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Wavelet to Multiwavelet Packet for Noisy Image Compression: A Comparative study

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Abstract:
Many methods have been presented over the past years to perform image compression. These have one common goal i.e. to alter the representation of information contained in an image. Advances in wavelet transforms and quantization methods have produced algorithms capable of surpassing the existing image compression standards like the Joint Photographic Expert Group (JPEG) algorithm [28,27]. Even though, some images have high frequency information (which is how noise may appear in an image) that is not preserved well in standard Wavelet based compression algorithms. However, the design possibilities for wavelets are limited because they cannot simultaneously posses all of the desirable properties. To this end, relatively new transforms are developed i.e. Wavelet Packet, Multiwavelet, Multiwavelet packet [27,1,40]. The performance of Wavelet, Multiwavelet, Multiwavelet Packet depends on image content. This performance varies for natural and synthetic image for low and high frequency contents. Multiwavelet offers more design options and are able to combine several desirable transform features. Theoretical and experimental results in the study of Multiwavelets have been steadily progressing and all of the key components for the applications of Multiwavelets to image compression are now in place. Our objective in this paper is to study and analyze the various views on the latest wavelet based algorithms and to represent in some subjective comparative form, calibrate measures and to generalize the theory in the presence of noise. We will also try to focus their utility and limitations pertaining to related applications. Multiwavelet approach compression techniques are helpful to get better performance for perfect reconstruction property, and it may be found suitable for enhancing the computability for compression of noisy images.

Keywords:
Wavelet, Wavelet Packet, Multiwavelet, Multiwavelet Packet, and Noisy image Compression.

1. Introduction:

“ A picture is worth a thousand words”. This is all the more true in the modern era (digital) in which information has become one of the most valued assets. A thousand words stored in a digital computer requires very little capacity, but a single pictures can require a very large storage capacity, while the advancement of information storage technology continues rapid pace, a means of reducing the storage requirements of a still image is badly needed in most situations. Therefore, image compression plays an important role in tele-video conferencing, remote sensing, document and medical imaging FAX, and control of remotely piloted vehicles in military etc., [35,7]. Many methods for digital image compression have been studied over the past decades. Advances in wavelet transforms and quantization methods have produced algorithms capable of surpassing the existing image compression standards like the Joint Photographic Expert Group (JPEG) algorithm [28][27]. Even though, some images have high frequency information that is not preserved well in standard Wavelet based compression algorithms. However, the design possibilities for wavelets are limited because they cannot simultaneously posses all of the desirable properties. The relatively new field of wavelet Wavelet Packet and Multiwavelet show good promise obviating some of the limitations of Wavelet. Multiwavelet offers more design options and is capable of combining several desirable transform features. The study of Multiwavelets have been steadily progressing and all of the key components for the applications of Multiwavelets to image compression are now in place.

2. Background:

"Data compression is the art and Science of representing information in compact form" [20]. The reason we need data compression is that more and more of the information that we generate and use in digital form, is in the form of numbers represented by bytes of the data. And the number of bytes required to represent multimedia data can be huge. Compression of data is very essential ingredient in any data storage to cut, large transmission bandwidth and long transmission time for image, Audio and Video data [48]. By compression we mean removal of redundant information in any data. There are many compression algorithms already developed. These algorithms fall into two broad classes 1. Lossless algorithms. 2. Lossy algorithms.

2.1 Lossless compression:

In Lossless data compression, the original data can be recovered exactly from the compressed data. It is generally used for application where any difference between the compressed and the reconstructed data is not allowed. Variable length coding, Huffman coding, Arithmetic coding, LZW coding, Bit plane coding, Lossless Predictive coding are the most commonly used coding techniques for Lossless data compression and normally providing a compression ratio of 2 to 10 and they are equally applicable to both binary to gray scale images [20][34].

2.2 Lossy compression:

Lossy compression techniques involve some loss of information and the data cannot be recovered in the same form. These methods are used where the some loss of data acceptable. However in Lossy compression we can generally obtain a higher compression ratio than the Lossless compression methods. Some of the common algorithms for Lossy compression are Lossy predictive coding, Transform coding, Zonal coding, Wavelet coding, Image compression standard etc [34]. Lossy compression techniques are much more effective at compression than the Lossless methods. And also these techniques give substantial image compression with very good quality reconstruction.

