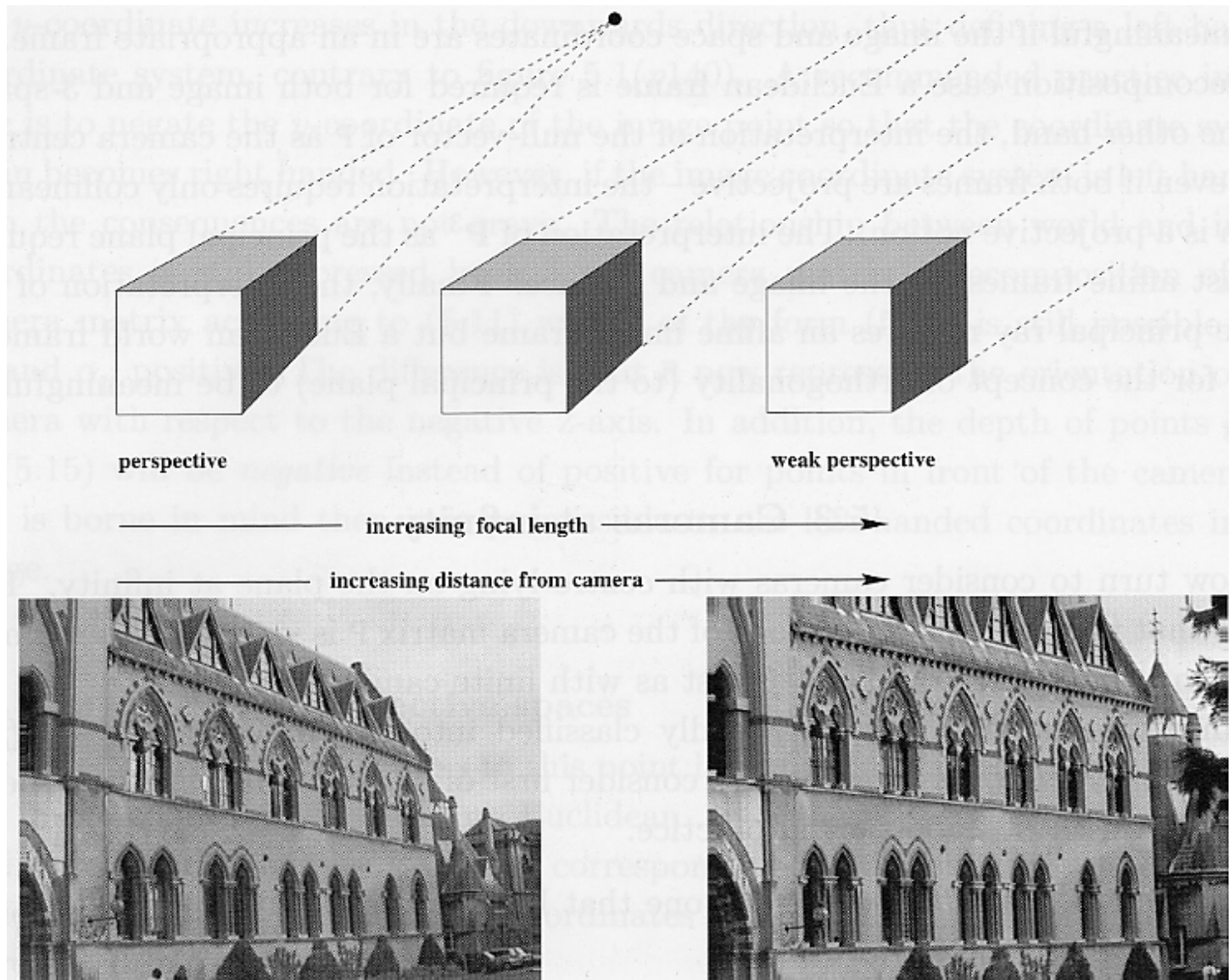


Large FOV, small f
Camera close to car



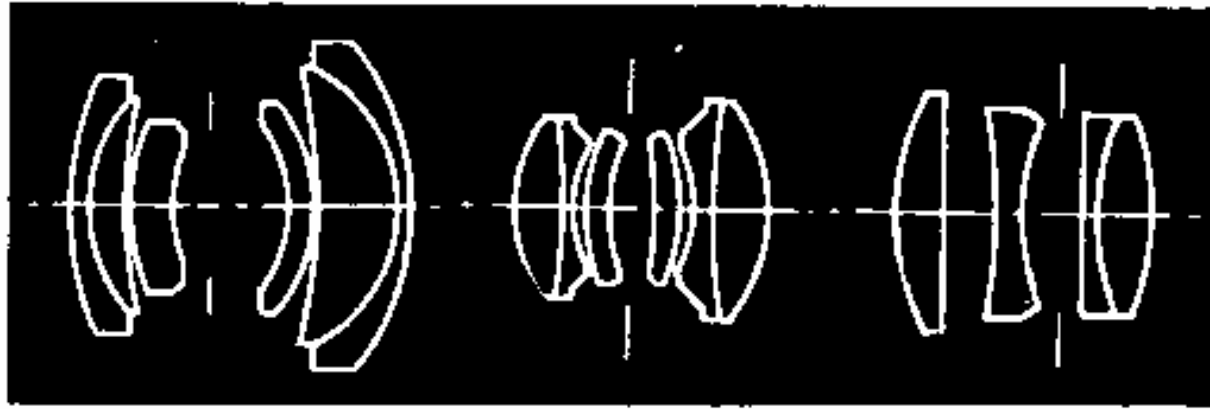
Small FOV, large f
Camera far from the car

Approximating an affine camera



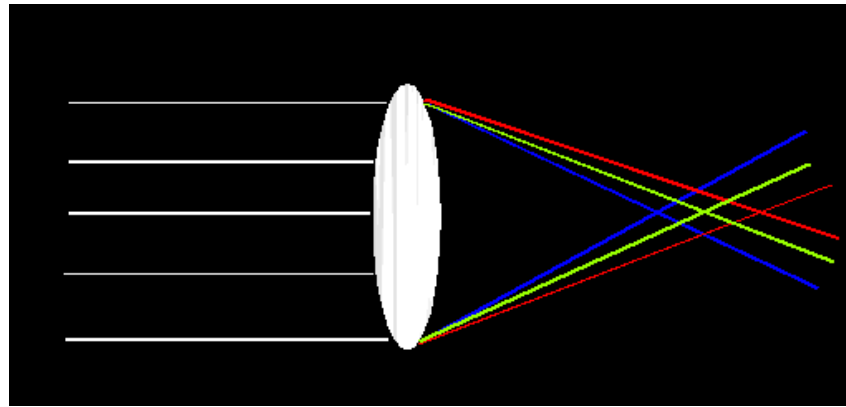
Source: Hartley & Zisserman

Real lenses



Lens Flaws: Chromatic Aberration

Lens has different refractive indices for different wavelengths: causes color fringing