2.3 Performance Criteria in Image Compression:

Once we have developed a data compression scheme, we need to be able to measure its performance. We have to measure the relative complexity of algorithm, memory required to implement the algorithm, how fast the algorithm performs on a given machine, the amount of compression, and how closely the reconstruction resembles the original. The two main criteria of measuring the performance of an image compression algorithm thus are *Compression Efficiency*, *Distortion caused by the compression algorithm* and *the Speed of the compression and Decompression process*. The logical way of measuring how well a compression algorithm compresses a given set of data is to look at the compression ratio, compression rate, fidelity, quality etc. [2][24][23][43][37] The higher the compression ratio, the more the noise added to the data, which affects the visual quality of the image and vice versa [20] [47][29].

3. Principles behind Compression:

A Common characteristic of most images is that neighboring pixels are correlated and there exists redundant information. Two fundamental components of compression are redundancy and irrelevancy reduction. Redundancy reduction aims at removing duplication from the signal source (Image and Video). Irrelevancy reduction omits parts of the signal that will not be noticed by the signal receiver, namely the Human Visual System (HVS). In general, three types of redundancy are identified.
-Spatial redundancy.
-Spectral redundancy.
-Temporal redundancy.
Image compression research aims at reducing the numbers of bits needed to represent an image by removing the spatial and spectral redundancies as much as possible [48].

4. Image Compression Methodologies

Many methods have been presented over the past years to perform image compression having one common goal: to alter the representation of information contained in an image, so that it can be represented sufficiently well with less information, regardless of the details of each compression method. These methods fall into two broad categories 1. Lossless algorithms. 2. Lossy algorithms.

In Current methods for lossless image compression, such as the one used in Graphical Interchange Format (GIF), image standard typically uses some form of Huffman or Arithmetic Coder or Integer-to-Integer Wavelet Transform [26]. Unfortunately, current lossless algorithms provide relatively, small compression factors compared with Lossy methods. To achieve a high compression factor, a lossy method must be used. The most popular current lossy image compression methods use a transform-based scheme as shown in figure-1. It consists of three components namely: 1) Source Encoder 2) Quantizer 3) Entropy Encoder.

4.1 Source Encoder.

Over the past years, a variety of linear transforms have been developed, and the choice of the transform used depends on a number of factors, in particular, computational complexity and coding gain [26]. The most commonly used transforms today are Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT), Continuous Wavelet Transform (CWT), Generalized Lapped Orthogonal Transform (Gen LOT)[48,26]. Noisy images can be compressed by removing Gaussian noise and this can be removed using decomposition technique. Discrete Wavelet Transform (DWT) and Discrete Cosine Transform (DCT) are used to decompose noisy images [10]. The performance of DCT and Wavelet transforms is presented and also it is noted that DCT used in JPEG standard is less computationally complex than wavelet transform for a number of image samples [48,26].

4.2 Quantizer:

A quantizer simply reduces the number of bits needed to store transformed coefficients by reducing the precision of those values. Quantization can be performed on each individual coefficient i.e. Scalar Quantization (SQ) or it can be performed on a group of coefficients together i.e. Vector Quantization (VQ). Many methods be proposed to perform quantization of the transform coefficients [44] [8][19]. Even so, quantization remains an active field of research and some new results show a greater promise for wavelet based image compression [26][54][32], the choice of a good quantizer depends on the transform that is selected while transforming, some quantization methods perform better with particular transform. Also, perceptual weighting of coefficients in different sub bands can be used to improve subjective image quality.

Quantization methods used with wavelet transforms fall into two general categories: Embedded and Non Embedded quantizers. They determine bit allocation based on a specified bit budget, allocating bit across a set of quantizers. Corresponding to the image sub bands embedded quantization scheme [26].

4.3 Entropy Coding:
This process removes redundancy in the form of repeated bit patterns in the output of the quantizer. The most commonly used entropy coders are the Huffman Coding, Arithmetic Coding, Run Length Encoding (RLE) and Lempel-Ziv(LZ) algorithm[26]., although for applications requiring fast execution, simple Run Length (RLE) has proved very effective[48]. Consequently arithmetic codes are most commonly used in wavelet-based algorithms [4][14][18][9].