Near Lens Center



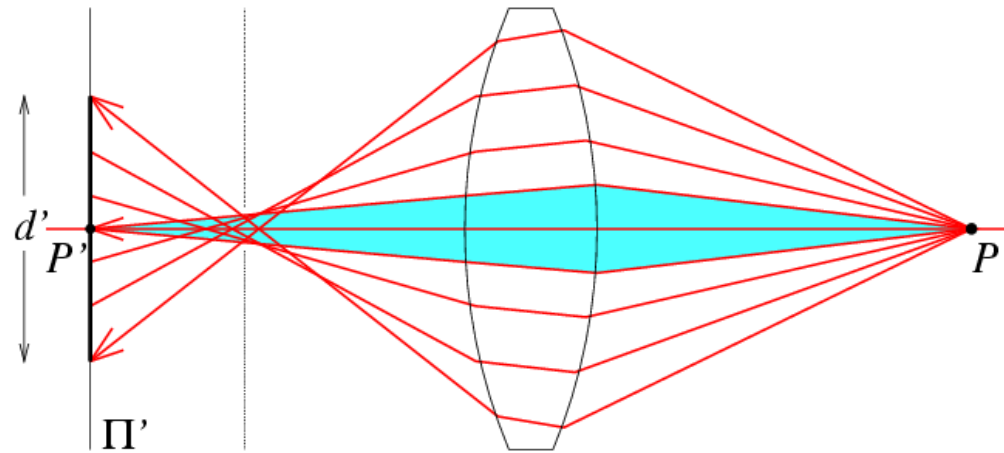
Near Lens Outer Edge



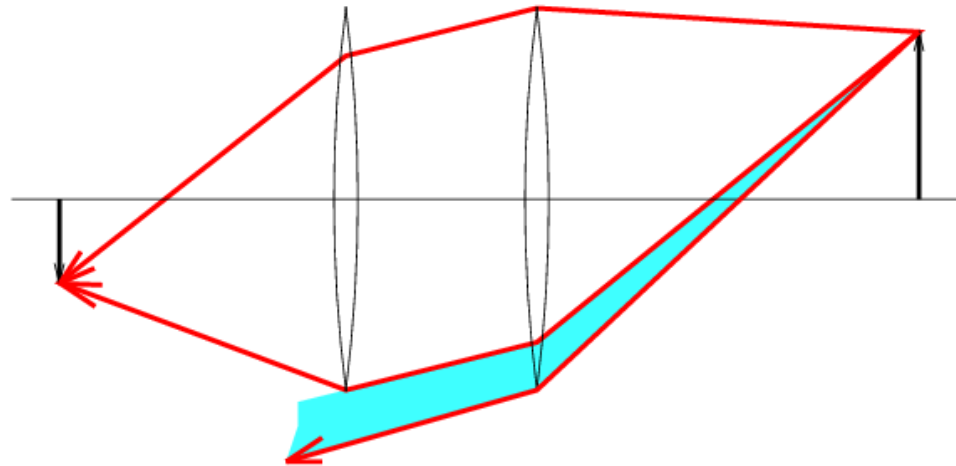
Lens flaws: Spherical aberration

Spherical lenses don't focus light perfectly

Rays farther from the optical axis focus closer

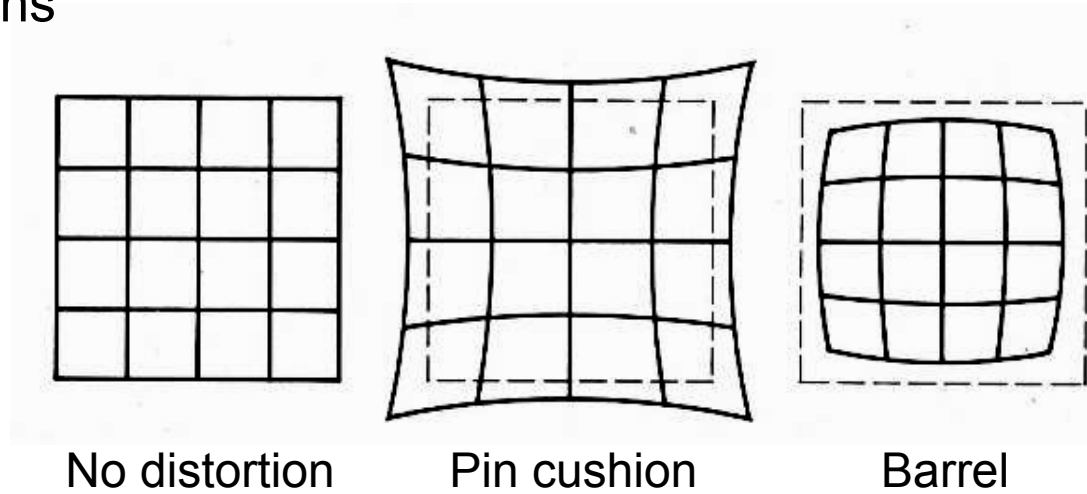


Lens flaws: Vignetting

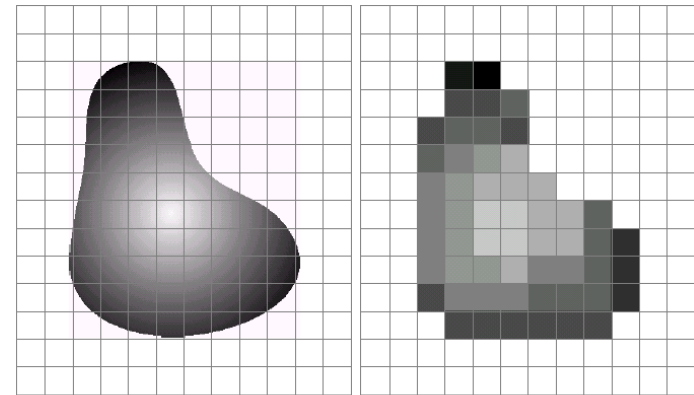


Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



Digital camera



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

A digital camera replaces film with a sensor array

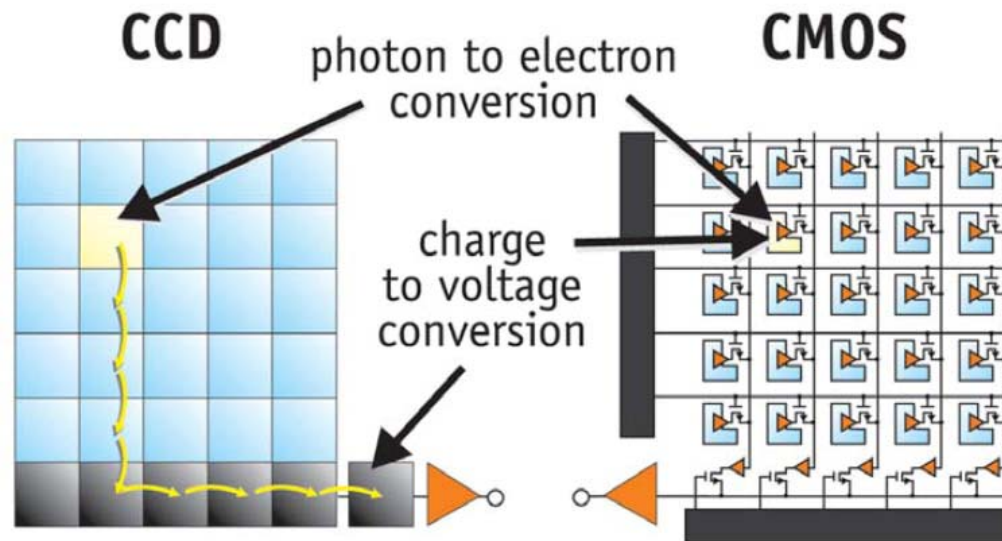
- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types
 - **Charge Coupled Device (CCD)**
 - **Complementary metal oxide semiconductor (CMOS)**
- <http://electronics.howstuffworks.com/digital-camera.htm>

CCD vs. CMOS

CCD: transports the charge across the chip and reads it at one corner of the array.
An **analog-to-digital converter (ADC)** then turns each pixel's value into a digital value by measuring the amount of charge at each photosite and converting that measurement to binary form

CMOS: uses several transistors at each pixel to amplify and move the charge using more traditional wires. The CMOS signal is digital, so it needs no ADC.

<http://electronics.howstuffworks.com/digital-camera.htm>

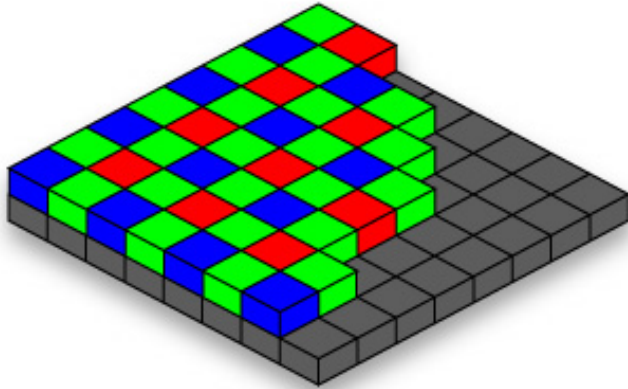


CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

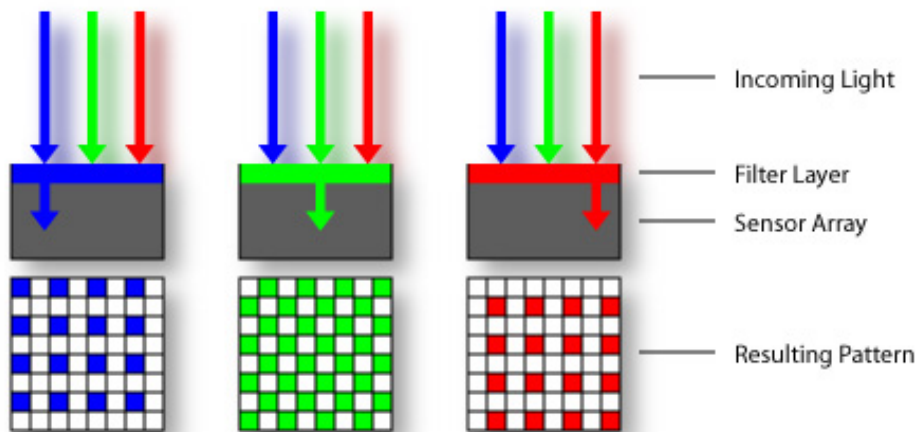
[http://www.dalsa.com/shared/content/pdfs/CCD vs CMOS Litwiller 2005.pdf](http://www.dalsa.com/shared/content/pdfs/CCD_vs_CMOS_Litwiller_2005.pdf)

Color sensing in camera: Color filter array

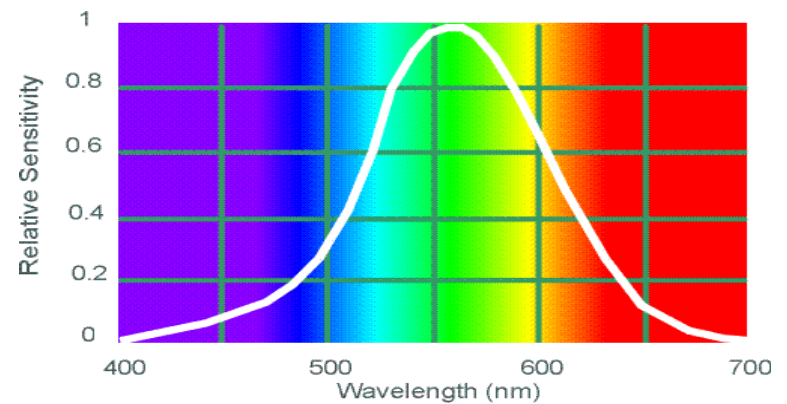
Bayer grid



Estimate missing components from neighboring values (demosaicing)



Why more green?