5. Wavelets and Image Compression:
It was shown that the most commonly used image compression methods use three steps: Transform, Quantization and Entropy Coding. Now we shall discuss how the first step , i.e. the transform, may be accomplished using Wavelets. The theory of Wavelets starts with the concepts of Multi Resolution Analysis (MRA)[39, 16, 52, 5, 6,31, 25, 13] and the detailed theory of Multiresolution is found at [34,41]. In many applications wavelets based schemes outperform other coding schemes like the one based on DCT. Since there is no need to block the input image and its basis function have variable length, wavelet coding schemes at higher compression avoid blocking artifacts [53], wavelet based coding is more robust under transmission and decoding error and facilitates progressive transmission of images. Wavelet coding schemes are especially suitable for applications where scalability and tolerable degradation is important. Advances in wavelet transforms and quantization methods have produced algorithms capable of surpassing the existing image compression standard like JPEG algorithm [27].

5.1 Structure of Wavelet
Wavelet transform is a pair of filters. The way we compute the wavelet transform by recursively averaging and differentiating coefficients is called the filter bank [12], where one is a low pass filter (lpf) and the other is a high pass filter (hpf). Each of the filters is downsampled by two. Each of those two output signals can be further transformed. Similarly, this process can be repeated recursively several times, resulting in a tree structure called the decomposition tree. Wavelet transform can be used to analyze or decompose signals and images, which is called decomposition [51][3][49][33]. The same components can be assembled back into the original signal without loss of information called reconstruction or synthesis and the same has been shown in figure 2.

The structure of Wavelet can be represented as a four channel perfect reconstruction of filter bank. Now each filter is 2D with subscript indicating the type of filter (HPF or LPF) for separation of horizontal and vertical components. The resulting four-transform components consist of all possible combinations of high and low pass filtering in the two directions. By using these filters in one stage an image can be decomposed into four bands. There are three types of detail of images for each resolution Diagonal (HH), Vertical (LH) and Horizontal (HL). The operations can be repeated on the low low (LL) i.e. on approximation band using the second identical filters bank [45]. The decomposition process can be iterated, with successive approximations being decomposed. However, in practice, more than one decomposition level is performed on the image data. Successive iterations are performed on the low pass coefficients (approximation) from the previous stage to further reduce the number of low pass coefficients. Since the low pass coefficients

contain most of the original signal energy, this iteration process yields better energy compaction. The quality of compressed image depends on the number of decomposition. Compression of an image can be obtained by ignoring all coefficients less than the threshold value. If we use decomposition iteration, it will be more successful in resolving DWT coefficient because Human Visual System (HVS) is less sensitive to removal of smaller details. Decomposition iterations depend on the filter order. Higher order does not imply better image quality because of the length of the wavelet filter. This becomes a limiting factor for decomposition. Usually, Five levels of decompositions are used in current wavelet based image compression [26][46]
. The maximum levels of Decomposition of any image can be determined by using the formula

Maximum Levels of Decomposition = $\log_2 x_{max}$
Where x_{max} is the maximum size of given image

5.2 Wavelet Packet:
The Wavelet Transform is one type of signal transform that may be used in image compression. But the Wavelet Transform (WT) often fails to accurately capture high frequency information especially at low bit rates where such information is lost in quantization noise. Hence another transform method must be employed. Coifman, Meyer and Wick Hauser have developed such technique based on the wavelet transform and it is called Wavelet Packet (WP)[36]. A single level of standard wavelet decomposition splits the input signal into low pass and high passes coefficients. Though the filtering and downsampling a multilevel wavelet filter bank involves iterating the low pass –high pass filtering and down sampling procedure only on the output of the low pass branch of the previous stage Coifman et.al., formulated an extension of the octave band wavelet decomposition by allowing the low pass – high pass filtering and down sampling procedure to be iterated also on high pass branches in the tree [36]. They defined new basis function called wavelet packet as follows
Let $\phi(t)$ and $\varphi(t)$ be the scaling and wavelets function respectively, which obey the two scale Equation

$$\phi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} hk\ \phi(2t-k) \qquad (1)$$

$$\varphi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} gk\ \phi(2t-k) \qquad (2)$$

Where {hk} and {gk} are the scaling and wavelet filter coefficients. Now let u0=ϕ(t) and u1 (t)=φ(t) and define

$$U_{2n}(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} hkUn(2t-k) \qquad (3)$$

$$U_{2n+1}(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} gkUn(2t-k) \qquad (4)$$