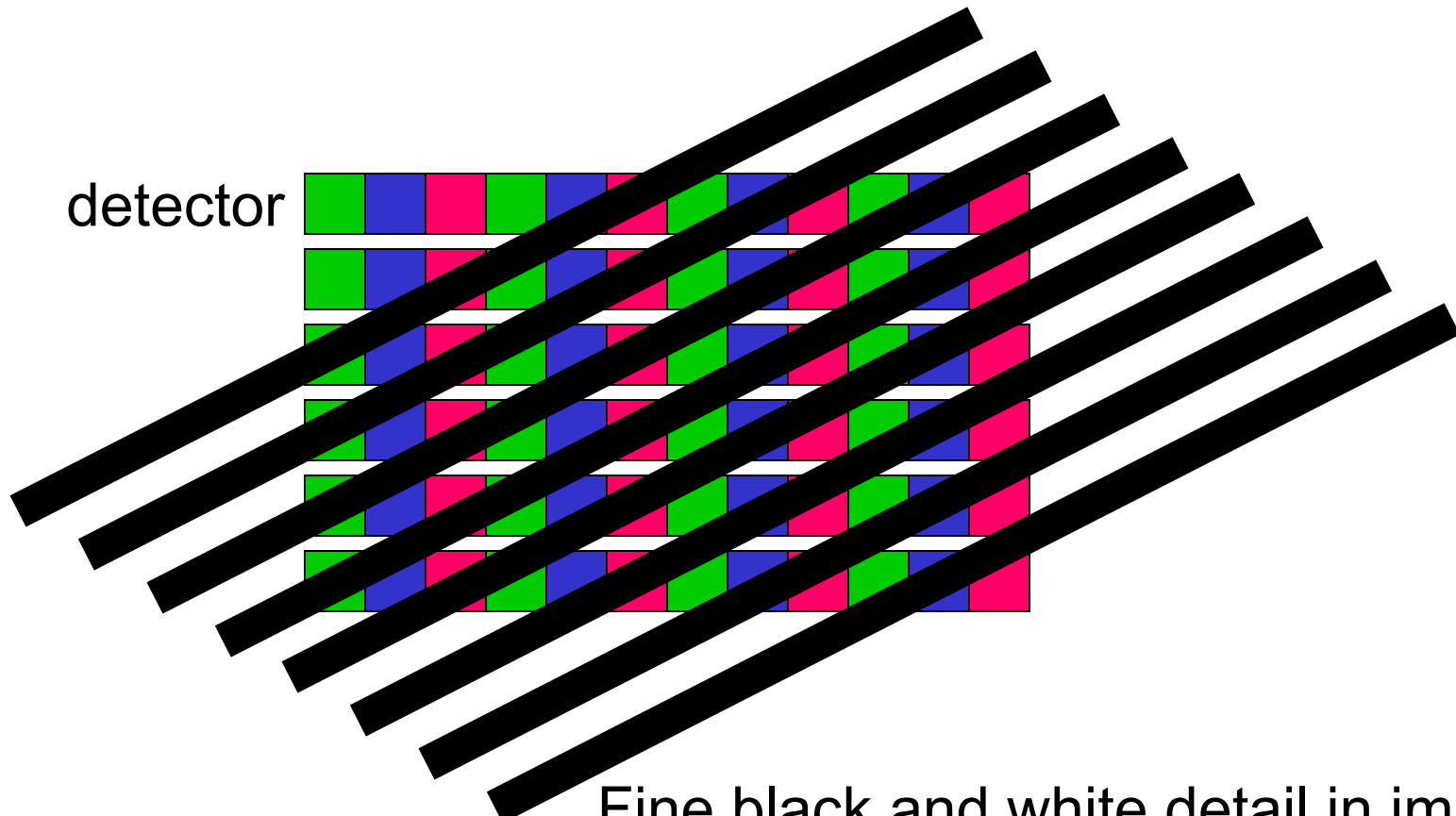


Human Luminance Sensitivity Function

Problem with demosaicing: color moire



The cause of color moire

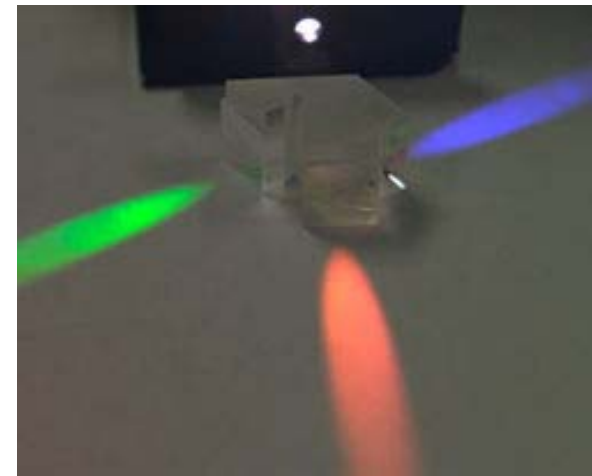
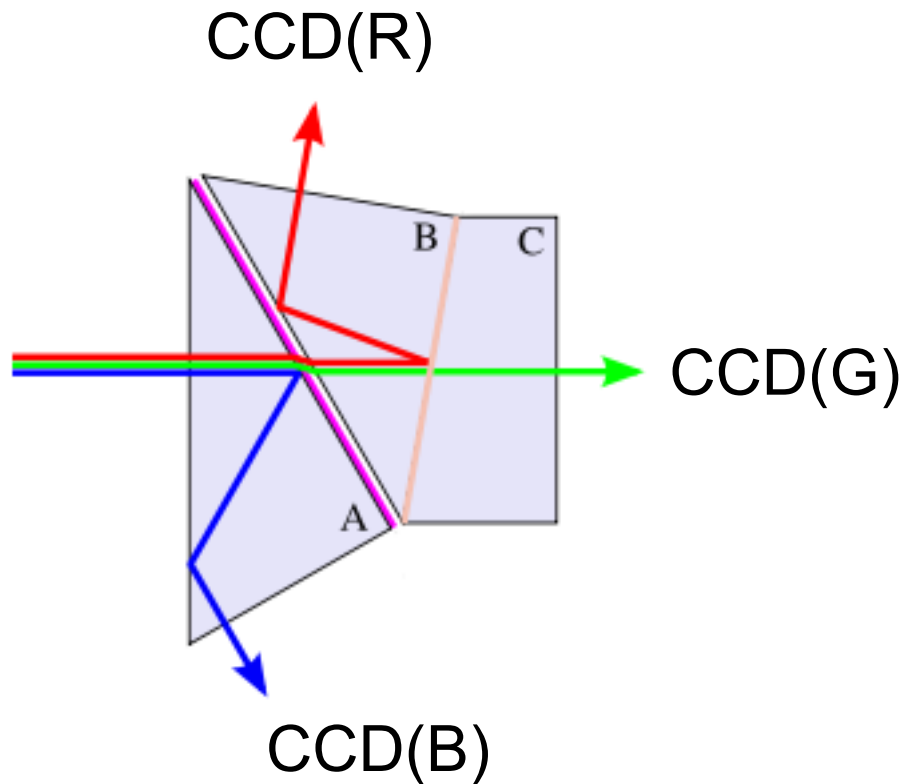