Taking dyadic rescaling translation of these functions yields the library functions {2-j/2 Un (2^{-j}t-k)}. This library is over complete, but a proper complete basis can be found by selecting a subset of the library with right set of parameters {n,j,k}[27].
Wavelet packet performs significantly better than wavelet for compression of images with large amount of textures, such as commonly used in Barbara image. For example Meyer et. al.[9] show that wavelet packet techniques applied to image with texture patterns can give over 0.5 db improvement in same cases over the SPHIT algorithm

results[4]. The authors also point out that the perceived image quality is significantly improved using wavelet packet instead wavelets, especially in the textured regions of the images [26]. Xiong et.al. Show similar results using wavelet and wavelet packets both the SPHIT & their own SFQ (Space Frequency Quantization) method [54]. Regardless of the choice of the quantizer, they show wavelet packets often-outperforming wavelets by 0.5-1.00 db across the bit rates for the Barbara image, i.e. how the image contains large amount of high frequency (which is how noise appear in the image) which is either mostly unstructured (as in Goldhill & Mandrill test images) or geometric or irregular in nature (e.g. Testpat2 & IC). The Testpat2 & IC images were taken from the MATlab image processing toolbox; these were chosen to represent “Synthetic” image type in contrast to the “Natural” images character of the other images [27].

5.3 Multiwavelet

The wavelet transform is a type of signal transform that is commonly used in image compression. A newer alternative to the wavelet transform is the Multiwavelet transform. Multiwavelets are similar to the wavelets but have some important differences. In particular, whereas wavelets have an associated scaling function $\phi(t)$ and wavelet functions $\varphi(t)$, Multiwavelets have two or more scaling functions [15].

For notational convenience, $\varphi(t)$ = the set of scaling functions can be written using vector notation$\Phi(t) \equiv [\ \phi1(t), \phi2(t), \phi3(t)\ldots\ldots\ldots\ldots\ldots \phi r(t)]^T$

Where $\Phi(t)$ is called the multi scaling functions likewise, the Multiwavelet functions is defined form the set of $\varphi(t)$= wavelet functions as

$\Psi(t)=[\ \varphi1(t), \varphi2(t), \varphi3(t),\ldots\ldots\ldots\ldots \varphi r(t)]^T$

Where r=1, $\Psi(t)$ is called scalar wavelet of simply wavelet while in principle r can be arbitrarily large the Multiwavelets studied to date are primarily for r=2

Multiwavelet two scale equations resembles those for scalar wavelets

$$\Phi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} Hk\Phi(2t-k) \qquad (5)$$

$$\Psi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} Gk\Phi(2t-k) \qquad (6)$$

Note that however {Hk} and {Gk} are the matrix filters i.e. Hk and Gk are rXr matrix for each integer k. The matrix elements in those filters provide greater degree of freedom than the traditional scalar wavelet [28]. This degree of freedom can be used to incorporate useful properties is to the Multiwavelet filters such as orthogonality, Symmetry, and high order approximation [27]. In contrast to the limitations of scalar wavelets, Multiwavelets are able to posses the best of all these properties simultaneously [26]. Multiwavelets offer more design options; and are able to combine several desirable transform features [22]. The previously published results of Multiwavelets based image compression mostly fall on of the performance enjoyed by the current wavelet method. The two new techniques for improving the decomposition iteration and zero tree based quantization for Multiwavelets are presented [28], Multiwavelet properties on image compression results are the best Multiwavelet image compression performance reported to date [21].

5.4 Multiwavelet Packet:

Just as with scalar wavelets, the Multiwavelet filter bank procedure involves iterating the filtering operations on the low pass channel of the filter bank. And just as with scalar wavelets, iterating on the high pass channel as well can produce new basis function, this approach combines the wavelet packet decomposition with

Multiwavelet filter and hence we call it the Multiwavelet Packet in a manner analogous to the wavelet packet in the last section [28][27]

Let U0(t)= Φ(t) and U1(t) =Ψ(t), and define

$$U2n = \sqrt{2}\sum_{k=-\infty}^{\infty} HkUn(2t-k) \qquad (7)$$

$$U2n+1(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} GkUn(2t-k) \qquad (8)$$

The difference between Wavelet packet and Multiwavelet packet is that each branching in the tree structure creates four new channels instead of two; due to the duel channel nature of Multiwavelets filter banks. The computational complexity for Multiwavelet packet may be higher than the wavelet packet [27].