detector

Fine black and white detail in image
misinterpreted as color information

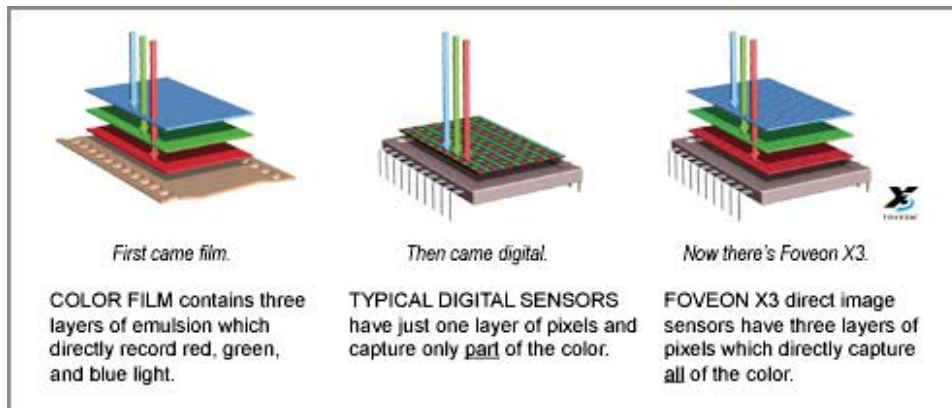
Color sensing in camera: Prism

- Requires three chips and precise alignment
- More expensive

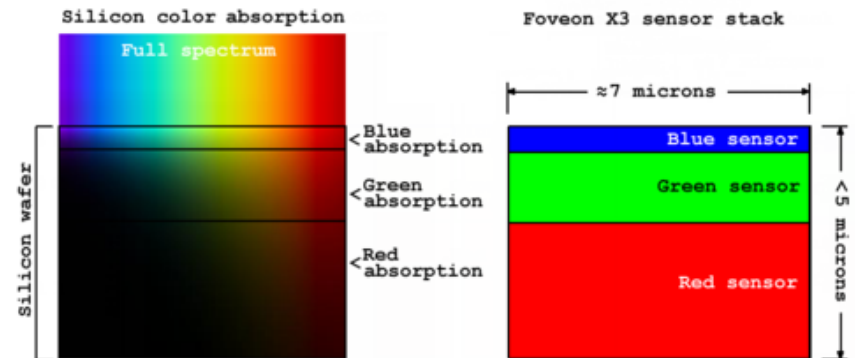


Color sensing in camera: Foveon X3

- CMOS sensor
- Takes advantage of the fact that red, blue and green light penetrate silicon to different depths

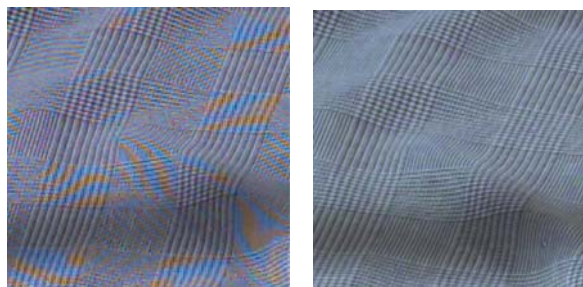


<http://www.foveon.com/article.php?a=67>



http://en.wikipedia.org/wiki/Foveon_X3_sensor

better image quality



Source: M. Pollefeys

Issues with digital cameras

Noise

- low light is where you most notice [noise](#)
- light sensitivity (ISO) / noise tradeoff
- stuck pixels



Resolution: Are more megapixels better?

- requires higher quality lens
- noise issues

In-camera processing

- oversharpening can produce [halos](#)



RAW vs. compressed

- file size vs. quality tradeoff

Blooming

- charge [overflowing](#) into neighboring pixels



Color artifacts

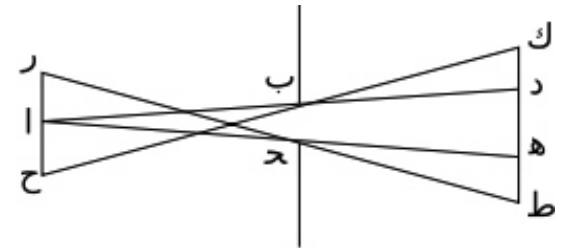
- [purple fringing](#) from microlenses, artifacts from [Bayer patterns](#)
- white balance

More info online:

- <http://electronics.howstuffworks.com/digital-camera.htm>
- <http://www.dpreview.com/>

Historical context

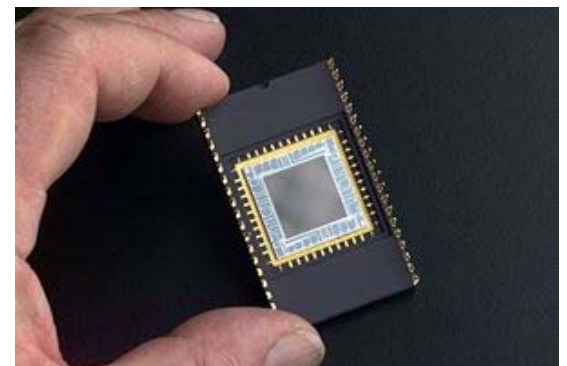
- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicéphore Niépce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD:** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes



Niepce, "La Table Servie," 1822



CCD chip

Next time

Light and color