6. Discussion:

Many different methodologies for noise reduction (or de-nosing) giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data given its degraded version have been presented, Various Thresholding techniques based on wavelet domain-filtering techniques such as SUREthresh, Visuthresh and Bayesthresh etc. out of which SUREthresh is one of the best tools for de-noising operation i.e. Wavelet Thresholding is an effective method of de-noising noisy signals and will take an important role in de-noising the image [17][38, 30, 11, 50]. Even though some images have high frequency information that is not preserved well through standard wavelet based compression algorithms. However, the design possibilities for wavelets are limited because they cannot simultaneously posses all the desirable properties. Relatively new field of Wavelet packet and Multiwavelet holds promise in obviating some of the limitations of wavelet. The results confirm the ability of wavelet packets to outperform wavelets in some image compression situations. Now we will consider an alternative approach to improving wavelet based image compression i.e. Multiwavelet and Multiwavelet Packets, which may work well for compression of noisy images. An observations on wavelets and Multiwavelets suggests that different strength and weakness [28]. Multiwavelets give the best performance on the “synthetic” images and “Natural” images with significantly high frequency content, such as Goldhill, Mandrill and Fingerprint images. Natural images with mostly low frequency content (e.g. Lena and Barbara images) are best Compressed with scalar wavelets [28][27][26]. Multiwavelets gives better results for color images Vs Scalar wavelets in terms of perfect reconstruction property (PR) i.e. subjective quantity and Peak Signal to Noise Ratio (PSNR) [42]. An observations [28] suggest that the PSNR (in db) results for wavelet packet and Multiwavelet Packets:, is as follows Multiwavelet Packets performance is mixed, while Multiwavelet Packet typically give the best results for synthetic images, wavelet packet gives the best results for natural images with few expectations [28][27][26]. Multiwavelet packet gave predominantly better results on synthetic images. Relatively Multiwavelet packet performance is poor on natural images because Multiwavelet transform produces a different sub band structure than the wavelet transforms. Wavelet Packet gives the best results for canonical images [27].

7. Conclusion:

Through this paper we have studied different techniques used to compress noisy images. The compression ratio is different for Synthetic and Natural images. The Wavelet and wavelet packets result is also discussed in this article. The performance of Wavelet, Multiwavelets, and Multiwavelet packet depends on image content and varies for natural and synthetic image for high and low frequency contents [1]. The contents remain to be developed for the compression and reconstruction of images with predominantly low frequency and as well as high frequency. Multiwavelet packet gives good results for de-noising 1-D and 2-D signals, combined with the successes shown here for Multiwavelet image compression, it seems likely that Multiwavelet may work well for compression of noisy images [28][26]. An image may contain different noises like impulse noise, pepper & salt noise, Gaussian noise etc., due to acquisition and relevant environmental problems. However, the current data compression methods might be far away from the ultimate limits imposed by the underline structure of specific data sources such as images. Interesting issue like obtaining accurate models of images, optimal representation of such model and rapid computation such as optimal representation are the “grand challenges” facing the data compression community [48].

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Figures: 1 and 2

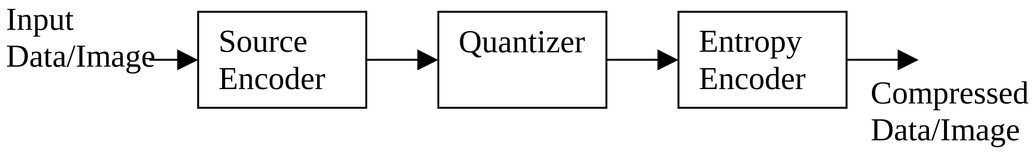

Figure-1

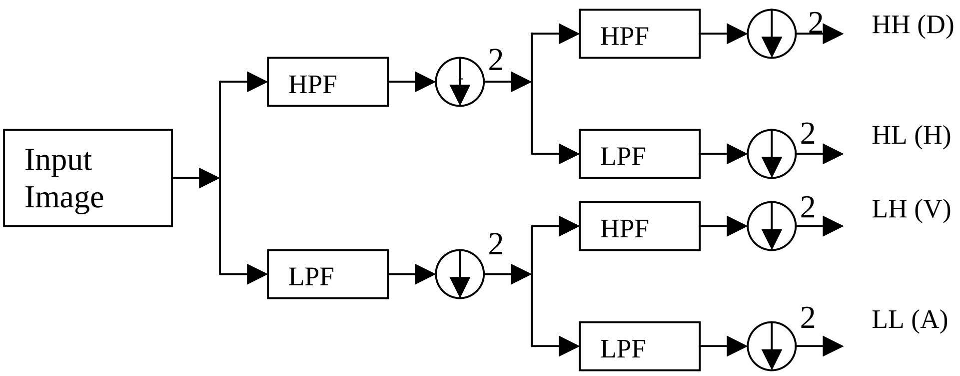

Figure 2 Analysis of 2D DWT shows one stage filter